

Department of Mathematics
University of Notre Dame
Math 119, Calculus A
Fall Semester 2004

Name: _____

Instructor: _____

Exam 2

Thursday October 28th 2004

This Examination contains **14** problems, worth a total of **100** points, on **10** sheets of paper including the front cover. The first **10** problems (Part I) are multiple choice with no partial credit, and each is worth **6** points. You have **75** minutes to work on the exam. No books, notes, calculators or other aid of any kind is allowed. Two extra pages are supplied for rough work.

You must record here your answers to problems 1–10. Place an \times through your answer to each problem.

- | | |
|------------------------|-------------------------|
| 1. (a) (b) (c) (d) (e) | 6. (a) (b) (c) (d) (e) |
| 2. (a) (b) (c) (d) (e) | 7. (a) (b) (c) (d) (e) |
| 3. (a) (b) (c) (d) (e) | 8. (a) (b) (c) (d) (e) |
| 4. (a) (b) (c) (d) (e) | 9. (a) (b) (c) (d) (e) |
| 5. (a) (b) (c) (d) (e) | 10. (a) (b) (c) (d) (e) |

All work in this course is conducted under the Honor Code. Please sign the pledge:
“On my honor, I have neither given nor received unauthorized aid on this Exam.”

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| MC. | |
| 11. | |
| 12. | |
| 13. | |
| 14. | |
| Tot. | |

Part I: Multiple choice. There are 10 problems in this part, each worth 6 points. There is only one correct answer to each problem. **You must record your final answer on the title page.**

1. (6 pts) Find the limit

$$\lim_{x \rightarrow \infty} \frac{x^2 - 1}{\sqrt{2x^4 + 1}}.$$

- (a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 0 (d) ∞ (e) Does not exist

2. (6 pts) Find $\frac{dy}{dx}$

$$xy + y - x^2 = 1.$$

- (a) $\frac{2x - y}{x + 1}$ (b) $\frac{2x}{y - 1}$ (c) $2x - y$ (d) $2x$ (e) $\frac{2x - y + 1}{x}$

3. (6 pts) Find the limit.

$$\lim_{x \rightarrow \infty} \frac{2 \cos(x)}{x^2}.$$

- (a) 1 (b) $\frac{1}{2}$ (c) 0 (d) ∞ (e) Does not exist

4. (6 pts) The function

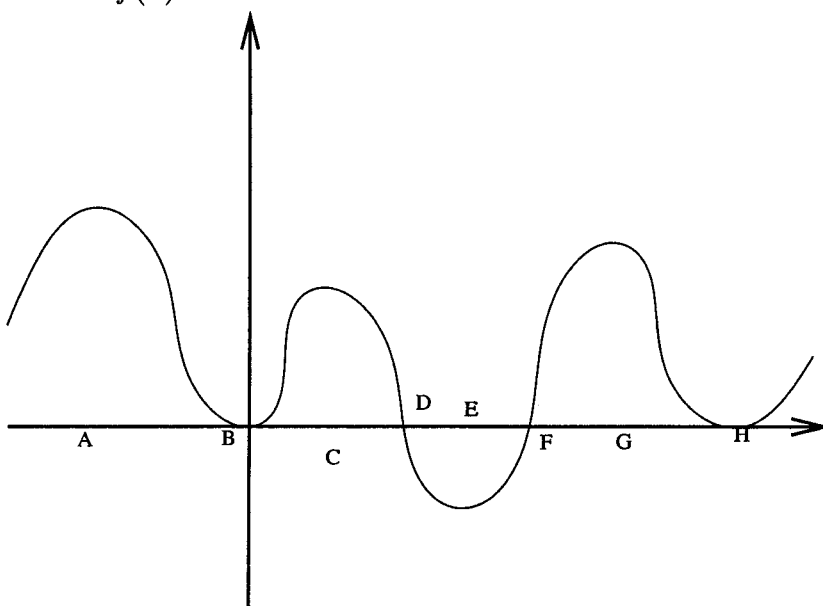
$$f(x) = \frac{x^3 + 2}{3x - x^3}$$

has:

- (a) One horizontal asymptote $y = \frac{1}{3}$.
(b) One vertical asymptote $x = 2$.
(c) One horizontal asymptote $y = -1$.
(d) Exactly two horizontal asymptotes $y = 1$ and $y = -1$.
(e) No horizontal asymptotes.

5. (6 pts) Let $f(x) = x^2 + 1$. On which interval can Rolle's theorem be applied to $f(x)$?
- (a) $(-1, 2)$ (b) $[0, 1]$ (c) $[-1, 1]$
 (d) $[0, \infty)$ (e) None of them

6. (6 pts) The graph in the diagram below is that of the **derivative of the function** $f(x)$. What are the x -coordinates of the relative minima of $f(x)$?



- (a) B, E, H (b) B, F (c) F
 (d) B, H (e) D

7. (6 pts) Find the derivative of $\tan^2 3x$.

- (a) $3 \sec^2 3x$ (b) $3 \tan 3x \sec^2 3x$ (c) $6 \tan 3x \sec 3x$
(d) $2 \sec^2 3x$ (e) $6 \tan 3x \sec^2 3x$

8. (6 pts) Consider the three functions given by

$$f(x) = x^3 - 3x, \quad g(x) = \sin x - x^2 + 7x \quad \text{and} \quad h(x) = -x^2 + 7x.$$

Which of these functions is concave up and increasing on $(2, \infty)$?

- (a) f only (b) g only (c) h only
(d) All of them (e) None of them

9. (6 pts) Find the absolute maximum and minimum values of $f(x) = \frac{|x-2|}{x^2}$ on $[1, 3]$.
- (a) there is no maximum or minimum.
 - (b) the min is 0, the max is 1.
 - (c) the min is 1, the max is 3.
 - (d) the max is 1, there is no minimum.
 - (e) the min is 1, the max is 2.

10. (6 pts) Let $x^2 - xy = 0$. Find $\frac{d^2y}{dx^2}$ at the point $(1, 1)$.
- (a) 1
 - (b) 0
 - (c) -2
 - (d) 2
 - (e) -1

Part II: Partial credit. There are 4 problems in this part, each worth 10 points. For a complete solution, you must show all your work.

11. (a) (4 pts) State the Mean Value Theorem:

(b) (6 pts) A plane leaves at 1pm on a 2,400 miles flight. The plane arrives at its destination at 7pm. Use the Mean Value Theorem to show that there was at least one time during the flight when the speed of the plane was 400 miles per hour.

12. (10 pts) The volume of a cube is expanding at a rate of $15 \text{ cm}^3/\text{sec}$. How fast is the surface area changing when the side is 2 cm?

13. (10 pts) Sketch the graph of a continuous functions f that has the following characteristics:

$f(x)$ is defined on the interval $(-\infty, 2)$:

(a) $\lim_{x \rightarrow -\infty} f(x) = 1.$

(b) $\lim_{x \rightarrow 0} f(x) = -3.$

(c) $\lim_{x \rightarrow 2^-} f(x) = \infty.$

(d) $f''(x) > 0$ for $x \geq 0.$

(e) $f'(0) = 0.$

14. Let $f(x) = 3x^4 - x^3 + 2$.

(a) (2 pts) Find $f'(x)$ and $f''(x)$.

(b) (2 pts) List all the open intervals on which the function is increasing. If there are none write NONE.

(c) (2 pts) List all the local maxima. If there are none write NONE.

(d) (2 pts) List all the open intervals on which the function is concave down. If there are none write NONE.

(e) (2 pts) List all the x -coordinates of the inflection points. If there are none write NONE.

