The dynamics of the combustion products behind plane and spherical detonation fronts in explosives

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The flow behind the detonation front of an explosive contained in a tube strong enough to confine the motion to one dimension is shown to be a progressive wave of finite amplitude of the type studied by Riemann. The wave is similar at all stages of its progress if the initiation of the explosion is instantaneous, the linear scale of the whole field of flow increasing at a uniform rate. If the products of combustion obey the law $p\rho^{-\gamma}$ = constant the distribution of gas velocity along the tube is linear. If the initiation end of the tube is closed a fixed proportion of the whole detonating column is at rest.

This last case has an analogy in three dimensions. The dynamics of spherical detonation from a point in an explosive is analyzed. As in the one-dimensional case, a fixed proportion of the whole volume of burnt gas is at rest. The radial rate of change of the variables, velocity, pressure and density become infinite at the detonation front, but it is unlikely that this result would be true in a real explosive where the time of reaction is not zero.

The results are applied in both linear and spherical cases to the detonation of T.N.T., using data given by Jones & Miller (1948).

Introduction

The existing hydrodynamical and chemical theory of detonation has been developed by supposing that the whole chemical reaction in the explosive takes place within a narrow region, the explosive being at rest before this thin detonation front reaches it. The equations of continuity, momentum and energy when applied to the two sides of the front, together with an equation of state for the products of decomposition, do not suffice to determine the five unknowns: U, u_1, p_1, ρ_1, T_1. Here U is the velocity of the detonation front, u_1, p_1, ρ_1 and T_1, are the velocity, the pressure, density and temperature of the gas close behind the front. A fifth equation is needed, and it is clear that this equation must in some way represent the reaction of the gas streaming away behind the front on the gas immediately behind it. That this must be so can be seen by imagining the case where a rigid plane moves forward behind the detonation front confining the burnt gas between it and the front. If this plane were to move fast enough it must be able to increase the value of U beyond the value found when the burnt gas can escape freely.

The Chapman-Jouget condition

To determine the fifth condition, therefore, it might be thought that it would be necessary to study the movement behind the detonation wave. The necessity for doing this, however, has been avoided by making the hypothesis that small disturbances in the gas immediately behind the detonation front travel at the same speed, U, as the front itself. The equation representing this hypothesis is

$$U = u_1 + c_1, \tag{1}$$

where c_1 is the velocity of sound in the gas immediately behind the front.

The progressive wave behind the detonation front

Though this hypothesis, which was applied in the first instance in different forms by Chapman (1899) and Jouguet (1905–6), has been confirmed indirectly by experiment, and has formed the basis of much fruitful work on explosives, no one before the late war seems to have discussed the hydrodynamics of the burnt gas and the distribution of pressure behind the detonation front. Consider the detonation of explosive contained in a tube so strong that the motion is confined to one dimension. The motion of a gas for which ρ is a function of p only has been completely analyzed by Riemann, so that if u and p are known at all points at one instant they are known at all other times. Riemann's analysis is a generalization to disturbances of finite amplitude of the simple analysis of arbitrary small disturbances in one dimension into two sound waves moving in opposite directions. When only one wave is present* the analysis is very simple and u is found to be a function of ρ only, as it is in the limiting case of a progressive sound wave, which moves in one direction only. In fact, a progressive wave of finite amplitude may be regarded as being composed of an infinite number of superposed infinitesimal progressive sound waves. Each of these sound waves changes p, ρ and u, so that

$$\delta p = \rho c \delta u = c^2 \delta \rho, \tag{2}$$

where c is the velocity of sound through the gas when its density is ρ. Since the sound waves are superposed, a change in u is always associated with the change in ρ given by (2), and since c also is a function of ρ only, u is found by integrating (2). Thus

$$u - u_1 = \int_{\rho}^{\rho_1} c \frac{d\rho}{\rho}. \tag{3}$$

Since small disturbances are propagated relative to fixed axes with velocity $u+c$ it will be seen that if p, ρ and u are known at one point P at one instant, they will have the same value at time t later at a point distant $(u+c)\,t$ from P. It will be seen therefore that a progressive wave of finite amplitude can exist behind a detonation front provided that the Chapman-Jouguet condition (1) is satisfied and the conditions behind the detonation front remain constant as the front progresses. Except for the point immediately behind the detonation front where the velocity and density are fixed by the detonation conditions, u can have any arbitrary distribution at one given instant, and the Riemann analysis will determine the motion at all subsequent times till a shock wave appears somewhere in the field. On the other hand, when the wave has progressed a distance which is large compared with the initial length in which detonation is being built up, it will tend to a definite limiting form. This may most conveniently be illustrated by plotting u against r/R or r/Ut, where r is the distance from the point of initiation. The limiting ordinates in this curve are 0 and 1, and the initial portion where the steady detonation conditions had not been attained continually shrinks into the region near $r/R = 0$ as t increases. The limiting form to which the disturbance tends as it increases is that which occurs at all times if the explosive is at rest and detonation starts instantaneously at time $t = 0$ at a point $r = 0$. This form will be discussed in two cases.

