

## EE 563: Exam 1, Fall 1993

Note: Vectors will appear in bold, and random entities will be capitalized.

1. Personal problems:

(a) (8 pts.) If you wait long enough, it is likely that someone will steal your car. Regardless of when the theft occurs, assume the replacement cost of the car is \$2,000. The time at which the theft occurs has the PDF

$$f_T(t) = 1/10[U(t) - U(t - 10)],$$

with the present moment assumed to be  $t = 0$  and time measured in years. You can buy insurance for replacement of the automobile at any time for \$40 per month, in minimum 1 month increments. How long would you wait before buying the insurance, assuming you want a positive expected gain from the insurance? Note that as you wait, at any time you're deciding, you are conditioning on the fact that the theft hasn't occurred yet at that point.

(b) (10 pts.) Repeat the problem in (a), again with  $t$  in years, but with

$$F_T(t) = 1 - e^{-0.2t}U(t).$$

2. Let a sample space ( $\Omega$ ) related to your gambling adventure be {win,lose,draw}.

(a) (7 pts.) Construct a complete probability space using this  $\Omega$ . Define a random variable ( $X$ ) on your probability space, with only the following restriction:  $X$  must take on at least two nonzero values, each occurring with nonzero probability.

(b) (7 pts.) Plot the distribution function and density function of  $X$ , and calculate its variance.

3. (16 pts.) Suppose that

$$f_{XY}(x, y) = \begin{cases} 1/\pi & x^2 + y^2 \leq 1, \\ 0 & \text{elsewhere.} \end{cases}$$

The RV  $V$  is defined as

$$V = X^2 + Y^2$$

Find  $f_V(v)$ .

4. The mean vector and covariance matrix of the RV  $\mathbf{X} = [X_1, X_2, X_3]^T$  are

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

(a) (6 pts.)  $\mathbf{Y} = G\mathbf{X}$ , with  $G = [1 \ 2 \ -1]$ . Find  $E[Y]$ ,  $\sigma_Y^2$ , and  $E[Y^2]$ .

(b) (5 pts.) Now suppose that  $\mathbf{X}$  is jointly Gaussian. Find  $f_{X_2}(x)$ .

(c) (5 pts.) Compute  $E[(X_1 + X_2)^2]$ .

(d) (5 pts.) Find the correlation matrix of  $\mathbf{X}$ .

5. A student awakes at a random time ( $T_r$ ) in the morning, with  $T_r$  having the CDF

$$F_{T_r}(t) = \begin{cases} 0 & t \leq 8 \\ \frac{(t-8)}{5}, & 8 < t < 12 \\ 1, & t \geq 12 \end{cases}$$

Given that she awakes at  $T_r = t$ , the time of her appearance at school ( $T_a$ ) is uniformly distributed on the interval  $[t, 13]$ .

(a) (8 pts.) What is the expected time of her arrival ( $E[T_a]$ )?

(b) (10 pts.) Our heroine shows up at 12:30. Conditioned on this fact, what is the probability that she arose before 12?

6. (13 pts.) The RVs  $X$  and  $Y$  are independent, Poisson-distributed RVs, with

$$P(X = k) = \frac{e^{-\alpha} \alpha^k}{k!}, \quad k \geq 0$$

$$P(Y = k) = \frac{e^{-\beta} \beta^k}{k!}, \quad k \geq 0$$

for two constants  $\alpha$  and  $\beta$ . Find the distribution of  $Z = X + Y$ .

Joint Gaussian RVs:

$$f_{\mathbf{x}}(\mathbf{x}) = \frac{1}{(2\pi)^{N/2} |\mathbf{K}_{\mathbf{x}}|^{1/2}} \exp \left\{ -1/2 (\mathbf{x} - \mu_{\mathbf{x}})^T \mathbf{K}_{\mathbf{x}}^{-1} (\mathbf{x} - \mu_{\mathbf{x}}) \right\}$$

$$\phi_{\mathbf{x}}(\omega) = \exp \left\{ j\omega^T \mu_{\mathbf{x}} - 1/2 \omega^T \mathbf{K}_{\mathbf{x}} \omega \right\}$$

Characteristic function:  $\phi_{\mathbf{x}}(\omega) = E[e^{j\omega^T \mathbf{x}}]$