The Monty Hall Debate

The New York Times, July 21, 1991, Sunday, Late Edition - Final

SECTION: Section 1; Part 1; Page 1; Column 5; National Desk

LENGTH: 1859 words

HEADLINE: Behind Monty Hall's Doors: Puzzle, Debate and Answer?

BYLINE: By JOHN TIERNEY, Special to The New York Times

DATELINE: BEVERLY HILLS, Calif., July 20

BODY:

Perhaps it was only an illusion, but for a moment here it seemed that an end might be in sight to the debate raging among mathematicians, readers of Parade magazine and fans of the television game show "Let's Make a Deal."

They began arguing last September after Marilyn vos Savant published a puzzle in Parade. As readers of her "Ask Marilyn" column are reminded each week, Ms. vos Savant is listed in the Guinness Book of World Records Hall of Fame for "Highest I.Q.," but that credential did not impress the public when she answered this question from a reader:

"Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the other doors, opens another door, say No. 3, which has a goat. He then says to you, 'Do you want to pick door No. 2?' Is it to your advantage to take the switch?"

Since she gave her answer, Ms. vos Savant estimates she has received 10,000 letters, the great majority disagreeing with her. The most vehement criticism has come from mathematicians and scientists, who have alternated between gloating at her ("You are the goat!") and lamenting the nation's innumeracy.

Her answer -- that the contestant should switch doors -- has been debated in the halls of the Central Intelligence Agency and the barracks of fighter pilots in the Persian Gulf. It has been analyzed by mathematicians at the Massachusetts Institute of Technology and computer programmers at Los Alamos National Laboratory in New Mexico. It has been tested in classes from second grade to graduate level at more than 1,000 schools across the country.

But it was not until Thursday afternoon that a truly realistic simulation of the problem was conducted. The experiment took place at the Beverly Hills home of Monty Hall, the host of 4,500 programs of "Let's Make A Deal" from 1963 to 1990. In his dining room Mr. Hall put three miniature cardboard doors on a table and represented the car with an ignition key. The goats were played by a package of raisins and a roll of Life Savers.

After Mr. Hall allowed this contestant 30 independent attempts to win the car, two conclusions seemed clear: Ms. vos Savant's vitriolic critics, including the mathematics professors, are dead wrong. But Ms. vos Savant is not entirely correct either, because there is a small flaw in her wording of the problem that was detected not only by Mr. Hall but also by some of the experts. Despite her impressive analysis and 228-point I.Q., she was not as quick as Mr. Hall in understanding the psychological dimensions of the problem.

'So Easy' to Blunder

A few mathematicians were familiar with the puzzle long before Ms. vos Savant's column. They called it the Monty Hall Problem -- the title of an analysis in the journal American Statistician in 1976 -- or sometimes Monty's Dilemma or the Monty Hall Paradox.

An earlier version, the Three Prisoner Problem, was analyzed in 1959 by Martin Gardner in the journal Scientific American. He called it "a wonderfully confusing little problem" and presciently noted that "in no other branch of mathematics is it so easy for experts to blunder as in probability theory."

The experts responded in force to Ms. vos Savant's column. Of the critical letters she received, close to 1,000 carried signatures with Ph.D.'s, and many were on letterheads of mathematics and science departments.

"Our math department had a good, self-righteous laugh at your expense," wrote Mary Jane Still, a professor at Palm Beach Junior College. Robert Sachs, a professor of mathematics at George Mason University in Fairfax, Va., expressed the prevailing view that there was no reason to switch doors.

"You blew it!" he wrote. "Let me explain: If one door is shown to be a loser, that information changes the probability of either remaining choice -- neither of which has any reason to be more likely -- to 1/2. As a professional mathematician, I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error and, in the future, being more careful."

The criticism has continued despite several more columns by Ms. vos Savant defending her choice. "You are utterly incorrect," insisted E. Ray Bobo, a professor of mathematics at Georgetown University.

"How many irate mathematicians are needed to get you to change your mind?"

'The Henry James Treatment'

Mr. Hall said he was not surprised at the experts' insistence that the probability was 1 out of 2. "That's the same assumption contestants would make on the show after I showed them there was nothing behind one door," he said.

"They'd think the odds on their door had now gone up to 1 in 2, so they hated to give up the door no matter how much money I offered. By opening that door we were applying pressure. We called it the Henry James treatment. It was 'The Turn of the Screw.' "

Mr. Hall said he realized the contestants were wrong, because the odds on Door 1 were still only 1 in 3 even after he opened another door. Since the only other place the car could be was behind Door 2, the odds on that door must now be 2 in 3.

Sitting at the dining room table, Mr. Hall quickly conducted 10 rounds of the game as this contestant tried the non-switching strategy. The result was four cars and six goats. Then for the next 10 rounds the contestant tried switching doors, and there was a dramatic improvement: eight cars and two goats. A pattern was emerging.

"So her answer's right: you should switch," Mr. Hall said, reaching the same conclusion as the tens of thousands of students who conducted similar experiments at Ms. vos Savant's suggestion. That conclusion was also reached eventually by many of her critics in academia, although most did not bother to write letters of retraction. Dr. Sachs, whose letter was published in her column, was one of the few with the grace to concede his mistake.

"I wrote her another letter," Dr. Sachs said last week, "telling her that after removing my foot from my mouth I'm now eating humble pie. I vowed as penance to answer all the people who wrote to castigate me. It's been an intense professional embarrassment."

