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# Simulation of Power Gain and Dissipation in Field-Coupled Nanomagnets

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**Abstract.** Coupled nanomagnetic dots were proposed as a promising way to realize computing devices. This paper investigates power flow phenomena in these structures. Using micromagnetic simulations we will demonstrate that the power dissipation of those devices is close to the theoretical lowest limit of any computation and that coupled nanomagnets exhibit power gain, i.e. they are active devices. These results suggest that magnetic computing could be a functionally equivalent replacement of transistor-based circuits in signal processing applications, where robust, low-power operation is crucial.

**Keywords:** nanomagnetics, field-coupled computing, active circuits, adiabatic clocking

## 1. Introduction

Field-coupling [1] is a novel computing paradigm, which is radically different from 'traditional' current-based solid-state electronic information processing: instead of electrical currents it uses electric or magnetic excitations to propagate and process information. Magnetic field-coupling, which is one particular implementation of this concept, utilizes the always-present magnetic stray field of nanoscale magnets for interconnection. It uses magnetic solitons [2], propagating domain walls [3] or antiferromagnetically coupled single-domain dots as [4] an alternative of 'wiring' the devices.

Moore's law (the exponential-rate miniaturization of solid-state electronic devices) is expected to continue in the coming decades and end-of the roadmap silicon devices will probably lie in the atomic dimensions. Although it is unlikely that any alternative paradigm will outperform silicon-based devices in general signal-processing tasks, magnetic field-coupling architectures can have significant benefits in some applications.

We will demonstrate that very low (and tunable) power dissipation is one of those advantages. Minimizing power dissipation is an important figure of merit as dissipation became a 'bottleneck' in the downscaling of high-speed MOS circuits.

Magnetic field-coupled devices show power gain of purely magnetic origin. This active behavior is required for the scalability of any computing architecture: if a complex system cannot recover inevitable dissipative losses in the signal path, then it will always fall in its thermodynamic ground state and unable to perform useful computation [5,6].

This paper investigates power flow phenomena (dissipation and power gain) in magnetic field-coupled devices. In Section 2 we introduce magnetic logic gates built from pillar-shaped permalloy nanoparticles, we demonstrate how those arrangements can perform elementary Boolean functions and how adiabatic clocking can control the dynamic behavior of the system. The third section will describe a simple method for extracting power flow parameters from micromagnetic simulations and will apply this procedure to investigate power dissipation in multi-domain magnetic structures. Section 4 will investigate stand-alone and coupled single-domain particles (magnetic logic gates);

showing that their dissipation can be controlled by the choice of the reversing field, and in the case of adiabatic switching the total dissipation can be very low. Section 5 is devoted to the power gain characteristics: by means of numerical simulation we will prove that field-coupled nanomagnets are active devices, capable of directing energy from the external pumping field to a magnetic signal path and compensating for the effects of the dissipation. To qualitatively interpret the numerical results and make a step towards the design of active field-coupled devices, we will draw an analogy between the origin of power gain in field-coupled nanomagnets and the once widely used magnetic amplifier circuits.

## 2. Construction of Nanomagnetic Logic Devices

Small nanomagnetic dots (in the size regime of approximately sub-hundred nanometer) exhibit single-domain behavior: they contain no internal domain walls and the net magnetic moment of the particle is constant (always the volume times the saturation magnetization $M = V \cdot M_S$). If such particle is elongated then it shows shape anisotropy: due to the interaction with its own magnetic field the ground-state magnetization is parallel to the longest axis [7]. The nanoparticle behaves as a bistable system, and a logic value of ‘1’ and ‘0’ can be assigned to the upward and downward pointing states, respectively [4,8]. Two of those nanomagnets placed close to each other prefer antiparallel alignment of their magnetic moments due to their dipolar interaction. An arrangement of even number of pillars (as sketched in Fig. 1(a)) can be viewed as a ‘nanomagnet wire’, which transmits the magnetization state of the leftmost dot to the rightmost dot, while odd number of dots behaves as inverters. Figure 1(b) illustrates the magnetic majority gate: setting the magnetization state of the inputs (dots A, B and C) the ground-state magnetization of the output dot is antiparallel to the majority of the inputs. This device behaves as a logical AND or OR gate between A and B depending on the state of C.

