

Sociology 592 - Research Statistics I
Exam 1
September 25, 1991

Where appropriate, show your work - partial credit may be given. (On the other hand, don't waste a lot of time on excess verbiage.) Do not spend too much time on any one problem. It is legitimate (and probably essential) to refer to results that have previously been proven in class or homework, without re-proving them - for example, you wouldn't need to prove that $P(-1.96 \leq Z \leq 1.96) = .95$, since we have already shown that in class. Likewise, you are free to refer to anything that was demonstrated in the homework or handouts.

1. (5 points each, 20 points total). Indicate whether the following statements are true or false. If you think the statement is false, indicate how the statement could be corrected. For false statements, do not just say that you could substitute "not equals" for equals. For example, the statement $P(Z \leq 0) = .7$ is false. To make it correct, don't just say $P(Z \leq 0) <> .7$, instead say $P(Z \leq 0) = .5$ or $P(Z \leq .525) = .7$.

A. The population is 40% Protestant, 30% Catholic, and 30% Other. Twenty-five percent (25%) of the Protestants and 20% of the Catholics smoke, while 10% of the population consists of Others who Smoke. This means that

$$P(\text{Smoke}) = .40 * .25 + .30 * .20 + .30 * .10 = .19$$

B. A fair coin is tossed 12 times. The probability of getting 5 to 7 heads is .6124.

C. A defense contractor claims that only 20% of its weapons are defective. Military critics believe that the reliability of the weapons is overstated. A random sample of 100 weapons finds 11 that are defective. If $\alpha = .05$, the null hypothesis should be rejected.

D. $P(Z \leq -.90) = .8159$

2. (10 points each, 30 points total) Answer three of the following. The answers to most of these are fairly straightforward, so do not spend a great deal of time on any one problem. NOTE: I will give up to 5 points extra credit for each additional problem you do correctly.

a. It is election day 1992. After once trailing by 40 points in the polls, Democratic Presidential nominee Robert Kerrey of Nebraska and his running mate Jerry Brown of California have turned this into one of the closest elections in history. The electorate has polarized into 6 distinct and non-overlapping groups. Here is a description of each, and how they voted: [HINT: while this question is amusingly written and offers subtle political and social satire, all the stuff you really need to know is in boldface]

Hollywood star gazers (10% of the population): This group votes for whichever candidate will bring the most Hollywood glamour and dazzle to the White House. Traditionally a Republican stronghold, the movie-star looks of Kerrey and Brown have helped keep this group undecided for most of the campaign. For them, the turning point comes 3 days before the election, when it is announced that Kerrey has eloped with actress Debra Winger. **Seventy percent (70%) vote Democratic.**

Willy Horton Haters (10% of the population): This group believes that the nation's top priority, indeed its only priority, is making sure that convict Willy Horton never gets out of jail. As in 1988, **100% vote Republican.**

Football Fanatics (12% of the population): These voters believe that a President is only as good as the College football team behind him. Although Republican George Bush claims Texas and 27 other states as his home, not one of them has a team that can compare with Kerrey's Nebraska Cornhuskers, who in 1992 are storming towards their first national championship in over 20 years. **Seventy-five percent (75%) vote Democratic.**

Impoverished graduate students (4% of the population): This group originally intended to vote for whichever candidate would do the most to raise graduate student stipends. When both candidates promised to eliminate all forms of graduate student support from the federal budget, most

graduate students voted on the basis of their second biggest concern: football. **Seventy-five percent (75%) vote Democratic.**

Serious issue voters (30% of the population): This group carefully examines the issues raised by each candidate and votes for the person they feel offers the best and most thoughtful program for America. Unfortunately, extended discussion of Willy Horton, flag burning, Debra Winger, Linda Ronstadt, Governor Moonbeam, Dan Quayle's high school grades, and other similar matters have once again made it impossible for the candidates to go over issues this year. Hence, **Serious issue-voters split evenly between the Republicans and the Democrats.**