* This case was investigated as early as 1858 by Earnshaw (1860).

Case of gas for which $p\rho^{-\gamma}$ is constant

In this case
$$c = c_1\left(\frac{\rho}{\rho_1}\right)^{\frac{1}{2}(\gamma-1)}. \tag{4}$$

Inserting (4) in (3) and integrating
$$u = u_1 - \frac{2c_1}{\gamma-1}\left\{1-\left(\frac{\rho}{\rho_1}\right)^{\frac{1}{2}(\gamma-1)}\right\}. \tag{5}$$

It will be noticed that u and c are linearly related, so also are u and $u+c$, in fact
$$u+c = u_1+c_1-\frac{\gamma+1}{2}(u_1-u). \tag{6}$$

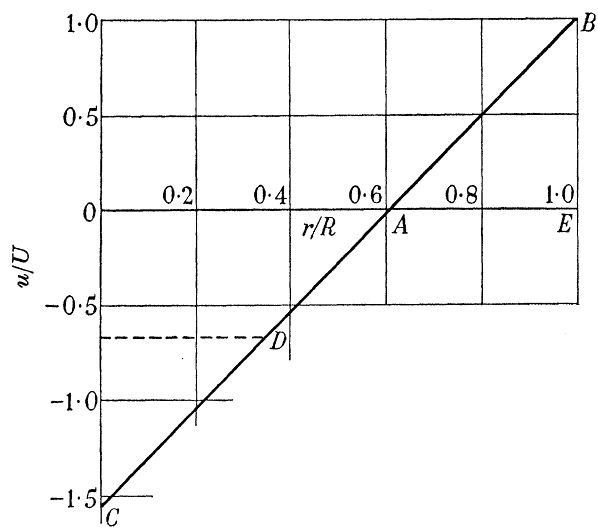

FIGURE 1. Distribution of velocity behind a plane detonation front when $p\rho^{-1\cdot 3}$ = constant.

If the distribution of u or c in space at any time is plotted against $r = (u+c)t$ a straight line will result. Figure 1 shows a particular case in which the conditions at the detonation front were such that $u_1 = \frac{1}{3}U$ and $c = \frac{2}{3}U$. If the detonation starts at one end $r = 0$ of a tube the gas velocity, u_2 at this end is found by setting $u+c = 0$ in (6), hence
$$u_2 = u_1 - \frac{2}{\gamma+1}(u_1+c_1), \tag{7}$$

and since $\gamma > 1$, u_2 must be negative, so that the gas flows backwards from the ignition point as would be expected. u_2 is represented in figure 1 by the point C. In this discussion it has been assumed that the gas can flow freely backwards through the end of the tube. It may happen that the pressure in the atmosphere outside the tube is greater than that calculated using the equation $p\rho^{-\gamma} = p_1\rho_1^{-\gamma}$. In that case a solution of the problem is found by assuming that u and p decrease with distance from the detonation front in accordance with the calculation for a Riemann progressive wave. At the point, D, in the tube where the outside atmospheric pressure is reached, the velocity becomes constant and remains constant between D and the end of the tube. A special case of some interest is when the rear end of the tube is blocked up so that $u = 0$. In that case the gas is at rest between the end and the point A, figure 1, where the line BAC cuts the axis $u = 0$. To find the ratio of the length of the portion where the gas is at rest to the whole column, i.e. OA/OE in figure 1, set $u = 0$ in (6). The ratio is

$$\frac{c}{U} = \frac{u_1 + c_1}{U} - \frac{\gamma+1}{2U} u_1 = 1 - \frac{\gamma+1}{2}\left(\frac{u_1}{U}\right). \tag{8}$$

In the particular case when $u_1 = \frac{1}{3}U$ it is $1 - \frac{1}{6}(\gamma+1)$, which is 0·616 when $\gamma = 1{\cdot}3$. This is the case shown in figure 1.

It is worth noticing that when the end of the tube is open the condition $u + c = 0$ which has been applied there is the same as Reynold's condition when air at high pressures flows through a constriction in a tube. The speed at this point is equal to the speed of sound there.

Determination of detonation velocity

In the hydrodynamical theory of detonation, a detonation front is assumed to move with uniform velocity U into a stationary explosive. The equations which represent the conservation of mass, momentum and energy are

$$\rho_1(U - u_1) = \rho_0 U \qquad \text{(mass)}, \tag{9}$$

$$p_1 + \rho_1(U - u_1)^2 = p_0 + \rho_0 U^2 \quad \text{(momentum)}, \tag{10}$$

$$\frac{p_1}{\rho_1} + \tfrac{1}{2}(U - u_1)^2 - E_1 = \frac{p_0}{\rho_0} + \tfrac{1}{2}U^2 - E_0 \quad \text{(energy)}. \tag{11}$$

E_0 and E_1 are the chemical plus heat energies per unit mass, p_1, ρ_1, u_1 and p_0, ρ_0, 0, are the pressures, densities and velocities on the two sides before and after passing the shock wave.

