NAME:
AE 360
Examination 1
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1. Consider the non-conservative form of the compressible, viscous, one-dimensional energy equation:

$$\rho \frac{\partial e}{\partial t} + \rho u \frac{\partial e}{\partial x} = -\frac{\partial q_x}{\partial x} - P\frac{\partial u}{\partial x} + \tau_{xx}\frac{\partial u}{\partial x}$$

   Showing all steps and utilizing the appropriate mass and linear momentum equations,

   - write this equation in *full conservative form.*

2. To account for real gas behavior at elevated temperatures, one can model the internal energy of an *ideal* gas as a quadratic function of temperature:

$$e(T) = e_o + c_{vo}T + aT^2$$

   For air, $e_o = -19546 \ \frac{J}{kg}, c_{vo} = 731.33 \ \frac{J}{kg \ K}, a = 0.055648 \ \frac{J}{kg \ K^2}$ gives accurate results for $300 \ K < T < 3000 \ K$

   - Give a symbolic expression for the sound speed of this calorically imperfect *ideal* gas
   - Evaluate the sound speed of ideal calorically imperfect air ($R = 287 \ \frac{J}{kg \ K}$) when $P = 800 \ kPa$, $\rho = 1.2 \ \frac{kg}{m^3}$.
   - Evaluate the sound speed at the same pressure and density using a calorically perfect ideal assumption with $\gamma = \frac{7}{5}, R = 287 \ \frac{J}{kg \ K}$.

3. Calorically perfect air with $\gamma = \frac{7}{5}$, $R = 287 \ \frac{J}{kg \ K}$ enters a combustion chamber at a static pressure of $P_1 = 100 \ kPa$ and static temperature of $T_1 = 300 \ K$ and velocity $u_1 = 200 \ \frac{m}{s}$. The duct has a constant cross sectional area of $A = 0.03 \ m^2$. The duct has a length of $L = 0.3 \ m$. We can model the combustion process as that of uniformly distributed heat flux through the side walls with $q_w = 3 \ \frac{MW}{m^2}$. Find

   - the inlet Mach number,
   - the stagnation temperature at the inlet,
   - the stagnation temperature at the outlet,
   - the outlet Mach number
   - the outlet temperature

4. Calorically perfect air with $\gamma = \frac{7}{5}$, $R = 1717 \ \frac{ft^2}{s^2 \ R}$ flows isentropically into a diverging nozzle of cross sectional area $A_1 = 2 \ ft^2$ with an inlet Mach number of $M_1 = 1.3$. The air has stagnation pressure of $P_o = 100 \ psia$ and stagnation temperature of $T_o = 500 \ R$. The area at the outlet is $A_2 = 5 \ ft^2$. Find

   - the mass flow rate
   - the Mach number at the outlet
   - the stagnation pressure at the outlet
   - the static temperature at the outlet