Math 10350 Fall 07 – Handout 1

1. Consider the function $f(x) = \dfrac{x^2 - 2x - 3}{x - 3}$.

a. What is the value of $f(x)$ at $x = 3$?

b. Complete the table below and discuss the behavior of $f(x)$ as x approaches 3

x	2.97	2.98	2.99	3	3.01	3.02	3.03
$f(x) = \dfrac{x^2 - 2x - 3}{x - 3}$				?			

c. Using algebra, make precise your observation in Part 1(b).
d. Give a sketch of the graph of $f(x)$. Use your graph to explain an important property about the limit of a function.

2. The graph of a function f is shown in Figure 1. By inspecting the graph, find each of the following limits if it exists. If the limit does not exist, explain why.

(i) $\lim_{x \to 4} f(x) \stackrel{?}{=}$

(ii) $\lim_{x \to -1} f(x) \stackrel{?}{=}$

(iii) $\lim_{x \to 2} f(x) \stackrel{?}{=}$

(iv) $\lim_{x \to 0} f(x) \stackrel{?}{=}$

(v) $\lim_{x \to 3} f(x) \stackrel{?}{=}$

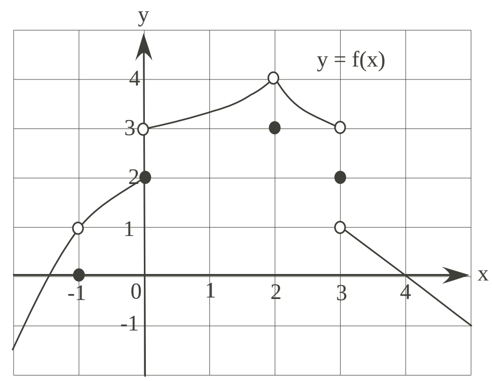

Figure 1

3. If the graph of $f(x)$ is given in Figure 1 and $g(x) = 3x + 2$, find the following limits using the properties of limits:

a. $\lim_{x \to 2} [2f(x) + 3g(x)] \stackrel{?}{=}$ **b.** $\lim_{x \to 2} [f(x) \cdot g(x)] \stackrel{?}{=}$ **c.** $\lim_{x \to 2} \dfrac{g(x)}{f(x) - 4} \stackrel{?}{=}$

d. $\lim_{x \to 2} \dfrac{g(x)}{f(x) + 4} \stackrel{?}{=}$ **e.** $\lim_{x \to 2} [f(x) - g(x)]^4 \stackrel{?}{=}$ **f.** $\lim_{x \to 2} \sqrt{f(x)} \stackrel{?}{=}$

4. Using geometry, explain why $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$. Hence deduce that $\lim_{x \to 0} \dfrac{1 - \cos x}{x} = 0$.

5. Determine the following limits (if it exists) using the properties of limits (i.e. limit laws) and simplifying the expression, if necessary.

a. $\lim_{x \to 2} \dfrac{2 - x}{x^2 - 4}$ **b.** $\lim_{x \to 0} \dfrac{\sqrt{2 + x} - \sqrt{2}}{x}$ **c.** $\lim_{x \to 4} \dfrac{x^2 - 9}{x - 3}$

d. $\lim_{x \to 0} \dfrac{\sin 5x}{6x}$ **e.** $\lim_{x \to 0} \dfrac{\sin 7x}{\sin 5x}$ **f.** $\lim_{x \to 0} \dfrac{|x|}{x}$

6a. Give the formal definition of the limit $\lim_{x \to c} f(x) = L$.
6b. Find the value of $L = \lim_{x \to 2} (x^2 - 3)$ then find $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - 2| < \delta$.

7. Using limits find the slope of the graph of $f(x) = x^2$ at $x = 2$. Draw pictures to illustrate your work.