EE 60563: Random Vectors, Detection and Estimation Fall 2007
Homework 6, Due 14 November, in class

1. A hypothesis testing problem involves random observations of $\mathbf{Y} = S\mathbf{1} + \mathbf{W}$, where $\mathbf{1}$ is a vector of N ones and $\mathbf{W}$ is a vector of N independent, zero-mean Gaussian RVs, each of variance σ^2. S and $\mathbf{W}$ are independent.
(a) The hypotheses are

$$\begin{aligned} H_1 : S &= \beta \\ H_0 : S &= 0, \end{aligned}$$

for a known constant β. For the case $N = 2$, sketch the level curves of the two conditional PDFs and the decision regions for the minimum probability of error detector when $P(H = H_0) = 0.7 = 1 - P(H = H_1)$, $\beta = 1$ and $\sigma^2 = 0.1$.
(b) Suppose the SNR $(10 \log_{10} \left(\frac{\beta^2}{\sigma^2}\right)$) is -30dB. If we want $P_F = 10^{-3}$ and $P_D \geq 0.99$, how many samples must be collected in $\mathbf{Y}$?
(c) Now find the BLS estimate of S given $\mathbf{Y} = \mathbf{y}$, with the *a priori* probabilities in (a).

2. A conditionally, jointly Gaussian vector $\mathbf{X}$ has covariance matrix

$$\Lambda_{\mathbf{X}} = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

under both hypothesis H_0 and H_1. Its mean under H_0 is $m_0 = [2\ 0]^t$ and under H_1 it is $m_1 = [-2\ -2]^t$.
(a) Design the Neyman-Pearson detector for $P_F = 0.05$. You may use Matlab or similar package for any matrix manipulation.
(b) Sketch accurately the decision regions in the 2D sample space of $\mathbf{X}$, with level curves for $p(\mathbf{x}|H_i), i = 0, 1$.

3. An 8-ary hypothesis test features two observations Y_1 and Y_2 which are, conditioned on $\{H = H_i\}$, independent Gaussians with equal variance of 1. The mean vectors for the 8 hypotheses are: (-3,1), (-1,1), (1,1), (3,1), (-3,-1), (-1,-1), (1,-1) and (3,-1). $P(H = H_i) = 0.125$ for each i.
(a) For the MAP detector, find the exact probability of error.
(b) Compare the exact probability of error to the probability computed by a form the union bound which consists of using strictly binary hypothesis testing events as in the classic bound, but which also takes advantage of such binary error events which are subsets of one another, as we discussed in class.

4. Let $Y = S + N$, where S and N are independent RVs with

$$\begin{aligned} p_S(s) &= 2e^{-2s}u(s) \\ p_N(n) &= e^{-n}u(n) \end{aligned}$$

(a) Compute the Bayesian estimates of S from Y for all three cost functions we developed in class.
(b) Find the LLS estimator of $\hat{S}_{LLS}(Y)$.