Eliminating U and u_1 between (9), (10) and (11) the Rankine-Hugoniot equation is obtained, namely,

$$\tfrac{1}{2}(p_1 + p_0)\left(\frac{1}{\rho_0} - \frac{1}{\rho_1}\right) = E_0 - E_1. \tag{12}$$

$E_1 - E_0$ may be regarded as known if p_1, ρ_1 and T_1 the temperature, are known. T_1 can be eliminated by using the equation of state, thus (12) may be regarded as a single equation between p_1 and ρ_1. If this relationship between p_1 and ρ_1 is exhibited as a curve on a diagram for which p_1 is the ordinate and $1/\rho_1$ the abscissa the curve so formed is called the Rankine-Hugoniot curve. The initial condition of the explosive is represented by the point $(p_0, 1/\rho_0)$. The Chapman-Jouguet condition is satisfied by the point on the Rankine-Hugoniot curve where the tangent from the point $(p_0, 1/\rho_0)$ touches it. Eliminating u_1 from (9) and (10) it is found that

$$U^2 = \frac{p_1 - p_0}{1/\rho_0 - 1/\rho_1}, \tag{13}$$

and this is equal to $\tan\theta$, where θ is the angle which the tangent line described above makes with the abscissa axis. The velocity, U, which corresponds with the point on the Rankine-Hugoniot curve which satisfies the Chapman-Jouguet condition is therefore the minimum of all the velocities which are possible. The computation of the Rankine-Hugoniot curve in any particular case is complicated because the relationships between energy, temperature, pressure and density depend on the chemical composition of the constituents of the burnt gas. If these are in chemical equilibrium the composition of the mixture depends on temperature which is only known when the problem has been solved. It is therefore necessary to make a number of subsidiary calculations to find the compositions corresponding with a number of points on the Rankine-Hugoniot curve. In the case of certain mixtures (see Lewis & Friauf 1930) of oxygen and hydrogen with excess oxygen, hydrogen or nitrogen the calculations have been carried out, and the velocity of detonation found. The observed velocities agree well with those so calculated.

Application to T.N.T.

The calculations for the products of detonation of T.N.T. packed to density $\rho_0 = 1{\cdot}51$ have been performed by Jones & Miller (1948), who found $U = 6380$ m./sec., $u_1/U = 0{\cdot}2424$, $\rho_1 = 2{\cdot}00$. Jones also calculated the succession of equilibrium compositions and energies and also p and ρ as they change during an isentropic expansion after detonation. His results* are here given in the first three columns of table 1 and in columns 4, 7 and 8 of table 2. The data are therefore available for calculating the pressure distribution and gas flow in a tube sufficiently strong to withstand detonation of T.N.T.

TABLE 1. MOTION BEHIND PLANE DETONATION WAVE IN T.N.T. IN A CLOSED TUBE

$p \times 10^{-10}$ (dynes/sq.cm.)	$v\ (=1/\rho)$ (cm.^{3}/g.)	c/U	u/U	r/R
15	0·5	0·757	0·242	1·00
14	0·513	0·735	0·225	0·960
12	0·542	0·715	0·187	0·902
10	0·574	0·705	0·147	0·852
9	0·591	0·700	0·129	0·829
7	0·629	0·687	0·085	0·772
5	0·676	0·625	0·034	0·659
4	0·710	0·571	0·006	0·577
3·8	—	0·550	0	0·577
3·8	—	0·550	0	0·550

$U = 6380$ m./sec. Density of T.N.T. 1·51.

The results of using Jones's data in integrating (3) numerically to find u are given in column 4 of table 2 and the corresponding value of $c = (dp/d\rho)^{\frac{1}{2}}$ are given in column 3 of table 1. If R is the total distance through which the detonation has

* The figures given in columns 4, 7 and 8 of table 2 and in columns 1 and 2 of table 1 are taken from an unpublished report by H. Jones to the Ministry of Home Security. They are not identical with the revised figures published by Jones & Miller in table 3, p. 497 of their 1948 paper but the difference was not thought to be sufficiently great to warrant the recalculation of the spherical detonation wave.

travelled since it was initiated, we have seen that p, ρ and u are functions at any time of $(u+c)$ only. At all times, therefore, they are functions of $\frac{u+c}{U} = \frac{(u+c)\,t}{R}$ or r/R only. The values of r/R are given in column 5 of table 1.

