

**Part I. Multiple Choice – 5 points each.**

1. If  $\log_3 x + \log_3(x - 2) = 1$ , then

- (a)  $x = 2$                       (b)  $x = -1$  or  $x = -3$                       (c)  $x = 3$   
(d)  $x = -1$                       (e)  $x = 1$  or  $x = 2$

2. If  $f(x) = \frac{1}{1+2^x}$  with domain  $x$  any real number, then the inverse of  $f(x)$  is

- (a)  $f^{-1}(x) = 1 + 2^x$       (b)  $f^{-1}(x) = \log_2\left(\frac{1}{x} - 1\right)$       (c)  $f^{-1}(x) = \frac{1}{1-2^x}$   
(d)  $f^{-1}(x) = \frac{1}{1-\log_2 x}$       (e)  $f(x)$  has no inverse on the given domain.

3. Let  $f(x)$  be a differentiable function with inverse  $f^{-1}(x)$ . Suppose that  $f(0) = 3$ ,  $f(1) = 0$ ,  $f(3) = -1$ ,  $f'(0) = 2$ ,  $f'(1) = 4$  and  $f'(3) = 1$ . Find  $(f^{-1})'(0)$ .

- (a)  $1/4$       (b)  $4$       (c)  $1/2$       (d)  $1$       (e)  $-1/2$

4. The value of a car depreciates over time, and at any given time, the rate of decrease in value is proportional to the value of the car. If after 2 years, the car is worth  $3/4$  its original value, what fraction of its original value will it be worth in 3 years?

- (a)  $3/8$       (b)  $9/16$       (c)  $9/32$       (d)  $9/8$       (e)  $3\sqrt{3}/8$

5. A lunar module weighs 45 tons on the surface of the earth. Determine the work done in propelling the module to a height of 500 miles above the surface of the earth. (Hint: The radius of the earth is 4000 miles.)

- (a) 20,000 mile-tons      (b) 126,000 mile-tons      (c) 5,000 mile-tons  
(d) 22,500 mile-tons      (e) 202,500 mile-tons

6. Evaluate the following integral

$$\int_3^4 \frac{x}{\sqrt{x^2 - 4}} dx$$

- (a)  $2\text{arcsec}4 - 2\text{arcsec}3$       (b)  $2\sqrt{3} - \sqrt{5}$       (c)  $\sqrt{2} - \sqrt{5}$   
(d)  $(1/2)(\text{arcsec}4 - \text{arcsec}3)$       (e)  $2 \tan 4 - 2 \tan 3$

7. Find the fluid force on the vertical side of a tank wall where the wall is a semicircle of radius 4ft with the flat side facing up. Assume that the tank is full of a liquid and that the weight density of the liquid is  $w$  pounds per cubic foot.

The force is on the tank wall is:

- (a)  $4w/3$  pounds      (b)  $16w$  pounds      (c)  $128w/3$  pounds  
(d)  $64w$  pounds      (e)  $16w/3$  pounds

8. Find the area of the region bounded by the curves  $y = x$ ,  $y = x \ln x$ , and  $x = 1$ .

- (a)  $e^2/2 - 1$       (b)  $9/4 - 2 \ln 2$       (c)  $(e^2 - 3)/4$       (d)  $3/4$       (e)  $\ln 2$

9. The derivative of the function  $f(x) = \arctan(\ln x)$  is:

- (a)  $\frac{\ln x}{1+x^2}$       (b)  $\arctan\left(\frac{1}{x}\right)$       (c)  $\frac{1}{x} \cdot \frac{1}{(\ln x)^2 + 1}$   
(d)  $\frac{1}{x+x^3}$       (e)  $\frac{1}{\sqrt{x - (\ln x)^2}}$

10. Evaluate:

$$\int \frac{1}{\sqrt{3-x^2+2x}} dx$$

- (a)  $\operatorname{arcsec}\left(\frac{x-1}{2}\right)$       (b)  $\arcsin\left(\frac{x-1}{2}\right)$       (c)  $\frac{1}{2} \arcsin(x-1)$   
(d)  $(x-1)\operatorname{arcsec}\left(\frac{x-1}{2}\right)$       (e)  $\ln(\sqrt{3-x^2+2x})$

11. Find the area of the plane region bounded by the graphs of the functions  $y = x^2$  and  $y = x + 2$ .

- (a)  $\frac{9}{2}$       (b)  $\frac{15}{2}$       (c) 3      (d) 12      (e)  $-\frac{15}{2}$

12. Let  $R$  be the plane region bounded by the graph of the function  $y = \frac{1}{x}$  and the lines  $x = 2, y = 3$ . Let  $S$  be the solid obtained by rotating  $R$  around the  $y$ -axis. Which of the following integrals represents the volume of  $S$  computed using the disc method?

