Professor Jensen   Spring 2011

DUE: Wednesday, February 9

1. A country has an exhaustible natural resource whose stock at time t is x(t), where time is continuous and the initial stock is $x(0) = x_0 > 0$. Let q(t) denote the quantity extracted at time t, so that $x'(t) = - q(t)$. Also assume the inverse demand for this resource is given by $P(q) = A - q$, where $A > 0$ is a constant, and the cost of extraction is $C(q,x) = q^2 - cx$, where $c > 0$ is a positive constant. Finally, assume the market interest rate is $r > 0$, and that in each section below the objective is to extract the resource by time T, or $x(T) = 0$, where T is a choice variable.

a. What are the time paths of extraction and the resource stock when extraction occurs under conditions of perfect competition?

b. What are the time paths of extraction and the resource stock when the resource is owned by a monopolist whose objective is to maximize the present discounted value of its stream of profits?

c. What are the socially optimal time paths of extraction and the resource stock?

2. A community borders a lake which is teeming with fish. Let N(t) be the population of fish at time t. Without harvesting, the fish population growth rate is $N'(t) = gN(t)\{1- [N(t)/K]\}$ where g and K are positive constants (g is the growth rate and K is the carrying capacity). Fish can be harvested at the rate c(t), the catch at t, giving utility U(c(t)) to the community and reducing the growth rate of the fish population accordingly, where $U'(c) > 0 > U''(c)$. The community's problem is to choose a fishing plan to maximize the present value of the discounted stream of utilities, where time is continuous and $r > 0$ is the interest rate:

$$\text{maximize } \int_0^\infty e^{-rt} U(c)dt \quad \text{subject to } N' = gN(t)\{1- [N(t)/K]\} - c, N(0) = N_0 > 0.$$
$$c$$

Provide a steady state analysis of the optimal fishing (consumption) plan.