These results are plotted in figure 2. The broken curve representing the calculated values of u/U is no longer straight, as it was for a gas in which $p\rho^{-\gamma}$ is constant, but the curve has the characteristic that $u = 0$ at a value of r/R which in this case is 0·550. The calculations have not been carried beyond this point though this could easily be done. The results therefore represent those for a closed tube and they show that in such a tube the pressure is uniform from the point of detonation to $r/R = 0{\cdot}550$ and that it has a value of about $3{\cdot}8 \times 10^{10}$ dynes or 250 tons/sq.in. Beyond that point the pressure rises to that behind the detonation front, namely the value given in Jones's calculations of about 1000 tons/sq.in.

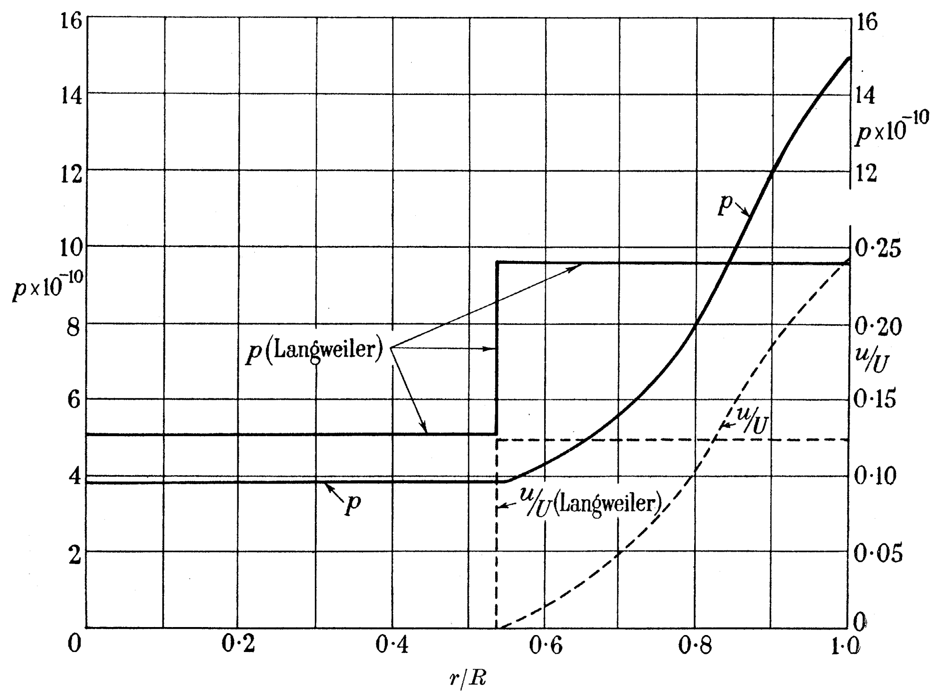

FIGURE 2. Plane detonation wave in T.N.T. (p expressed in dynes/sq.cm.).

Comparison with previous work

In his original paper (1899) Chapman recognized that when detonation occurs in a tube so that the motion is confined to one dimension the detonation wave must be followed by a region of forward-moving gas and that the length of this region must continually increase. The only attempt which has been made to calculate this length seems to be that of Langweiler (1938) who assumed that the burnt gases preserve the velocity u_1 which they acquire at the detonation front until the passage of a rarefaction shock wave reduces the velocity to zero. Langweiler's distribution of velocity is shown in figure 2 for comparison with the correct distribution. If W is the velocity of the assumed rarefaction shock wave and p_2 the pressure behind it, Langweiler uses equations which are identical with the equations (9) to (11) to determine W and p_2. He recognizes that a rarefaction shock wave cannot occur and he tries to overcome this difficulty by saying that the region of transition where the velocity is reduced from u_1 to 0 may be assumed small compared with the distances travelled by the detonation and rarefaction waves. That this assumption is untrue can be seen by inspection of figure 2, where the transition is shown extending through the whole of the forward-moving column of gas. It has been remarked by many writers that though the Rankine-Hugoniot relation appears at first sight to be applicable both to waves of condensation and to waves of rarefaction, the energy and momentum conditions being satisfied in both cases, yet the fact that there is an irreversible rise in temperature in a condensation wave precludes the possibility of reversing the motion.

Langweiler applied his calculations to T.N.T. of density 1·59. His results are shown in figure 2 for comparison with the present calculations for T.N.T. of density 1·51.

Langweiler considers also the effect of reduction in pressure at the end of the tube below p_2, the pressure which reduces the motion behind the wave to rest and therefore corresponds with a closed end. He states that the effect of this reduction would be to increase the speed of the rarefaction wave, thus reducing the thickness of the forward-moving gas which he calls the detonation head. It will be seen from figure 2 and the discussion which figure 2 illustrates that this is a mistake. Reduction of pressure behind the wave below the value p_2 leaves the forward-moving part of the detonation head unchanged but increases the total thickness of the transition layer till, when the pressure is reduced to a certain calculable value, the transition layer fills the whole tube right back to the point at which the detonation was initiated.