The correct operation of those logic gates requires driving the magnets reliably into their ground state. This can be achieved by the pumping scenario sketched in Fig. 2: applying an external clocking field along the hard axis of the magnets puts them into a logically neutral state (erases their memory): slowly reducing this clocking field to zero the magnetic systems goes to its ground state. Clocking has other benefits as well: it can be used to realize sequential circuits and ensure that signals propagate from the inputs toward the outputs and not the other way [9].

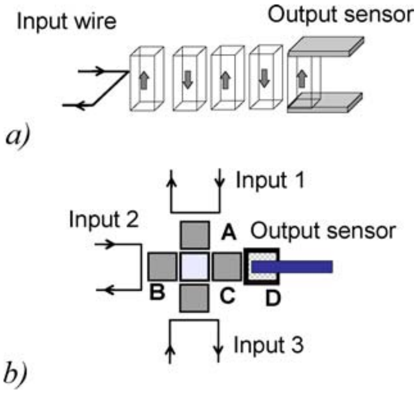

*Figure 1*. Basic nanomagnetic logic devices built from bistable, pillar-shaped dots. Part (a) shows a nanomagnet wire, and part (c) is a nanomagnet majority gate (part (c). The inputs are provided by electric wires, but they could also be the output of a previous gate.

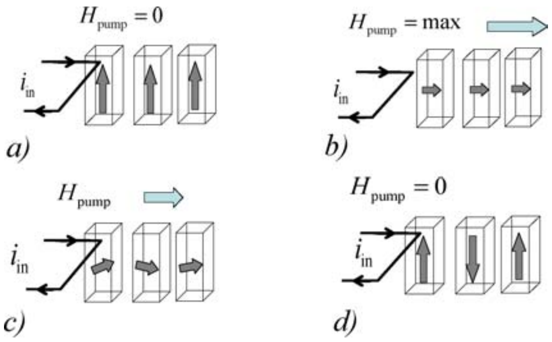

*Figure 2*. These schematics illustrate the clocking of the nanomagnet wire. An initial metastable state (part a) is wiped out by a strong hard axis field (part b) and while this field is slowly released (part c), the magnets flip into their ground state.

The above described scheme uses magnetic dipolar coupling between pillar-shaped dots. It is also possible to use dipolar coupling between planar dots [10], which show in-plane magnetization. Other approaches (such as propagating magnetic solitons [2] or nanoscale domain-wall conductors [3]) are also promising.

## 3. Calculating Energy Flow and Dissipation from Micromagnetic Simulations

The dynamics of a magnetization distribution under the influence of an external field is governed by the

Landau-Lifshitz equations [7]:

$$\frac{\delta \mathbf{M}(\mathbf{r},t)}{\delta t} = -\gamma \mathbf{M}(\mathbf{r},t) \times \mathbf{H}_{\text{eff}}(\mathbf{r},t) - \frac{\alpha\gamma}{M_s}[\mathbf{M}(\mathbf{r},t) \times (\mathbf{M}(\mathbf{r},t) \times \mathbf{H}_{\text{eff}}(\mathbf{r},t))] \tag{1}$$

where $\mathbf{M}(\mathbf{r}, t)$ is the time and position dependent magnetization distribution, $\alpha$ is a phenomenological damping constant and $\gamma$ is the gyromagnetic ratio and saturation magnetization, respectively. The equation describes a damped precession of magnetic moments around the effective field ($\mathbf{H}_{\text{eff}}$). This effective field is a superposition of 'real' magnetic fields and fields classically representing quantum effects (i.e. exchange interaction and crystalline anisotropies). Micromagnetic simulator codes (such as the widely used OOMMF code [11]) numerically solve the (1) partial differential equations for arbitrary initial conditions and external field sequences.

In sufficiently small (below 100 nm) particles the exchange field (which does not depend on the particle volume) overwhelms the demagnetizing field and sets the magnetization homogenous inside the nanomagnet. The magnetization distribution becomes a single vector and (1) simplifies to the single-domain Landau-Lifshitz equation.