Manipulating a Choice

Actually, many of Dr. Sachs's professional colleagues are sympathetic. Persi Diaconis, a former professional magician who is now a Harvard University professor specializing in probability and statistics, said there was no disgrace in getting this one wrong.

"I can't remember what my first reaction to it was," he said, "because I've known about it for so many years. I'm one of the many people who have written papers about it. But I do know that my first reaction has been wrong time after time on similar problems. Our brains are just not wired to do probability problems very well, so I'm not surprised there were mistakes."

After the 20 trials at the dining room table, the problem also captured Mr. Hall's imagination. He picked up a copy of Ms. vos Savant's original column, read it carefully, saw a loophole and then suggested more trials.

On the first, the contestant picked Door 1.

"That's too bad," Mr. Hall said, opening Door 1. "You've won a goat." "But you didn't open another door yet or give me a chance to switch." "Where does it say I have to let you switch every time? I'm the master of the show. Here, try it again."

On the second trial, the contestant again picked Door 1. Mr. Hall opened Door 3, revealing a goat. The contestant was about to switch to Door 2 when Mr. Hall pulled out a roll of bills.

"You're sure you want Door No. 2?" he asked. "Before I show you what's behind that door, I will give you $3,000 in cash not to switch to it."

"I'll switch to it."

"Three thousand dollars," Mr. Hall repeated, shifting into his famous cadence. "Cash. Cash money. It could be a car, but it could be a goat. Four thousand."

"I'll try the door."

"Forty-five hundred. Forty-seven. Forty-eight. My last offer: Five thousand dollars."

"Let's open the door."

"You just ended up with a goat," he said, opening the door.

The Problem With the Problem

Mr. Hall continued: "Now do you see what happened there? The higher I got, the more you thought the car was behind Door 2. I wanted to con you into switching there, because I knew the car was behind 1. That's the kind of thing I can do when I'm in control of the game. You may think you have probability going for you when you follow the answer in her column, but there's the pyschological factor to consider."

He proceeded to prove his case by winning the next eight rounds. Whenever the contestant began with the wrong door, Mr. Hall promptly opened it and awarded the goat; whenever the contestant started out with the right door, Mr. Hall allowed him to switch doors and get another goat. The only way to win a car would have been to disregard Ms. vos Savant's advice and stick with the original door.

Was Mr. Hall cheating? Not according to the rules of the show, because he did have the option of not offering the switch, and he usually did not offer it.

And although Mr. Hall might have been violating the spirit of Ms. vos Savant's problem, he was not violating its letter. Dr. Diaconis and Mr. Gardner both noticed the same loophole when they compared Ms. vos Savant's wording of the problem with the versions they had analyzed in their articles.

"The problem is not well-formed," Mr. Gardner said, "unless it makes clear that the host must always open an empty door and offer the switch. Otherwise, if the host is malevolent, he may open another door only when it's to his advantage to let the player switch, and the probability of being right by switching could be as low as zero." Mr. Gardner said the ambiguity could be eliminated if the host promised ahead of time to open another door and then offer a switch.

Ms. vos Savant acknowledged that the ambiguity did exist in her original statement. She said it was a minor assumption that should have been made obvious by her subsequent analyses, and that did not excuse her professorial critics. "I wouldn't have minded if they had raised that objection," she said Friday, "because it would mean they really understood the problem. But they never got beyond their first mistaken impression. That's what dismayed me."

Still, because of the ambiguity in the wording, it is impossible to solve the problem as stated through mathematical reasoning. "The strict argument," Dr. Diaconis said, "would be that the question cannot be answered without knowing the motivation of the host."

Which means, of course, that the only person who can answer this version of the Monty Hall Problem is Monty Hall himself. Here is what should be the last word on the subject:

"If the host is required to open a door all the time and offer you a switch, then you should take the switch," he said. "But if he has the choice whether to allow a switch or not, beware. Caveat emptor. It all depends on his mood.

"My only advice is, if you can get me to offer you $5,000 not to open the door, take the money and go home."

GRAPHIC: Photo: "One mathemetician spent an hour explaining to me why it paid to switch," said Monty Hall. (George Brich for The New York Times) (pg. 20)

Chart: "To Switch or Not?"

Monty Hall, a thoroughly honest game-show host, has randomly placed a car behind one of three closed doors. There is a goat behind each of the other two doors.

"First you point to a door," he explains. "Then I'll open one of the other doors to reveal a goat. After I've shown you the goat, you make your final choice, and you win whatever is behind that door."

You want the car very badly. You point to Door Number 1. Mr. Hall opens another door to show you a goat. Now there are two closed doors remaining, and you have to make your choice: Do you stick with Door 1? Or do you switch to the other door? Or doesn't it matter?

To see which strategy works best you can try playing the game over and over to see which wins most often. (You might try turning over playing cards instead of opening doors, and using the ace of spades as the prize.) You should get the same results as Monty Hall did when he conducted the experiment himself last week.

The results contradict most people's intution that, when there are only two unopened doors left, the odds on each one must be 1/2. But the fact that Mr. Hall opens another door doesn't affect the odds on Door 1: You had a one-third chance of being right to begin with, and you still have a one-third chance after he opens, say, Door 3. You knew he was going to open another door and reveal a goat regardless of what was behind Door 1, so his action provides no new information about Door 1. Therefore, since the odds on Door 1 are still one-third, and the only other place the car could be is behind Door 2, the odds on Door 2 must now be two-thirds.

Drawing illustrating choices of 'door' game. (pg. 20)

Drawing (pg. 1)