HOMEWORK 3

Due Tuesday June 23

(1) Verify the center of D_4 is $\langle r^2 \rangle$, where r is the rotation by $\pi/4$.

(2) Let T be the 12 element group given as the isometries (rotations) of the tetrahedron. Show T has 8 elements of order 3.

(3) Let G be a group such that $G/Z(G)$ is cyclic. Show that G is Abelian. Is the same true if $G/Z(G)$ is only Abelian?

(4) Let $N < G$ of index 2. Let $a \in G$ with $a^5 \in N$. Show that $a \in N$.

(5) Let $G = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in \mathbb{Q},\ ac \neq 0 \right\} \subset GL_2(\mathbb{Q})$

and $N = \left\{ \begin{pmatrix} 1 & b \\ 0 & c \end{pmatrix} : b, c \in \mathbb{Q},\ c \neq 0 \right\} \subset G$. Show that

(a) N is normal in G.

(b) $G/N \cong \mathbb{Q} \setminus \{0\}$.

Due Wednesday June 24

(1) Let G be a group and H a subgroup of index n. Let $N = \bigcap_{a \in G} aHa^{-1}$. Show that N is the largest normal subgroup of G contained in H and that G/N is isomorphic to a subgroup of S_n.

(2) Let G be a finite group and let p be the smallest prime dividing the order of G. Show that every subgroup of index p is normal.

(3) Prove the cyclic group of order 6 is the group defined by generators a, b and relations $a^2 = b^3 = a^{-1}b^{-1}ab = e$.