Math 10360 Calculus B Name:____________________

Exam III Instructor:____________________

April 25, 2006 Section:____________________

Calculators or personal electronic devices of any kind (including cellphones, pagers, and iPods) are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 1 hour and 15 minutes to do the test. You may leave earlier if you are finished.

Part I consists of 10 multiple choice questions worth 5 points each worth a total of 50 points. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Part II consists of 4 partial credit problems worth a total of 40 points. Write your answer and show **all** your work on the page on which the question appears.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1.	a	b	c	d	e	6.	a	b	c	d	e
2.	a	b	c	d	e	7.	a	b	c	d	e
3.	a	b	c	d	e	8.	a	b	c	d	e
4.	a	b	c	d	e	9.	a	b	c	d	e
5.	a	b	c	d	e	10.	a	b	c	d	e

For grading use:

1-10	
13	
14	
15	
16	
+10 Total	

Part I: Multiple choice questions (5 points each)

1. Decide whether the following sequence converges, and if it does converge, find the limit:

$$a_n = (-.9)^n + \frac{n}{n+1}.$$

(a) The sequence diverges (b) 1 (c) 0.1 (d) 0 (e) 1.9

2. Evaluate

$$\lim_{x \to 0} (1+x)^{\frac{1}{\sin x}}$$

.

(a) Does not exist (b) 0 (c) 1 (d) e (e) $e^{\frac{1}{\sin x}}$

3. Evaluate

$$\lim_{x\to+\infty} \frac{\ln(1+3e^x)}{x}.$$

(a) 1 (b) $\ln 3$ (c) $\frac{1}{3}$ (d) 0 (e) Does not exist

4. Decide whether the following series converges, and if so, find the sum:

$$\sum_{n=1}^{\infty} \frac{2^n+3^n}{4^n}$$

.

(a) The series diverges (b) Converges to $\frac{5}{4}$ (c) Converges to 2

(d) Converges to 0 (e) Converges to 4

5. Evaluate

$$\lim_{x\to 0} \frac{x \sin x}{\cos x}$$

.

(a) -2 (b) 2 (c) 1 (d) 0 (e) Does not exist

6. Evaluate the indefinite integral

$$\int \frac{x+3}{x^2+3x+2}\, dx.$$

(a) $2\ln|x+1| - \ln|x+2| + C$

(b) $\ln|x+1| - \ln|x+3| + C$

(c) $-\ln|x+1| + 2\ln|x+2| + C$

(d) $\ln|x+3| - \ln|x+1| - \ln|x+2| + C$

(e) $\frac{1}{2}\ln|x^2+3x+2| + C$

7. Evaluate the improper integral

$$\int_2^{+\infty} \frac{dx}{4+x^2}$$

(a) The integral diverges (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{2}$ (d) $\frac{1}{2}$ (e) 1

8. Decide whether the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+2}$$

converges or diverges. If it converges, find its limit.

(a) The series diverges (b) 1 (c) -1 (d) 0 (e) $\ln 2$

9. Decide whether the series

$$\sum_{n=1}^{\infty} \left[\frac{(2n+1)(3n+1)}{5n^2-1}\right]^n$$

converges, and if so, find its limit.

(a) The series diverges (b) 5 (c) 2 (d) 3 (e) $\frac{6}{5}$

10. Evaluate the indefinite integral

$$\int \cos^3 x \sqrt{\sin x}\, dx.$$

.

(a) $\frac{2}{3}(\sin x)^{3/2} - \frac{2}{7}(\sin x)^{7/2} + C$ (b) $\frac{2}{3}(\sin x)^{3/2}\ (\cos x)^2 + C$

(c) $\frac{1}{4}(\cos x)^4 + C$ (d) $\arcsin(x/2) + \cos^2 x + C$

(e) $\sin x - \frac{1}{3}(\sin x)^3 + C$

Part II: Partial credit questions (10 points each). Show your work.

11. Evaluate the indefinite integral

$$\int \frac{2(x+1)}{x^3 - x^2 + x - 1}\, dx$$

by following the given instructions.

(i) Let $Q(x) = x^3 - x^2 + x - 1$. By inspection, find a root of Q.

Answer: $r =$

(ii) Factor Q as $Q(x) = (x - r)$(quadratic polynomial):

$Q(x) =$

(iii) Using the factorization in part (iii), write the integrand as a sum of partial fractions:

$$\frac{2(x+1)}{x^3 - x^2 + x - 1} =$$

(iv) Evaluate the integral:

$$\int \frac{2(x+1)}{x^3 - x^2 + x - 1}\, dx =$$

12. (i) Does the series

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^2+n}$$

converge or diverge? In your explanation, be sure to cite which convergence test(s) you are using, and why it is applicable.

(ii) Same question for the series

$$\sum_{n=1}^{\infty} \frac{1}{3^n+n^2}.$$

13. Does the sequence

$$b_n = \sum_{i=1}^{n} \frac{1}{1+i^2}$$

converge or diverge? **Explain.**

14. (i) Explain why

$$\int_1^3 \frac{dx}{\sqrt[3]{x-2}}$$

is an improper integral.

(ii) Determine whether the improper integral in (i) converges or diverges. If it converges, evaluate it.