TO: Joseph M. Powers
FROM: Brian P.Rigney
DATE: 17 April 1997
RE: AE 360 Project: Part2

This memorandum describes the process used in developing an airfoil designed to lift a specified load. First, a brief problem description will be given, followed by a detailed look at the design process. The results of the analysis on the airfoil will then be discussed and finally a copy of a simplified computer code, using similar techniques to those employed during the airfoil analysis, will be given in an appendix.

An airfoil was to be designed which would provide sufficient lift to keep a 2000 kg load airborne. The ambient atmospheric pressure and temperate were 100 kPa and 300 K, respectively. The design Mach number was 2.2.

A computational fluid dynamics, or CFD, computer code was used in the design of this airfoil. The code first generated a grid around an object, using dimensions input into the program. After calculating the locations of the centers of each element within the grid, the mass, momentum, and energy conservation equations were solved at these distinct points. For this two-dimensional flow field, the mass, momentum, and energy conservation equations are:

$$\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right) + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \ \ , \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + \frac{\partial P}{\partial x} = 0 \ \ , \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) + \frac{\partial P}{\partial y} = 0 \ \ , \tag{3}$$

$$\rho\left(u\frac{\partial e}{\partial x} + v\frac{\partial e}{\partial y}\right) + P\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \ \ . \tag{4}$$

ρ represents the density of the fluid, u is the x-component of velocity, v is the y-component, P is the pressure, and e is the internal energy. By modeling the fluid as a calorically perfect ideal gas, the internal energy of the fluid can be expressed as

$$e = \frac{1}{\gamma - 1}\frac{P}{\rho} + e_o \ \ , \tag{5}$$

where γ is a constant for the fluid and e_o is the reference internal energy. By solving these equations in each finite volume, the solution for the entire flow field can be calculated.

The preliminary airfoil design was a diamond shaped airfoil, which was chosen because of the ease in calculating an analytic solution. This allowed for the code's output to be checked before optimizing the design. Using a trial and error procedure, each time changing the geometry input to the CFD code, an eventual optimum diamond shaped airfoil design was reached. The airfoil is pictured in Figure 1. The wedge angle of the airfoil, as defined in Figure 1, is 20°, with a chord length of 1.5 m and maximum thickness of 0.132 m. Because of the symmetry of the airfoil, no lift is produced at 0° angle of attack. Therefore, it was decided to place the airfoil at an angle of attack of 10°. This allowed for a large amount of lift without approaching the stall regime, a viscous phenomenon impossible to predict by this analysis. The mesh used for the CFD code is shown in Figure 2. The top surface was modeled with one trial and the bottom surface with another.

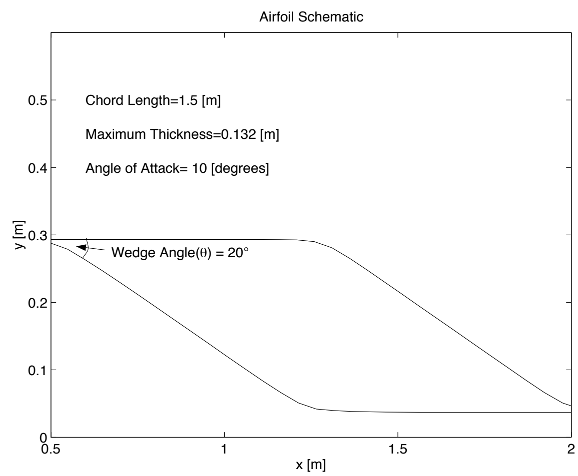

Figure 1: Schematic of diamond shaped airfoil.

