

HOMEWORK 4

Due Friday June 26

- (1) S_n is generated by the $n - 1$ transpositions $(12), (13), (14), \dots, (1n)$.
- (2) S_n is generated by the $n - 1$ transpositions $(12), (23), (34), \dots, (n - 1, n)$.
- (3) S_n is generated by (12) and $(123 \cdots n)$.
- (4) A_n is the only subgroup of S_n of index 2.
- (5) Show that $N = \{(1), (12)(34), (13)(24), (14)(23)\}$ is a normal subgroup of S_4 contained in A_4 .
- (6) Let H, K be subgroups of G of finite index show that $H \cap K$ has finite index.
- (7) Show by example that the index $[H : H \cap K]$ need not divide $[G : K]$.
- (8) Prove that every group of order p^2 for p prime is either cyclic or the product of two cyclic groups of order p .