The effective magnetic field is also defined through the total energy of the magnetic system:

$$\mathbf{H}_{\text{eff}} = \frac{1}{\mu}\frac{\partial E}{\partial \mathbf{M}}. \tag{2}$$

So one can immediately calculate the time derivative of the energy:

$$P(\boldsymbol{r},t) = \mu \mathbf{H}_{\text{eff}} \frac{\partial \mathbf{M}}{\partial t} \tag{3}$$

When we place a magnet to a time-varying external field, it exchanges energy with these fields and dissipates energy as well. Writing out (3) explicitly, one could separate the terms corresponding to energy transfer with different external field components and dissipation. For example, collecting all the terms containing $\alpha$ (the dissipation parameter), the total power dissipation can be determined. Performing the calculations in the single-domain approximation one can identify the fields arising from the interaction of two selected dots and calculate the energy exchange originating from this particular interaction. We used this procedure to analyze the solutions of the Landau-Lifshitz equations.

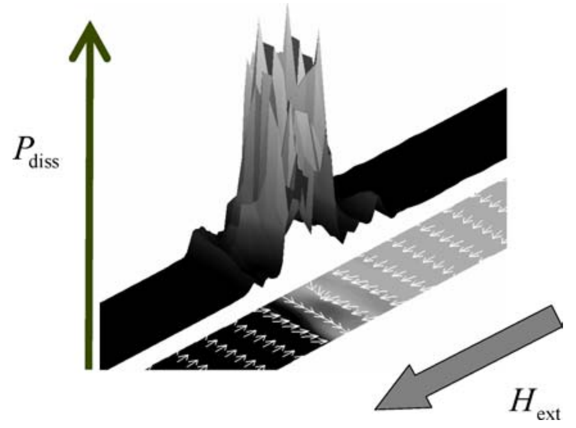

*Figure 3.* In a domain-wall conductor (like this 50 nm wide, 20 nm thick permalloy strip) heat is dissipated right around the propagating wall.

As an application example of this model we first consider a narrow, thin strip of ferromagnetic material (illustrated in Fig. 3), which behaves as a domain wall conductor. In fully magnetized state its magnetization points to the right. A weak external field can nucleate a domain wall at one end of the strip. This wall propagates with an approximately constant speed of 500 m/s. Power is dissipated only inside the wall, where the magnetization distribution rapidly changes—the amount of dissipation is $1.5 \times 10^{-7}$ W. Such domain-wall conductors are possible building blocks of magnetic computing devices [3]. An envisioned magnetic microprocessor, dissipating 150 W of power, could contain about a billion simultaneously propagating walls.

Similarly to domain-wall conductors, macro-scale (multi-domain) magnets dissipate power when their domain structures quickly rearrange in the presence of a changing external field. The dissipated energy can be calculated from the above microscopic model and will be the same as the area of their hysteresis curve [12].

## 4. Power Dissipation in Nanoscale Magnetic Dots

Single-domain nanomagnets contain no internal domain walls, and their switching dynamics can be entirely controlled by the applied external field sequence. One could expect that the energy they dissipate during switching depends on the external pumping scenario as well.

The energy landscape of a stand-alone, pillar shaped single-domain magnet is sketched on Fig. 4(a), by the

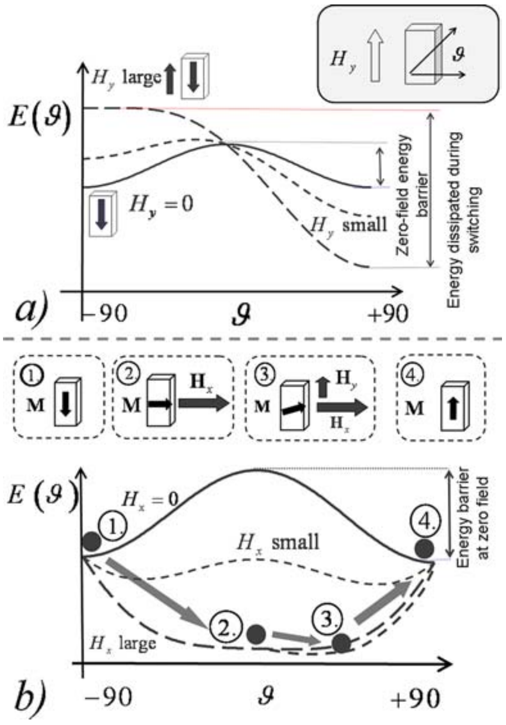

*Figure 4.* Energy landscape during the switching of a single domain nanomagnet by easy-axis field (part a) and adiabatically applied hard and easy axis fields (part b).