Not-a-dimes-worth-of-difference-between-them voters (34% of the population): As their name implies, this group believes there is not a dime's worth of difference between either Presidential candidate, hence they toss a nickel to determine their vote. **Fifty percent (50%) vote Democratic.**

Thus, with _____% of the vote, the next President of the United States is _____.

b. (For those who like card problems) There is a card in a hat. It is either the ace of spades or the king of spades, with equal probability. You take another identical ace of spades and throw it into the hat. You then choose a card at random from the hat. You see it is an ace. What are the odds the original card in the hat was an ace? [HINT: The answer to this problem is in your course packet somewhere - but you still have to provide a formal proof!]

c. If $\text{Income} \sim N(\$20,000, \$5000^2)$, how low does your income have to be for you to be among the poorest 25% of the population?

d. Prove Expectations rule #11 (course packet, p. 26):
If X and Y are independent, $\text{COV}(X,Y) = 0$.
You can use any of the first 10 rules in your proof.

e. Here are the results from last semester's first exam in statistics. Compute the mean and variance of the scores. As the frequencies show, there were 13 students in the course.

Score (X_i)	Frequency (f_i)
31	1
65	1
86	1
90	1
94	3
95	2
96	1
98	1
100	1
104	1

3. (25 points) According to the October 1991 issue of Working Mother Magazine,

During the recession, corporations laid off tens of thousands of workers and many restructured their businesses. But the policies, programs and benefits designed to support working parents not only escaped the ax, they have literally exploded.

A professor decides to study whether the fallout from this explosion has worked its way into academia yet. She draws a random sample of 500 universities, 100 of them private and 400 public. She classifies each university on two variables: (1) "family-friendly" or "not family-friendly" - based on whether the University offers such benefits as on-site child care, paid parental leave, job-sharing, etc.; and (2) "small endowment" vs. "large endowment" - based on the size of the school's endowment relative to the size of the student body.

She finds that 30% of the private schools and 75% of the public schools have small endowments. For Private schools, 1/3 of those with small endowments and 1/2 of those with large endowments are "family friendly." For Public schools, 5/6 of those with small endowments are not family friendly, while 40% of those with large endowments are.

a. Complete the following table. Remember that, as is already noted in the table, there are 100 private universities and 400 public.

	Private			Public		
Family friendly/Endowment	Small	Large	Σ	Small	Large	Σ
"Family Friendly"						
"Not Family Friendly"						
Σ			100			400

b. What percentage of the "family friendly" universities are private?

c. As these figures show, Public universities tend to be less family-friendly than Private universities. This may reflect the greater concern Private schools have for their employees. However, Public universities also tend to have smaller endowments than do Private schools, which may hamper their ability to afford benefits. Suppose that public universities were as wealthy as private universities, i.e. 70% (instead of the current 25%) had large endowments. Suppose further that it continued to be the case that Public Universities maintained their endowment-specific family-friendly rates, i.e. 1/6 of the schools with small endowments and 40% of the schools with large endowments were "family friendly". What percentage of Public Universities would then be "family-friendly?"

4. (25 points) Notre Dame takes great pride in the health and fitness of its students. According to the University, 1/3 of our undergraduates were captains of a high school sports team. A professor, noting how tired and sleepy students seem to be during his classes, suspects that the University must be exaggerating its claims. To check his suspicions, he draws a random sample of 72 students, and finds that 17 of them were captains of a high school sports team. Test the University's claim at the .05 level of significance. Be sure to indicate:

- The null and alternative hypotheses - and whether a one-tailed or two-tailed test is called for.
- The appropriate test statistic
- The critical region
- The computed value of the test statistic
- Your decision - should the null hypothesis be rejected or not be rejected? Why?

NOTE: You will receive partial credit if you can at least tell me, if the University is correct, what is the probability that a random sample of 72 students would contain 17 or fewer captains?