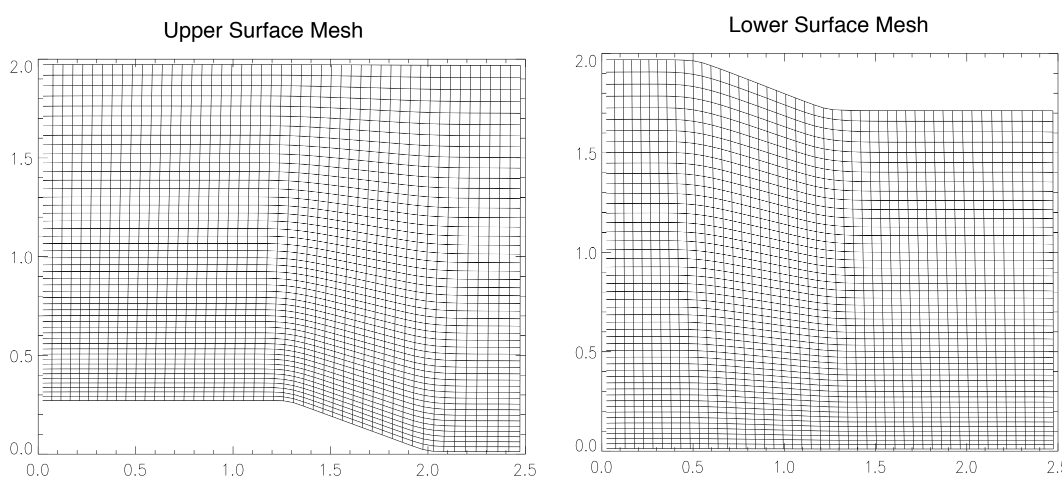

Figure 2: Mesh generated around the diamond shaped airfoil.

Figures 3, 4, and 5 show the pressure, temperature, and density contour plots over both the top and bottom half of the airfoil. The pressure, temperature, and density all experience a sharp gradient as they move through the oblique shock on the underside of the airfoil. All three of these variables increase as the flow moves through this shock. A Prandtl-Meyer expansion wave can be seen on both the top and bottom surfaces of the airfoil. Also, at the trailing edge, another oblique shock arises.

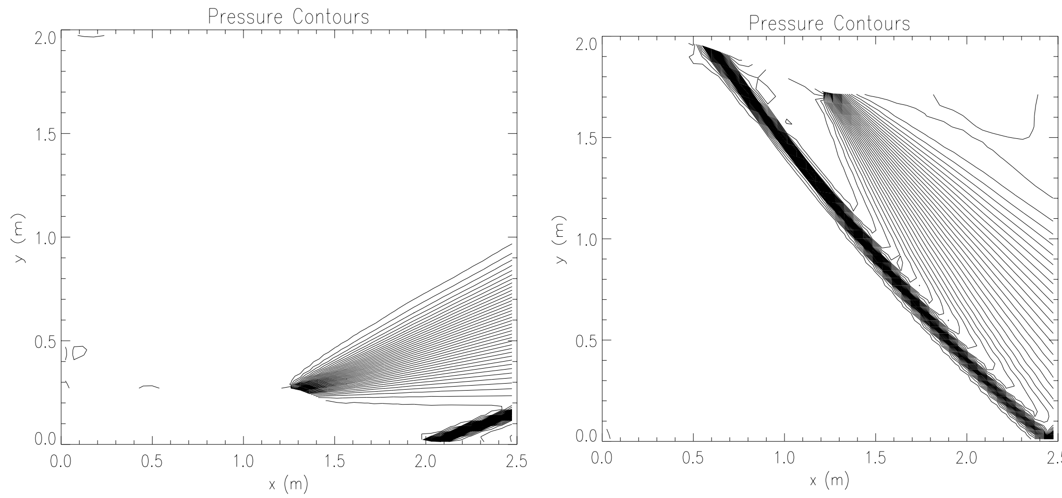

Figure 3: Contour plot of pressure over both the top and bottom surfaces of a diamond shaped airfoil.

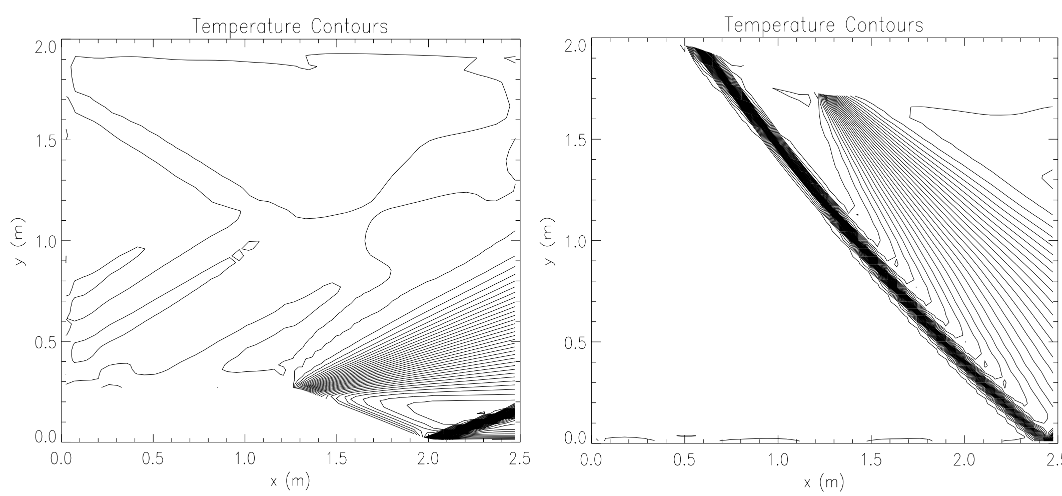

Figure 4: Contour plot of temperature over both the top and bottom surfaces of a diamond shaped airfoil.

