

Comparing Logit and Probit Coefficients between Models and Across Groups

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I. Comparing coefficients across models

$$V(y^*) = V(\alpha + x\beta) + V(\varepsilon_{y^*}) = V(\alpha + x\beta) + \pi^2 / 3 = V(\alpha + x\beta) + 3.29$$

```
. quietly logit ybinary x1
. listcoef, std
```

logit (N=500): Unstandardized and Standardized Estimates

Observed SD: .50035659
Latent SD: 2.3395663

Odds of: 1 vs 0

ybinary	b	z	P> z	bStdX	bStdY	bStdXY	SDofX
x1	0.73887	10.127	0.000	1.4777	0.3158	0.6316	2.0000

```
. quietly logit ybinary x2
. listcoef, std
```

logit (N=500): Unstandardized and Standardized Estimates

Observed SD: .50035659
Latent SD: 2.3321875

Odds of: 1 vs 0

ybinary	b	z	P> z	bStdX	bStdY	bStdXY	SDofX
x2	0.48868	10.134	0.000	1.4660	0.2095	0.6286	3.0000

```
. quietly logit ybinary x1 x2
. listcoef, std
```

logit (N=500): Unstandardized and Standardized Estimates

Observed SD: .50035659
Latent SD: 5.3368197

Odds of: 1 vs 0

ybinary	b	z	P> z	bStdX	bStdY	bStdXY	SDofX
x1	1.78923	9.815	0.000	3.5785	0.3353	0.6705	2.0000
x2	1.17314	9.714	0.000	3.5194	0.2198	0.6595	3.0000

II. Comparing coefficients across groups

Case 1: True coefficients are equal, residual variances differ

	Group 0	Group 1
True coefficients	$y_i^* = x_{i1} + x_{i2} + x_{i3} + \varepsilon_i$	$y_i^* = x_{i1} + x_{i2} + x_{i3} + 2\varepsilon_i$
Standardized Coefficients	$y_i^* = x_{i1} + x_{i2} + x_{i3} + \varepsilon_i$	$y_i^* = .5x_{i1} + .5x_{i2} + .5x_{i3} + \varepsilon_i$

Case 2: True coefficients differ, residual variances differ

	Group 0	Group 1
True coefficients	$y_i^* = x_{i1} + x_{i2} + x_{i3} + \varepsilon_i$	$y_i^* = 2x_{i1} + 2x_{i2} + 2x_{i3} + 2\varepsilon_i$
Standardized Coefficients	$y_i^* = x_{i1} + x_{i2} + x_{i3} + \varepsilon_i$	$y_i^* = x_{i1} + x_{i2} + x_{i3} + \varepsilon_i$

Case 3: True coefficients differ, residual variances differ even more

	Group 0	Group 1
True coefficients	$y_i^* = x_{i1} + x_{i2} + x_{i3} + \varepsilon_i$	$y_i^* = 2x_{i1} + 2x_{i2} + 2x_{i3} + 3\varepsilon_i$
Standardized Coefficients	$y_i^* = x_{i1} + x_{i2} + x_{i3} + \varepsilon_i$	$y_i^* = \frac{2}{3}x_{i1} + \frac{2}{3}x_{i2} + \frac{2}{3}x_{i3} + \varepsilon_i$

Allison's example: Apparent differences in effects across groups may be an artifact of differences in residual variability

Table 1: Results of Logit Regressions Predicting Promotion to Associate Professor for Male and Female Biochemists (Adapted from Allison 1999, p. 188)

Variable	Men		Women		Ratio of Coefficients	Chi-Square for Difference
	Coefficient	SE	Coefficient	SE		
Intercept	-7.6802***	.6814	-5.8420***	.8659	.76	2.78
Duration	1.9089***	.2141	1.4078***	.2573	.74	2.24
Duration squared	-0.1432***	.0186	-0.0956***	.0219	.67	2.74
Undergraduate selectivity	0.2158***	.0614	0.0551	.0717	.25	2.90
Number of articles	0.0737***	.0116	0.0340**	.0126	.46	5.37*
Job prestige	-0.4312***	.1088	-0.3708*	.1560	.86	0.10
Log likelihood	-526.54		-306.19			
Error variance	3.29		3.29			

* $p < .05$, ** $p < .01$, *** $p < .001$

Allison's solution: Add delta to adjust for differences in residual variability

Table 2: Logit Regressions Predicting Promotion to Associate Professor for Male and Female Biochemists, Disturbance Variances Unconstrained (Adapted from Allison 1999, p. 195)

Variable	All Coefficients Equal		Articles	
	Coefficient	SE	Coefficient Unconstrained	SE
Intercept	-7.4913***	.6845	-7.3655***	.6818
Female	-0.93918**	.3624	-0.37819	.4833
Duration	1.9097***	.2147	1.8384***	.2143
Duration squared	-0.13970***	.0173	-0.13429***	.01749
Undergraduate selectivity	0.18195**	.0615	0.16997***	.04959
Number of articles	0.06354***	.0117	0.07199***	.01079
Job prestige	-0.4460***	.1098	-0.42046***	.09007
δ	-0.26084*	.1116	-0.16262	.1505
Articles x Female			-0.03064	.0173
Log likelihood	-836.28		-835.13	