Spherical detonation waves

Detonation conditions in which the burnt gases are constrained to move in one dimension only can be realized experimentally in the case of a gaseous mixture which can be contained in a tube strong enough to resist the bursting pressure. It cannot be completely realized with a high explosive which exerts a pressure so high that any containing tube will burst. If spherical detonation starting from a point inside an explosive is dynamically possible it should be realizable experimentally.

If the arguments so far advanced by writers on the subject to justify the Chapman-Jouguet condition are valid, they should apply to a spherical detonation front provided the hydrodynamic flow conditions are capable of permitting a flow to exist for which $u_1 + c_1 = U$ at a sphere which is expanding with radial velocity U. Progressive spherical waves of finite amplitude analogous to progressive Riemann waves do not exist. To find out whether spherical detonation waves might be expected it is therefore necessary to find out whether it is possible to construct and solve the hydrodynamic equations for radial motion which satisfy the boundary condition $u + c = U$ at a sphere of radius $R = Ut$.

If this could be done, the Chapman-Jouguet condition would apply at the detonation front and the radial velocity U would be identical with that found using equation (12). The maximum pressure and speed of propagation would be the same as those occurring in a plane detonation, but the radial distribution of velocity and pressure would differ from those in the plane case.

The flow behind a spherical detonation wave will in general depend on the manner in which the detonation is set up, but if it could be assumed that detonation starts when the explosion has extended only a short distance from the point of initiation, then a steadily expanding regime would be set up analogous to the one-dimensional wave described in figure 1. In that case the radial velocity u and also p, ρ and c would be functions of r/t only. The equation of motion is

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}, \tag{14}$$

and the condition that u, p and ρ depend only on $x = r/t$ is

$$\left(\frac{\partial}{\partial t}+x\frac{\partial}{\partial r}\right)(u, p, \rho) = 0, \tag{15}$$

so that (14) becomes

$$(u-x)\frac{du}{dx}=-\frac{1}{\rho}\frac{dp}{dx}. \tag{16}$$

The equation of continuity is

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\left(\frac{\partial u}{\partial r}+\frac{2u}{r}\right)=0, \tag{17}$$

which in view of (15) may be written

$$\frac{u-x}{\rho}\frac{d\rho}{dx}+\frac{du}{dx}+\frac{2u}{x}=0. \tag{18}$$

Writing $c^2 = dp/d\rho$, where c is the velocity of sound, $dp/dx = c^2 d\rho/dx$ so that $d\rho/dx$ may be eliminated between (16) and (18). Hence

$$\frac{du}{dx}\left\{1-\left(\frac{x-u}{c}\right)^2\right\}=-2\frac{u}{x}. \tag{19}$$

At this stage it is of interest to compare (19) with the equivalent equation for detonation waves in one dimension. Equations (15) and (16) are identical in the two cases, the continuity equation differs only from (17) in that the term $2u/r$ is absent in the one-dimensional case. The equation equivalent to (19) is therefore

$$\frac{du}{dx}\left\{1-\left(\frac{x-u}{c}\right)^2\right\}=0. \tag{20}$$

Hence either $du/dx = 0$ or $x = u+c$. The condition $du/dx = 0$, i.e. u = constant, applies over the ranges represented in figure 1 by OA. The condition $x = u+c$ is identical with Riemann's condition in a one-dimensional wave. It applies to the region represented by AB in figure 1.

The adiabatic equation of state gives c as a function of ρ or conversely ρ as a function of c^2, thus in (18) $\dfrac{1}{\rho}\dfrac{d\rho}{dx}$ may be written $\dfrac{1}{\rho}\dfrac{d\rho}{dc^2}\dfrac{dc^2}{dx}$, since $\dfrac{1}{\rho}\dfrac{d\rho}{dc^2}$ may be regarded as a function of c^2. Equations (18) and (19) determine u and c as functions of x when the appropriate boundary conditions are satisfied.

These equations may be expressed in non-dimensional form in several ways. One method is to write

$$\xi = u/x, \quad \eta = c^2/x^2, \quad Z = \log_e x.$$

The resulting equations are

$$\frac{d\eta}{d\xi} = \frac{2\eta}{\xi}\frac{\eta-(1-\xi)^2+f\xi(1-\xi)}{3\eta-(1-\xi)^2}, \tag{21}$$

$$\frac{dZ}{d\xi} = -\frac{1}{\xi}\frac{\eta-(1-\xi)^2}{3\eta-(1-\xi)^2}, \tag{22}$$

where

$$f = \frac{\rho}{c^2}\frac{dc^2}{d\rho}. \tag{23}$$

This form is convenient when the burnt gases behave like a perfect gas. In that case

$$\frac{\rho}{c^2}\frac{dc^2}{d\rho^2} = \gamma - 1,$$

so that (21) determines η as a function of ξ.