Do not evaluate the integrals.

- (a)  $\pi \int_0^3 \left(2 - \frac{1}{y}\right)^2 dy$     (b)  $\pi \int_{1/3}^3 \left(9 - \frac{1}{y^2}\right) dy$     (c)  $\pi \int_{1/2}^2 \left(4 - \frac{1}{x^2}\right) dx$   
(d)  $\pi \int_0^{1/3} 9 dx + \pi \int_{1/3}^2 \frac{1}{x^2} dx$     (e)  $\pi \int_{1/2}^3 \left(4 - \frac{1}{y^2}\right) dy$

13. Find the coordinates  $(\bar{x}, \bar{y})$  of the center of mass for the lamina of uniform density  $\rho$  bounded by the graphs of the equations  $y = x^2$  and  $y = x$ .

(a)  $\left(\frac{1}{2}, \frac{2}{5}\right)$       (b)  $\left(\frac{1}{5}, \frac{1}{5}\right)$       (c)  $\left(\frac{1}{2}, \frac{1}{2}\right)$

(d)  $\left(\frac{1}{4}, \frac{1}{2}\right)$       (e)  $\left(\frac{2}{5}, \frac{2}{5}\right)$

14. Evaluate the following integral

$$\int_1^2 \frac{\ln x}{x^2} dx$$

(a)  $-\frac{1}{4} \ln 2 - \frac{7}{12}$       (b)  $-\frac{1}{4}(\ln 2 - 31)$       (c)  $\frac{8}{3} \ln 2 - \frac{7}{9}$

(d)  $\frac{1}{2}(1 - \ln 2)$       (e)  $\ln 2 + 1$

15. Find the area of the surface generated by revolving the region bounded by the curves  $y = \frac{x}{2}$ ,  $y = 0$  and  $x = 6$  about the  $x$ -axis.

- (a)  $9\sqrt{5}\pi$                       (b)  $3\pi$                       (c)  $\sqrt{5}$   
(d)  $\frac{3\pi}{5}$                               (e)  $\frac{1}{4}$

16. Find the arc length of the graph of the function  $f(x) = \frac{2}{3}x^{3/2} + 1$  on the interval  $[0, 3]$ .

- (a)  $\frac{14}{3}$                               (b)  $\frac{2\pi}{3}$                               (c)  $\frac{1}{3}$   
(d)  $2\sqrt{3}$                               (e)  $\sqrt{3}$

17. Use the shell method to find the volume of the solid generated by revolving the plane region bounded by the curves  $y = x + 1$ ,  $x = 0$  and  $y = 3$  about the  $y$ -axis.

- (a)  $\frac{23\pi}{3}$       (b)  $\frac{\pi}{6}$       (c)  $\frac{8\pi}{3}$       (d)  $2\pi$       (e)  $\frac{9\pi}{4}$

18. Find the indefinite integral:

$$\int \sin^2 x \cos^3 x \, dx.$$

The integral is equal to:

- (a)  $\frac{1}{4} \sin^4 x \cos^5 x + C$     (b)  $\frac{1}{5} \frac{(\sin x \cos x)^5}{\sin x} + C$     (c)  $\frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x + C$   
(d)  $\frac{1}{4} \sin^4 x - \frac{1}{3} \sin^6 x + \frac{1}{8} \sin^8 x + C$     (e)  $3 \cos^3 x \sin x - 4 \cos^4 x + C$

19. Find the indefinite integral:

$$\int \sec^2 x \tan^2 x \, dx.$$

The integral is equal to:

- (a)  $\frac{1}{3} \sec^3 x \tan^3 x + C$     (b)  $\sec^6 x - \sec^4 x + C$     (c)  $\frac{1}{3} \tan^3 x + C$   
(d)  $\frac{1}{3} \sec^3 x + C$     (e)  $\frac{1}{3} \sec^3 x - \frac{1}{3} \tan^3 x + C$

20. Find

$$\lim_{x \rightarrow \infty} \frac{x^3 - 1}{e^x + 1} :$$

- (a) 0    (b) 6    (c)  $\infty$     (d) -1    (e) 1

21. Find the indefinite integral:

$$\int \frac{x^2 + x + 2}{(x + 1)(x^2 + 1)} dx.$$

The integral is equal to:

- (a)  $\ln|x + 1| \ln|x^2 + 1| + C$       (b)  $(x + 1)^{-2} + (x^2 + 1)^{-2}$   
(c)  $\ln|x + 1| + \arctan(x^2 + 1) + C$       (d)  $\ln|x + 1| - \ln|x^2 + 1| + C$   
(e)  $\ln|x + 1| + \arctan x + C$

22. Find the power series centred at  $c = 0$  for  $f(x) = \frac{8}{4-x}$  and find the radius of convergence:

- (a)  $\sum_{n=0}^{\infty} \frac{4}{3^n} x^n, R = 3$       (b)  $\sum_{n=0}^{\infty} 8(x - 4)^n, R = \infty$   
(c)  $\sum_{n=0}^{\infty} \frac{2}{4^n} x^n, R = 4$       (d)  $\sum_{n=0}^{\infty} 2x^n, R = 1$   
(e)  $\sum_{n=0}^{\infty} 8(x - 3)^n, R = 3$

23. Find the limit

$$\lim_{x \rightarrow \infty} (1 + 3x)^{1/x}.$$

- (a) 0      (b) 1      (c)  $e$       (d)  $e^{1/2}$       (e)  $e^3$

24. Find whether the improper integral

$$\int_0^{\infty} e^{-x} dx$$

converges or diverges. If it converges, evaluate:

- (a) diverges      (b) converges, 1      (c) converges,  $-1$   
(d) converges, 0      (e) converges, 2.

25. Find whether the series:

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$$

converges or diverges. If it converges, evaluate:

- (a) 1      (b) the series diverges      (c)  $\frac{1}{4}$       (d) 2      (e)  $\frac{1}{2}$

26. Find the radius of convergence  $R$  of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!} x^n$$

- (a)  $R = 0$       (b)  $R = \infty$       (c)  $R = 1$       (d)  $R = 2$       (e)  $R = \frac{1}{2}$

27. Find the partial fraction decomposition of the rational function:

$$\frac{x^3 + 3x^2 + 2x + 5}{(x - 1)^2(x + 2)(x^2 + 1)}$$

(a)  $\frac{Ax+B}{(x-1)^2} + \frac{C}{x+2} + \frac{Dx+E}{x^2+1}$

(b)  $\frac{A}{(x-1)^2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+1}$

(c)  $\frac{Ax}{(x-1)^2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+1}$

(d)  $\frac{A}{x-1} + \frac{Bx+C}{(x-1)^2} + \frac{D}{x+2} + \frac{Ex+F}{x^2+1}$

(e)  $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} + \frac{Dx+E}{x^2+1}$

**Part II. Partial Credit Problems – 10 points each.**

1. Consider the following differential equation

$$\sqrt{x^2 - 9}y' = 5xy$$

with initial condition  $y = 2$  at  $x = 3$ . Solve for  $y$ .

2. Evaluate the integral

$$\int x3^x dx$$

3. Let  $R$  be the plane region bounded by the graphs of the functions  $y = x^2$  and  $y = x + 2$ . Find the volume of the solid obtained by rotating  $R$  around the  $x$ -axis.

4. Use the formula for the sum of a geometric series express  $.99\overline{43} = .9943434343\dots$  as a fraction.

5. Find whether each of the following series converge or diverge. Show the test used in each case.

(You do not need to evaluate the series, but show how the test works.)

$$\sum_{n=0}^{\infty} n^{\frac{1}{n+1}}$$

$$\sum_{n=0}^{\infty} \frac{2^n n^2}{5^n}$$