solid curve. The ground-state magnetization configurations (at $\vartheta = -90^\circ$ and $\vartheta = +90^\circ$) are separated by an energy barrier, typically a few hundred room-temperature $kT$ high. The most obvious way of switching this magnet is to apply a field which is antiparallel to its present magnetization state. This deforms the energy landscape of the magnet the way it is illustrated by the dashed curves of Fig. 4(a). When the field reaches a sufficiently large value, the barrier disappears and the magnet slides down its other magnetization state which is parallel to the external field. The energy dissipated in this process is approximately equal to the height of the energy barrier and also with the area of easy-axis hysteresis curve. The reversal occurs abruptly and this is a rather energy-wasting scenario.

A different way of switching is illustrated in Fig. 4(b). First a slowly increasing horizontal (hard axis) field is applied, which gradually sets the magnetization direction parallel to the pumping field and 'flattens' the energy landscape. A small vertical field then 'bends' the magnetization upwards and after releasing the external pumping field the magnet is set to the upward-pointing steady state. The path of the system on the energy surface is illustrated on the curves of Fig. 4(b). If the pumping is performed sufficiently slowly, then the dot always stays in its actual ground state and the system never 'falls down' an energy barrier like it did in the previous case. As one can see from the (1) Landau-Lifshitz equation, the farther the magnetization vector from its ground state, the larger the dissipation. If the effective field remains almost parallel to the magnetization direction during the entire switching process then the dissipation is minimized [12].[1]

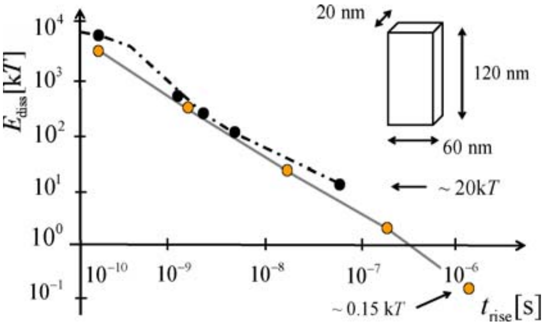

*Figure 5.* Energy dissipated during the switching of a perfect single domain (solid line) and a nearly-perfect single-domain particle (dotted line). The dissipation of the 'ideal' single domain magnet approaches zero. The line is a guide for the eye.

The tradeoff between clocking speed and dissipation is quantized in Fig. 5. For any experimentally reasonable external pumping rate the switching is adiabatic and for ideally single-domain magnets the dissipation is practically zero for pumping frequency in the MHz range. Real nanomagnets are never perfectly single-domain and the bending of magnetization, edge domains, surface roughness result in a minimum dissipation around a few room-temperature $kT$ per switching.

The above outlined method of adiabatic switching is essentially the same as the clocking scenario, which we proposed to reliably put the nanomagnet system into its ground state. Switching the nanomagnets this way not only results in predictable switching dynamics but also minimizes dissipation.

## 5. Power Gain in Coupled Nanomagnets

Consider a dot, which is part of a nanomagnet wire and exchanges energy with its right and left neighbors (besides the external field). We numerically solved the single-domain Landau-Lifshitz equations when the magnets 'flip' toward their ground state—the result (the

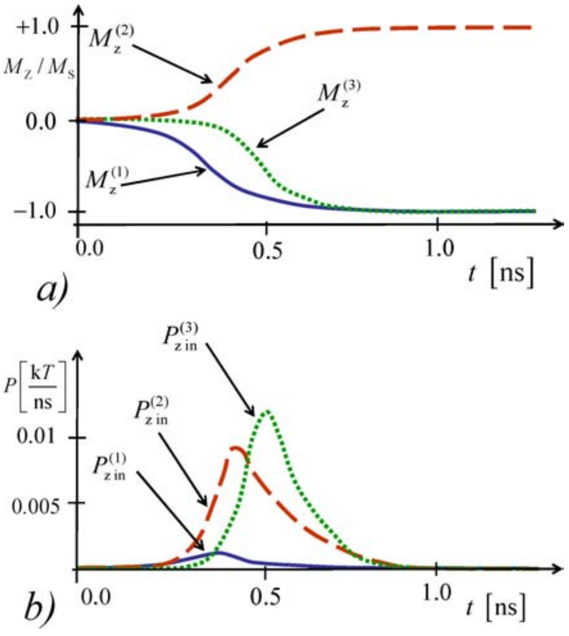

*Figure 6.* Magnetization dynamics and energy flow in a nanomagnet wire. Part (a) shows how the signal-representing magnetization components flip toward a defined state and part (b) is the corresponding energy flow.

