Preliminary Exam II – Math 119

1. Find $\frac{dy}{dx}$ by implicit differentiation given that

$$\cos(\pi y) - 3\sin(\pi x) = 1.$$

a. $\frac{-3\cos(\pi x)}{\sin(\pi y)}$ b. $\frac{-3\sin(\pi x)}{\cos(\pi y)}$ c. $1 + 3\sin(\pi x)$

d. $\frac{\sin(\pi y)}{3\cos(\pi x)}$ e. $\frac{\cos(\pi y)}{3\sin(\pi x)}$

2. An object is moving along the circle $x^2 + y^2 = 16$. If $\frac{dy}{dt} = 2$ when it reaches the point with coordinates $x = 3$ and $y = 1$, find $\frac{dx}{dt}$ at that point.

a. $\frac{-2}{3}$ b. $\frac{2}{3}$ c. $\frac{1}{3}$ d. $\frac{-1}{3}$ e. $\sqrt{3}$

3. Find the absolute extrema of the function

$$f(x) = x^3 - \frac{3}{2}x^2 + 5$$

on the closed interval $[-1, 2]$.

(a). max = 7, min = 5/2 (b). max = 7, min = 9/2 (c). max = 5, min = 9/2

(d). max = 5, min = 5/2 (e). max = 0, min = 1

4. Find all of the critical numbers of the function

$$f(x) = \frac{x^2 + 1}{x}.$$

a. $x = 0, 1, -1.$ b. There are no critical numbers. c. $x = 1, -1.$

d. $x = 0, 1.$ e. $x = 0.$

5. Compute $\frac{d^2y}{dx^2}$ for the curve $x^2 - y^2 = 25$.

(a). $\frac{1}{y} - \frac{x^2}{y^3}$ (b). $\frac{x}{y}$ (c). $\frac{1}{y} - \frac{x}{y^2}$ (d). $\frac{x - 2y}{y^2}$ (e). $\frac{x}{y} - \frac{1}{y^2}$

6. The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$. Air is escaping from a spherical balloon, and its volume is decreasing at a constant rate of 3 cm^3/sec. At what rate is the radius of the balloon changing when $r = 10$ cm?

(a). $\frac{-3}{400\pi}$ cm/sec (b). $\frac{4000\pi}{3}$ cm/sec (c). -400π cm/sec
(d). $\frac{1}{40\pi}$ cm/sec (e). 3 cm/sec

7. Let

$$f(x) = \frac{x^2}{4} + \sin(x)?$$

The point $(\pi/6, f(\pi/6))$ is:

(a). a point of inflection of f (b). a local minimum of f
(c). a local maximum of f (d). an absolute minmum of f
(e). an absolute maximum of f

8. Compute

$$\lim_{x\to\infty} \frac{1 + x^2 + 2x^{15}}{1 + x^{14} + 3x^{15}}$$

a. 2/3 b. 1 c. the limit doesn't exists d. 15 e. ∞

9. Suppose that the second derivative of the function $f(x)$ exists for all points in the interval $(0, 2)$, and that $f''(1) > 0$. Which of the following must be true?

I. If $f'(1) = 0$, then f has a local minimum at $x = 1$.

II. f' is an increasing function on the interval $(0, 2)$.

III. If $f''(-1) < 0$, the graph of f has a point of inflection between $x = -1$ and $x = 1$.

(a). I (b). I and II (c). II and III (d). III (e). I and III

10. The total costs per month to a certain company for making and storing x widgets is given by the function

$$C(x) = 4x + \frac{16,000}{x^2} \quad \text{dollars}$$

for $x \geq 1$. How many widgets should they manufacture to minimize the costs?

(a). 20 widgets per month (b). 40 widgets per month
(c). 1 widget per month (d). 16,000 widgets per month
(e). $\sqrt[3]{4,000}$ widgets per month

11. Suppose that the derivative of f exists for all numbers x. Assume that the only critical number of f is $x = 0$, and that $f'(1) = 1$. Which of the following **must** be true?

I. If $f'(-1) = -1$, then f has an absolute minimum at the point $x = 0$.
II. f is increasing on the interval $(0, \infty)$.
III. The graph of f has an inflection point at $(0, f(0))$.
IV. The graph of f is concave up on $(0, \infty)$.

(a). I and II (b). III and IV (c). II and III
(d). I and IV (e). II and IV

12. An object is dropped from a height of 1000 feet. The height of the object t seconds later is given by the function $h(t) = -16t^2+1000$ feet. Verify the mean value theorem by finding a time t between 0 and 3 seconds at which the instantaneous velocity of the object is equal to its average velocity during the first 3 seconds.

(a). 9/4 seconds (b). 1.5 seconds (c). 2 seconds
(d). 3/4 seconds (e). 9/2 seconds

Partial Credit.

13. a. Give a formula for the surface area of a cube whose edges have length x.
b. All edges of a cube are expanding at a constant rate of 7 cm per second. How fast is the surface area changing when each edge is 12 cm in length?

14. Find the slope of the tangent line to the curve

$$x^2 + xy + y^2 = 3$$

at the point $x = 1$, $y = 1$.

15. Consider the function

$$f(x) = x^4 - 2x^3 + 1$$

a. Find the critical numbers of f.
b. Find the intervals on which f is increasing or decreasing.
c. Find the points at which relative extrema of f occur.

16. Consider the function

$$g(x) = x^2 + \frac{1}{x}$$

a. Find $g''(x)$.
b. Find the intervals on which g is concave up or concave down.

17. An object is dropped from a height of 1000 feet. The height of the object t seconds later is given by the function $h(t) = -16t^2 + 1000$ feet. Verify the mean value theorem by finding a time t between 0 and 3 seconds at which the instantaneous velocity of the object is equal to its average velocity during the first 3 seconds.

(a). 9/4 seconds (b). 1.5 seconds (c). 2 seconds
(d). 3/4 seconds (e). 9/2 seconds

Partial Credit.

18. a. Give a formula for the surface area of a cube whose edges have length x.
b. All edges of a cube are expanding at a constant rate of 7 cm per second. How fast is the surface area changing when each edge is 12 cm in length?

19. Find the slope of the tangent line to the curve

$$x^2 + xy + y^2 = 3$$

at the point $x = 1$, $y = 1$.

20. Consider the function

$$f(x) = x^4 - 2x^3 + 1$$

a. Find the critical numbers of f.
b. Find the intervals on which f is increasing or decreasing.
c. Find the points at which relative extrema of f occur.

21. A man 6 feet tall is walks at a rate of 5 ft/sec toward a streetlight that is 30 feet high. When he is 10 feet from the base of the light,
(a) at what rate is the tip of his shadow moving?
(b) at what rate is the length of his shadow changing?