Another non-dimensional form is obtained by substituting $\eta = \xi^2/\psi^2$ so that $\psi = u/c$. The equations are then

$$\frac{d\psi}{dZ} = \frac{\psi\xi}{(1-\xi)^2-\xi^2}\{2\xi-(1-\xi)\,\psi^2 f\}, \tag{24}$$

$$\frac{d\xi}{dZ} = \xi\frac{3\xi^2-(1-\xi)^2\psi^2}{(1-\xi)^2\psi^2-\xi^2}. \tag{25}$$

Boundary conditions

To solve these equations in any particular case it is necessary to know the adiabatic p, ρ relationship in the burnt gases. $c^2 = dp/d\rho$ and $dc^2/d\rho$ are then calculated in terms of ρ so that f can be tabulated as a function of c.

The Chapman-Jouguet condition, together with the Rankine-Hugoniot equation, determines the detonation velocity U and also the value of $u/U = 1-1/\mu$ at the wave front. If R is the radius of the wave at any time $R = Ut$ and $x = r/t$ so that

$$\frac{x}{U} = \frac{r}{R}. \tag{26}$$

Since Z only enters into equations (24) and (25) as a differential its value may be taken as zero at $r = R$ so that

$$Z = \log_e(r/R), \tag{27}$$

$$u/U = \xi r/R, \tag{28}$$

$$c/U = \xi r/(\psi R). \tag{29}$$

If μ is the ratio of the density behind the detonation wave to the density of the explosive then at $r = R$

$$\xi = 1-1/\mu. \tag{30}$$

The Chapman-Jouguet condition, namely $u/U + c/U = 1$, gives

$$\psi = \frac{u}{c}\frac{u}{U} = \mu - 1. \tag{31}$$

As an example of the application of these equations the complete description of detonation initiated at a point in the interior of a mass of T.N.T. of density 1·51 has been worked out. The values of μ and U calculated by Dr H. Jones for T.N.T. of this density are $\mu = 1{\cdot}32$, $U = 6380$ m./sec.

The calculated adiabatic relationship between p and ρ, together with the corresponding values of f and c/U, are given in table 2* and the relationship between f and c/U is shown in figure 3.

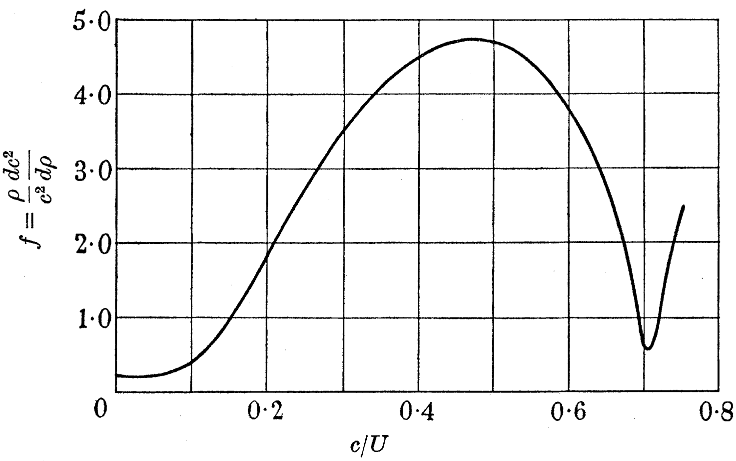

FIGURE 3

Equations (24) and (25) can only be integrated numerically step by step, starting from the spherical detonation surface and proceeding inwards. When the initial values $\xi = 1 - 1/\mu$, $\psi = \mu - 1$ are inserted in equations (24) and (25) it will be seen that both $d\psi/dZ$ and $d\xi/dZ$ are initially infinite because $(1-\xi)^2\psi^2 - \xi^2 = 0$. $d\psi/d\xi$ is, however, finite. The solution of (25) in the neighbourhood of $\xi = 1 - 1/\mu$ is

$$Z = -\frac{\mu^2}{\mu - 1}\left(\tfrac{1}{2} + \tfrac{1}{4}f\right)\left(\frac{\mu - 1}{\mu} - \xi\right)^2. \tag{32}$$

This solution may be used to find values of Z corresponding with values of ψ and ξ near the detonation surface. Using (27) to cover the region $r/R = 1{\cdot}0$ to $0{\cdot}9986$ the solution was carried step by step back to the centre by means of equations (19) and (20). At each stage the values of $d\xi/dZ$ and $d\psi/dZ$ were calculated and the increments in ξ and ψ corresponding with a small finite decrement in Z were taken, as first approximation, to be $\delta\psi_0 = (d\psi/dZ)_0\,\delta Z$, $\delta\xi_0 = (d\xi/dZ)_0\,\delta Z$, where $(d\psi/dZ)_0$, $(d\xi/dZ)_0$ represent the values calculated at the beginning of the interval δZ. The values $(d\psi/dZ)_1$, $(d\xi/dZ)_1$ at the end of the interval were then calculated using the

* The range covered in table 2 is limited to values which occur in and behind the detonation wave.

