HOMEWORK 2

Due Thursday June 18

(1) Let H and K be subgroups of a group G, with $K \leq H \leq G$. Then
$$[G : K] = [G : H][H : K].$$
(2) Prove every subgroup of index 2 is normal.
(3) Find a subgroup of index 3 which is not normal.
(4) Let $N \leq S_4$ consist of all permutations σ such that $\sigma(4) = 4$.
    (a) Prove N is a subgroup.
    (b) Is N normal in S_4?
(5) Let $H \leq G$; then the set aHa^{-1} is a subgroup for each $a \in G$ and $H \cong aHa^{-1}$.
(6) Let G be a finite group and H a subgroup of G of order n. If H is the only subgroup of G of order n, then H is normal in G.

Due Friday June 19

(1) Let H and K be subgroups of a group G. Show that the set of products HK is a subgroup of G if and only if $HK = KH$.
(2) Let H and K be normal subgroups of a group G with $H \cap K = \{e\}$. Show that
    (a) $hk = kh$ for every $h \in H,\ k \in K$;
    (b) HK is a subgroup of G with $HK \cong H \times K$.
(3) Prove that the center of the product of two groups is the product of their centers.

Due Monday June 22

(1) Show that $\text{Aut}(\mathbb{Z}/n\mathbb{Z}) = (\mathbb{Z}/n\mathbb{Z})^*$.
(2) What are the possible values of a^2 modulo 8?
(3) Prove $\mathbb{Z}/n\mathbb{Z}$ is an abelian group.
(4) Determine all groups of order 6.