$z$ component of magnetization) is plotted in Fig. 6(a). Using (3), we can determine how much power flows in from the first, second and third dot from their left neighbor. The result is plotted in Fig. 6(b). This numerical result indicates that each magnet passes more energy to its right neighbor than it receives from the left one. In other words the magnet behaves as an amplifier: it redirects energy from the horizontal pumping field to the signal path.

Transistors, tunnel diodes and most of today's active devices attain active behavior by converting the energy of a DC (operating point) current to the signal energy. Nanomagnets acquire the energy from a time varying magnetic field, which relates them to (magnetic) parametric amplifiers.

## 6. Magnetic Amplifiers at the Nanoscale

In the single-domain approximation the magnetization dynamics of a particular single-domain dot is entirely determined by the point value of magnetic fields at the center of the dot. If we could surround a nanomagnetic dot by current loops and apply appropriate current pulses to those loops, we can emulate the effect of neighboring dots. Such a hypothetical device is illustrated in Fig. 7. This device exchanges energy with the loops via the time-varying $\phi_{\text{in}}$ and $\phi_{\text{out}}$ fluxes, which generate voltage in the loops.

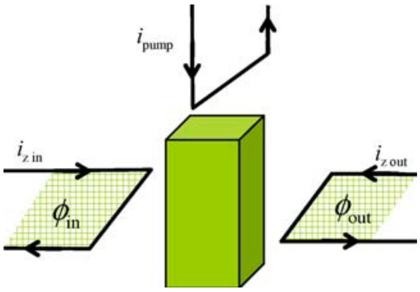

*Figure 7.* A single-domain nanomagnet coupled to current loops. This fictional nano-circuit is a model system of field-coupled nanomagnets.

The arrangement of Fig. 7 is a nanoscale realization of a magnetic amplifier, which was a widely used 'electronic' device before becoming obsolete by the 70's [13–15]. These devices used the nonlinearity of a magnetic core to feed energy from one coil to another, amplifying AC signals in very much the same way as transistor-based circuits do with DC power. The nano-magnetic amplifier of Fig. 7 has far better characteristics than its classical ancestors: in principle it can operate with speed in the hundred megahertz range with minimal dissipation (since nanomagnets does not necessarily exhibit hysteresis losses). Nevertheless, a nanoscale magnetic amplifier in the above form is not a very practical device: the value of inductances is extremely small and the fabrication difficulties are prohibitive.

Using the method of equivalent circuit construction, it can be shown that nanomagnets 'feel' their neighbors as if they were time-varying, nonlinear inductances connected to $i_{Z\,\text{in}}$ and $i_{Z\,\text{out}}$. The device of Fig. 7 is known to behave as an amplifier if it is connected to asymmetrical inductances. This qualitatively explains how the field-coupled nanomagnets can exhibit power gain as well.

## 7. Conclusions

This paper analyzed dissipation and power flow characteristics of field-coupled single domain nanomagnets, which were proposed as possible realization of novel computing devices. This architecture can offer significant improvement over MOS-based devices in terms of dissipated energy.

The dissipated power of a stand-alone magnet could be arbitrarily small if the clocking field is sufficiently slow and the magnet is ideally single-domain. In coupled systems, however, one should keep the coupling energy above the energy of thermal noise $(kT)^-$, otherwise both coupling and power gain will be washed out by thermal fluctuations. This means that the kink, which propagates the signal should have at least a few times $kT$ energy and the system cannot be arbitrarily close to ground state. The minimum dissipation is linked to the actual operating temperature, which is promising for low-temperature (e.g. space) applications.

The strong stray fields of nanoscale magnets can provide their interconnection without the need of electrically wiring them—in conventional magnetic amplifiers the magnetic signal have to be converted to current and then back to magnetic field again. Field-coupling permits considerably simpler circuits on the nanoscale without sacrificing fundamentally important device properties, such as active behavior.

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## Note

1. An alternative explanation is that the hard axis hysteresis of single-domain nanomagnets has no loop, so the area of the bfBH curve is zero.

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