approximate values of ξ and ψ thus found. A second approximation to ψ and ξ was then found using the formulae

$$\delta\psi = \tfrac{1}{2}\{(d\psi/dZ)_0 + (d\psi/dZ)_1\}\,\delta Z, \quad \delta\xi = \tfrac{1}{2}\{(d\xi/dZ)_0 + (d\xi/dZ)_1\}\,\delta Z.$$

The second approximations to ψ and ξ at the end of the interval were then used in equation (20) to calculate new values of $d\psi/dZ$ and $d\xi/dZ$ and these again were used to calculate a third approximation to ψ and ξ at the end of the interval. This process was repeated till the changes in ψ and ξ at the end of the interval due to proceeding to one further stage of approximation were negligible. The results of the calculations are given in table 2. The distribution of gas velocity is shown in figure 4 and for comparison the distribution in the one-dimensional case is shown in the same

TABLE 2. SPHERICAL DETONATION WAVE IN T.N.T.

r/R	ξ	ψ	c/U	f	u/U	$p \times 10^{-10}$	ρ
1·0	0·2424	0·320	0·7575	2·60	0·2424	15·00	2·000
0·99904	0·230	0·3084	0·742	2·06	0·230	14·45	1·972
0·99632	0·220	0·2986	0·733	1·80	0·219	13·90	1·942
0·99271	0·210	0·2874	0·724	1·20	0·208	13·34	1·928
0·9885	0·20	0·2748	0·719	1·00	0·1977	12·68	1·880
0·9821	0·19	0·2612	0·714	0·70	0·1866	12·11	1·850
0·9744	0·18	0·2469	0·710	0·60	0·1754	11·09	1·797
0·9656	0·17	0·2323	0·708	0·60	0·16415	10·59	1·773
0·9557	0·16	0·2177	0·702	0·60	0·1529	9·44	1·709
0·9448	0·15	0·2028	0·700	0·62	0·14172	8·91	1·683
0·9320	0·14	0·1879	0·694	0·80	0·1305	7·94	1·639
0·9178	0·13	0·1737	0·688	1·45	0·1193	7·08	1·595
0·8997	0·12	0·1601	0·675	1·97	0·1080	6·46	1·562
0·8804	0·11	0·1468	0·659	2·52	0·09684	5·875	1·531
0·8577	0·10	0·1337	0·641	3·00	0·08577	$5{\cdot}43_{3}$	1·506
0·8275	0·09	0·1205	0·618	3·40	0·07448	4·84	1·468
0·7960	0·08	0·1071	0·596	3·88	0·06368	$4{\cdot}41_{6}$	1·440
0·7584	0·07	0·09350	0·568	4·22	0·05309	3·98	1·407
0·7148	0·06	0·07966	0·538	4·50	0·04289	$3{\cdot}56_{5}$	1·372
0·6668	0·05	0·06514	0·512	4·65	0·03334	$3{\cdot}23_{6}$	1·342
0·6166	0·04	0·05084	0·487	4·72	0·02466	$2{\cdot}95_{1}$	1·316
0·5672	0·03	0·03673	0·461	4·74	0·01702	2·66	1·285
0·5186	0·02	0·02327	0·446	4·70	0·01037	2·48	2·265
0·4950	0·015	0·01695	0·441	4·70	0·00741	2·43	1·259
0·4723	0·010	0·01096	0·430	4·66	0·00472	2·34	1·249
0·4474	0·005	0·00527	0·425	4·64	0·00224	2·29	1·244
0·418	0	0	0·418	4·60	0	2·19	1·233

figure. Figure 5 shows the distribution of pressure in the spherical and the one-dimensional cases. It will be seen that the pressure and velocity drop very rapidly behind the detonation front. The pressure in fact drops to half its maximum value at a distance behind the front which is only $7\frac{1}{2}$ % of its radius. In this connexion it is worth noticing that photographs taken with a rotating mirror camera of the detonation of a cylindrical charge of T.N.T. show a narrow highly luminous band behind the detonation front, while similar photographs of the detonation of gases in tubes show a much broader luminous region.

The fact that the velocity drops to zero at some point between the detonation surface and the centre shows that a spherical detonation wave can maintain itself in the particular case investigated. It is not obvious whether this is true in all cases

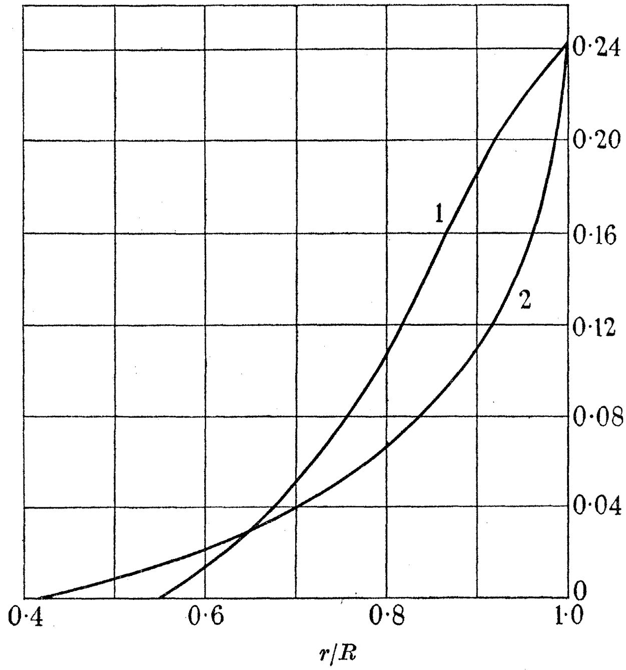

FIGURE 4. Velocity distribution (T.N.T.). Curve 1, plane detonation wave; curve 2, spherical wave.

