

Measurement Error Example (Supplemental)

Here we give a hypothetical example that illustrates the properties shown in the measurement handout. We create a data set where the true measures (Y_t and X_t) have a correlation of .7 with each other – but the observed measures (Y and X) both have some degree of random measurement error, and the reliability of both is .64. The way I am constructing the data set, using the `corr2data` command, there will be no sampling variability, i.e. we can act as though we have the entire population.

```
. matrix input corr = (1,.7,0,0\0.7 ,1,0,0\0,0,1,0\0,0,0,1)
. matrix input sd = (4,8,3,6)
. matrix input mean = (10,7,0,0)
. corr2data Yt Xt ey ex, corr(corr) sd(sd) mean(mean) n(500)

(obs 500)

. * Create flawed measures with random measurement error
. gen Y = Yt + ey
. gen X = Xt + ex
```

A & B. We see that the flawed, observed measures have the same means as the true measures – but their variances & standard deviations are larger:

```
. sum Yt Y Xt X
```

Variable	Obs	Mean	Std. Dev.	Min	Max
Yt	500	10	4	-2.639851	22.83863
Y	500	10	5	-3.706503	26.55569
Xt	500	7	8	-16.16331	28.80884
X	500	7	10	-23.81675	38.49127

C. Random measurement error in X , however, does NOT affect the covariance:

```
. corr Yt Xt X, cov
```

```
(obs=500)
```

	Yt	Xt	X
Yt	16		
Xt	22.4	64	
X	22.4	64	100

D. Nonetheless, the correlation is attenuated by random measurement error:

```
. corr Yt Xt X
```

```
(obs=500)
```

	Yt	Xt	X
Yt	1.0000		
Xt	0.7000	1.0000	
X	0.5600	0.8000	1.0000

Note that the correlation between X and Xt is .8 – and that the correlation between X and Yt (.56) is only .8 times as large as the correlation between Xt and Yt (.7). Also, the .8 correlation between X and Xt means that the reliability of X is .64.

E. In a bivariate regression, random measurement error in X causes the slope coefficient to be attenuated, i.e. smaller in magnitude. First we run the regression between the true measures, and then we run the regression of Yt with the flawed measure X:

```
. reg Yt Xt
```

Source	SS	df	MS			
Model	3912.16007	1	3912.16007	Number of obs =	500	
Residual	4071.84001	498	8.17638555	F(1, 498) =	478.47	
Total	7984.00008	499	16.0000002	Prob > F =	0.0000	
				R-squared =	0.4900	
				Adj R-squared =	0.4890	
				Root MSE =	2.8594	

Yt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
Xt	.35	.0160008	21.87	0.000	.3185627	.3814373
_cons	7.55	.169994	44.41	0.000	7.216006	7.883994

```
. reg Yt X
```

Source	SS	df	MS			
Model	2503.78247	1	2503.78247	Number of obs =	500	
Residual	5480.21761	498	11.004453	F(1, 498) =	227.52	
Total	7984.00008	499	16.0000002	Prob > F =	0.0000	
				R-squared =	0.3136	
				Adj R-squared =	0.3122	
				Root MSE =	3.3173	

Yt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
X	.224	.0148503	15.08	0.000	.1948231	.2531769
_cons	8.432	.1811488	46.55	0.000	8.07609	8.78791

Note that X has a reliability of .64 – and the slope coefficient using the flawed X (.224) is only .64 times as large as the slope coefficient using the perfectly measured Xt (.35).

F. Random measurement error in Y, however, does not cause the slope coefficient to be biased – but it does cause the standard error for the slope coefficient to be larger and the t value smaller. Again we run the true regression followed by the regression of Y with Xt.

. reg Yt Xt

Source	SS	df	MS	Number of obs =	500
Model	3912.16007	1	3912.16007	F(1, 498) =	478.47
Residual	4071.84001	498	8.17638555	Prob > F =	0.0000
				R-squared =	0.4900
				Adj R-squared =	0.4890
Total	7984.00008	499	16.0000002	Root MSE =	2.8594

Yt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
Xt	.35	.0160008	21.87	0.000	.3185627 .3814373
_cons	7.55	.169994	44.41	0.000	7.216006 7.883994

. reg Y Xt

Source	SS	df	MS	Number of obs =	500
Model	3912.16001	1	3912.16001	F(1, 498) =	227.52
Residual	8562.84011	498	17.194458	Prob > F =	0.0000
				R-squared =	0.3136
				Adj R-squared =	0.3122
Total	12475.0001	499	25.0000002	Root MSE =	4.1466

Y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
Xt	.35	.0232035	15.08	0.000	.3044111 .3955889
_cons	7.55	.2465171	30.63	0.000	7.065658 8.034342