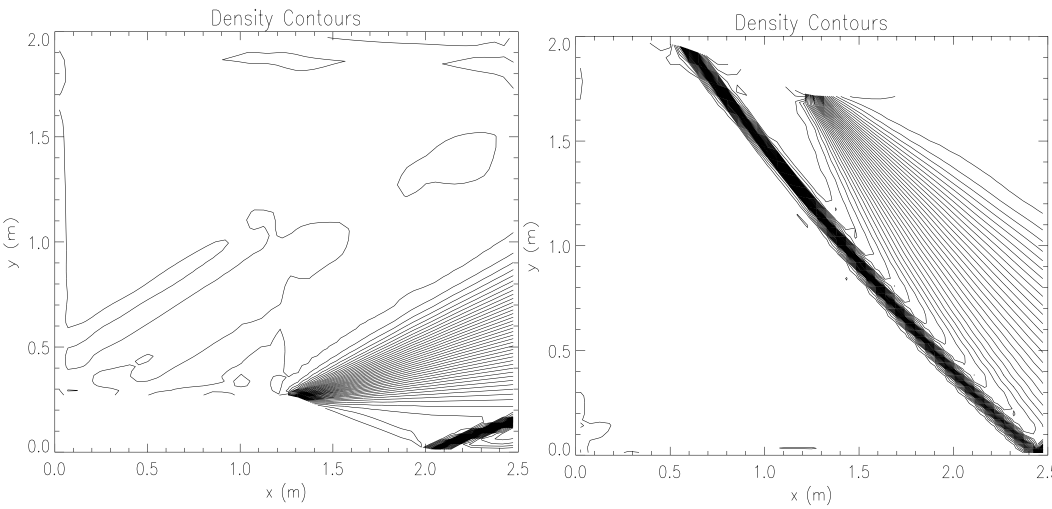

Figure 5: Contour plot of density over both the top and bottom surfaces of a diamond shaped airfoil.

Figure 6 shows the variation in pressure over both the upper and lower surfaces. Again, evidence of shock and rarefaction waves exists in this plot. The pressure sees a large increase on the bottom surface as it moves through the oblique shock and then decreases through the rarefaction. On the top surface, the pressure decreases through the rarefaction and then increases again due to the trailing edge shock.

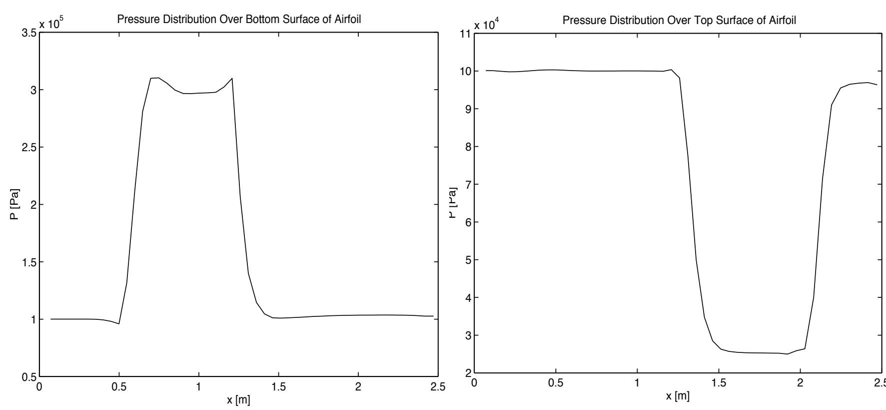

Figure 6: Pressure distribution over both the top and bottom surfaces of a diamond shaped airfoil.

In order to calculate the lift and drag forces, the pressures on the surface of the airfoil, output by the CFD code, were numerically integrated. These calculations resulted in:

$$L = 200.6 \ \ [KN/m]$$

$$D = 78.8 \ \ [KN/m] \ \ .$$

In order to calculated the non-dimensional force coefficients, C_l and C_d, the following equations are used:

$$C_l = \frac{L'}{\frac{1}{2}\rho u_\infty^2 c} \quad , \tag{6}$$

$$C_d = \frac{D'}{\frac{1}{2}\rho u_\infty^2 c} \quad . \tag{7}$$

where L' and D' are the lift and drag forces per unit span, u_∞ is the free stream velocity, and c is the chord length. The resulting values are

$$C_l = 0.4$$

$$C_d = 0.16 \quad .$$

With a span of only 1 m, this wing will easily carry the specified load.