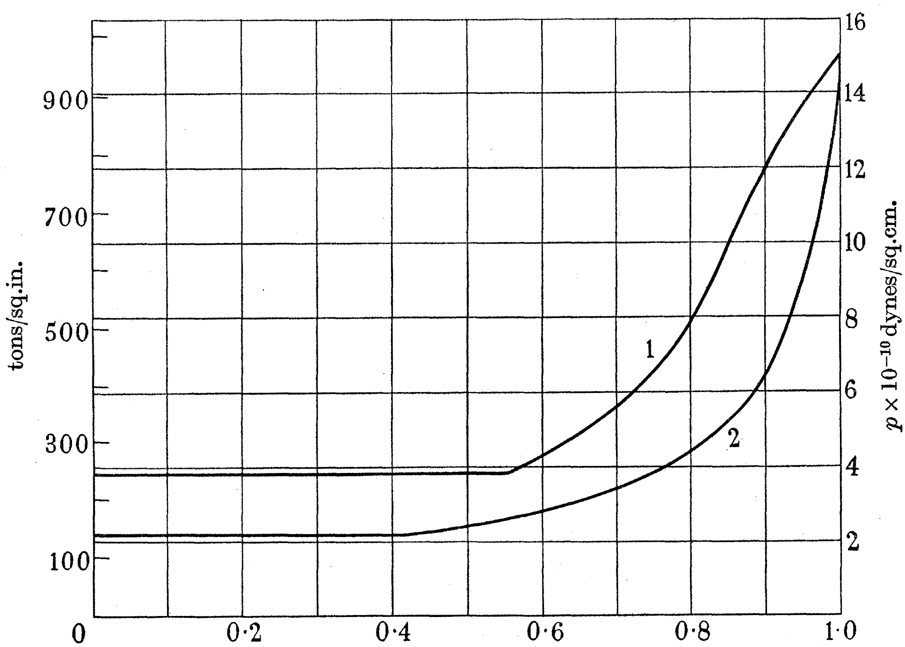

FIGURE 5. Pressure distribution (T.N.T.). Curve 1, plane; curve 2, spherical.

because the equations for the flow behind a spherical wave do not admit of a simple solution of the type obtained by Riemann for a plane wave.

Previous work on spherical detonation waves

The view here put forward is contrary to that expressed by Jouguet (1907) that spherical detonation waves moving with constant speed probably cannot exist. It seems that this view is founded partly on an expression connecting U with $u_1 + c_1$ which shows that if the pressure and velocity decrease with distance behind the detonation front then U must be greater than $u_1 + c_1$. Jouguet rejects all detonation waves for which $U > u_1 + c_1$ as being impossible to produce. This rejection, however, is based only on an unproved 'postulate', for which I see no adequate reason. In the present analysis (19) leads to the same conclusion that $U \geqslant u_1 + c_1$ for, if du/dx in (19) is positive, $1 - \left(\frac{x-u}{c}\right)^2$ must be negative so that $c < x - u$, and, since at the detonation surface $x = U$, $c = c_1$, $u = u_1$, therefore $U > u_1 + c_1$. Jouguet states also that his equation, which is analogous to (19), is not consistent with $U = u_1 + c_1$. It will be noticed that (19) might have been interpreted in this sense if the possibility that an infinite value of du/dx may occur is rejected. It is true that infinite values of du/dx cannot actually occur in real materials, and in this sense infinitely thin shock waves are not physically possible. The effects predicted by analysis which assumes infinitely thin shock waves are in fact observed, so that it seems probable that the spherical detonation wave calculated in the present paper can be propagated if it can be started. It seems likely that the peak pressure may not be attained though the predicted detonation velocity will be realized in a spherical wave.

The only worker who has experimented with spherical detonation waves seems to be Lafitte (1925) who found that ignition by a spark at the centre of a spherical glass bulb 24 cm. diameter, containing an explosive mixture of CS_2 with $3O_2$ did not produce detonation. When the same mixture was fired by means of a detonator containing 1 g. of fulminate of mercury a spherical detonation wave was produced. This wave travelled at the same speed as that found when the same mixture was exploded in a tube.

This paper was circulated under the title 'Detonation waves' in the Ministry of Home Security in January 1941. It has now been declassified.

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