* $p < .05$, ** $p < .01$, *** $p < .001$

Alternative (and broader) solution: Heterogeneous Choice Models

With heterogeneous choice (aka Location-Scale) models, the dependent variable can be ordinal or binary. For a binary dependent variable, the model (Keele & Park, 2006) can be written as

$$\Pr(y_i = 1) = g\left(\frac{x_i\beta}{\exp(z_i\gamma)}\right) = g\left(\frac{x_i\beta}{\exp(\ln(\sigma_i))}\right) = g\left(\frac{x_i\beta}{\sigma_i}\right)$$

In the above formula,

- g stands for the link function (in this case logit; probit is also commonly used, and other options are possible, such as the complementary log-log, log-log and cauchit).
- x is a vector of values for the i th observation. The x 's are the explanatory variables and are said to be the determinants of the choice, or outcome.
- z is a vector of values for the i th observation. The z 's define groups with different error variances in the underlying latent variable. The z 's and x 's need not include any of the same variables, although they can.
- β and γ are vectors of coefficients. They show how the x 's affect the choice and the z 's affect the variance (or more specifically, the log of σ).
- The numerator in the above formula is referred to as the choice equation, while the denominator is the variance equation. These are also referred to as the location and scale equations. Also, the choice equation includes a constant term but the variance equation does not.
- The conventional logit and probit models, which do not have variance equations, are special cases of the above, where $\sigma_i = 1$ for all cases.
- Allison's model is a special case of a heterogeneous choice model, where the dependent variable is a dichotomy and both the variance and choice equations include the same dichotomous grouping variable.

In Stata, heterogeneous choice models can be estimated via the user-written routine `oglm`.

```
. * oglm replication of Allison's Table 2, Model 2 with interaction added:
. use "http://www.indiana.edu/~jslsoc/stata/spex_data/tenure01.dta", clear
(Gender differences in receipt of tenure (Scott Long 06Jul2006))
. keep if pdatasample
(148 observations deleted)
. oglm tenure female year yearsq select articles prestige f_articles, het(female)
```

```
Heteroskedastic Ordered Logistic Regression      Number of obs   =      2797
                                                LR chi2(8)      =      415.39
                                                Prob > chi2     =      0.0000
Log likelihood = -835.13347                    Pseudo R2       =      0.1992
```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	

tenure						
female	-.3780597	.4500207	-0.84	0.401	-1.260084	.5039646
year	1.838257	.2029491	9.06	0.000	1.440484	2.23603
yearsq	-.1342828	.017024	-7.89	0.000	-.1676492	-.1009165
select	.1699659	.0516643	3.29	0.001	.0687057	.2712261
articles	.0719821	.0114106	6.31	0.000	.0496178	.0943464
prestige	-.4204742	.0961206	-4.37	0.000	-.6088671	-.2320813
f_articles	-.0304836	.0187427	-1.63	0.104	-.0672185	.0062514

lnsigma						
female	.1774193	.1627087	1.09	0.276	-.141484	.4963226

/cut1	7.365285	.6547121	11.25	0.000	6.082073	8.648497

```
. display "Allison's delta = " (1 - exp(.1774193)) / exp(.1774193)
-.16257142
```

```
. * Hauser & Andrew's original LRPC program
. * Code has been made more efficient and readable,
. * but results are the same. Note that it
. * actually estimates and reports
. * lambda - 1 rather than lamba.
. program define lrpc02
1.   tempvar theta
2.   version 8
3.   args lnf intercepts lambdaminus1 betas
4.   gen double `theta' = `intercepts' + `betas' + (`lambdaminus1' * `betas')
5.   quietly replace `lnf' = ln(exp(`theta')/(1+exp(`theta'))) if $ML_y1==1
6.   quietly replace `lnf' = ln(1/(1+exp(`theta'))) if $ML_y1==0
7. end
. * Hauser & Andrews original LRPC parameterization used with Allison's data
. * Results are identical to Allison's Table 2, Model 1
. ml model lf lrpc02 ///
>   (intercepts: tenure = male female, nocons) ///
>   (lambdaminus1: female, nocons) ///
>   (betas: year yearsq select articles prestige, nocons), max nolog
. ml display
```

```
Number of obs   =      2797
Wald chi2(2)    =      180.60
Prob > chi2     =      0.0000
Log likelihood = -836.28235
```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	

intercepts						
male	-7.490506	.6596634	-11.36	0.000	-8.783422	-6.197589
female	-6.230958	.6205863	-10.04	0.000	-7.447285	-5.014631

lambdaminus1						
female	-.2608325	.1080502	-2.41	0.016	-.4726069	-.0490581

betas						
year	1.909544	.1996937	9.56	0.000	1.518151	2.300936
yearsq	-.1396868	.0169425	-8.24	0.000	-.1728935	-.1064801
select	.1819201	.0526572	3.45	0.001	.0787139	.2851264
articles	.0635345	.010219	6.22	0.000	.0435055	.0835635
prestige	-.4462074	.096904	-4.60	0.000	-.6361357	-.2562791