Math 10360 Calculus B Name: ______________________

Exam II Instructor: ______________________

March 23, 2006 Section: ______________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 1 hour and 15 minutes to do the test. You may leave earlier if you are finished.

Part I consists of 12 multiple choice questions worth 5 points each worth a total of 60 points. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Part II consists of 4 partial credit problems worth a total of 40 points. Write your answer and show **all** your work on the page on which the question appears.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1. a b c d e
2. a b c d e
3. a b c d e
4. a b c d e
5. a b c d e
6. a b c d e
7. a b c d e
8. a b c d e
9. a b c d e
10. a b c d e
11. a b c d e
12. a b c d e

For grading use:

1-12	
13	
14	
15	
16	
Total	

Part I: Multiple choice questions (5 points each)

1. Which of the integrals below represents the area of a circle of radius 2 with center at the origin?

(a) $\pi \int_{-2}^{2} \sqrt{4-x^2}\,dx$ (b) $2\pi \int_{-2}^{2} \sqrt{4-x^2}\,dx$ (c) $\int_{-2}^{2} \sqrt{4-x^2}\,dx$

(d) $2 \int_{-2}^{2} \sqrt{4-x^2}\,dx$ (e) $2 \int_{-2}^{2} \sqrt{2-x^2}\,dx$

2. Let R be the region bounded by the graphs of the two curves $y = x^3$ and $y = x$. The are of the region R equals:

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) 0 (e) $\frac{1}{8}$

3. Planets are usually not a perfect sphere (ball) but an ellipsoid. An ellipsoid is obtained by rotation an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

around the x-axis. The volume, computed via the **disk method**, of the resulting ellipsoid is given by the integral:

(a) $\pi \frac{b^2}{a^2} \int_{-a}^{a} (a^2 - x^2)\, dy$

(b) $\pi \frac{b^2}{a^2} \int_{-a}^{a} (a^2 - x^2)\, dx$

(c) $\pi \frac{b^2}{a^2} \int_{-b}^{b} (a^2 - x^2)\, dx$

(d) $\pi \frac{b}{a} \int_{-b}^{b} \sqrt{a^2 - x^2}\, dx$

(e) $2\pi \frac{b}{a} \int_{-b}^{b} x\sqrt{a^2 - x^2}\, dx$

4. Let R be the region bounded by $y = \sqrt{x}$, the line $y = 16$ and the x-axis. Find the volume, using the **shell method**, of the solid obtained by revolving the region R about the y-axis. The volume V is given by the formula:

(a) $2\pi \int_0^{16} x^{3/2}\, dx$ (b) $2\pi \int_0^{16} x\, dx$ (c) $2\pi \int_0^{4} y^3\, dy$

(d) $2\pi \int_0^{16} y^2\, dy$ (e) $2\pi \int_0^{4} y^2\, dy$

5. Find the arc length of the graph of the function $y = \ln\cos x, 0 \le x \le \pi/4$. The arc length $\ell =$

(a) 0 (b) 1 (c) $\ln\sin\frac{\pi}{4}$ (d) $-\ln\sin\frac{\pi}{4}$ (e) $-\ln(\sec\frac{\pi}{4} + \tan\frac{\pi}{4})$

6. Find the **surface area** of the surface obtained by revolving the curve $y = 2\sqrt{x}, 0 \le x \le 3$ about the x-axis. The surface area $A =$

(a) $2\int_0^3 \sqrt{1 + \frac{1}{x}}\, dx$ (b) $2\int_0^3 \sqrt{1+x}\, dx$ (c) $\int_0^3 \sqrt{1+x}\, dx$

(d) 1 (e) $\int_0^3 \sqrt{1 + \frac{1}{x}}\, dx$

7. A 10-foot chain that weights 5 pounds per foot hanging from a winch 10 feet above ground level. Find the work done W by the winch in winding up the entire chain.

(a) 500 ft-lb (b) 2500 ft-lb (c) 5000 ft-lb (d) 7500 ft-lb
(e) 10000 ft-lb

8. Evaluate the definite integral

$$\int_0^1 \frac{e^x}{1+e^x}.$$

(a) $\ln \frac{1+e}{2}$ (b) $\ln 2$ (c) 1 (d) $e-1$ (e) $\ln(e-1)$

9. Evaluate the definite integral

$$\int_1^e \ln x dx$$

(a) $e-1$ (b) 0 (c) 1 (d) $\ln(e-1)$ (e) $e+1$

10. Evaluate the definite integral

$$\int_0^{\pi/2} e^x \cos x \, dx.$$

(a) $\frac{1}{2}(e^{\pi/2}-1)$ (b) $e^{\pi/2}-1$ (c) 0 (d) $\frac{1}{2}(e^{\pi/2}+1)$ (e) $e^{\pi/2}+1$

11. Evaluate the indefinite integral:

$$\int x\sqrt{x^2-1}\,dx.$$

(a) $\operatorname{arcsec} x + C$ (b) $\arcsin x + C$ (c) $\frac{1}{2}\ln(x^2-1)+C$ (d) $\sqrt{x^2-1}+C$
(e) $\frac{2}{3}(x^2-1)^{3/2}+C$

12. Find the fluid force on the rectangular plate in the diagram below. Assume that the plate is submerged vertically in a tank filled with water (density = 62.4 lb/cubic ft).

(a) (62.4)(5)(4)(11) lbs (b) (62.4)(7)(4)(11) lbs (c) (31.2)(7)(4)(11) lbs
(d) (31.2)(5)(4)(11) lbs (e) (62.4)(9)(4)(11) lbs.

Part II: Partial credit questions (10 points each). Show your work.

13. Evaluate the integral

$$\int_0^{\pi/2} x^2 \cos x \, dx.$$

14. Let R be the region bounded by $y = 3x^3, y = 0, x = 2$. Find the volume (via **disk method**) of the solid obtained by revolving the region R about the line $x = 2$.

(i) Sketch the region R (the region should be clearly indicated) together with the radius of rotation.

(ii) Indicate clearly on your sketch the radius of rotation. Express the radius of rotation r as a function.

(iii) Write down the integra representing the volume of the solid. The limit of integration and the variable (i.e. dx or dy) that you are integration should be clearly indicated.

(vi) Find the volume by evaluating the integral in (iii).

15. Let R be the region bounded by $y = \sqrt{r^2 - x^2}, y = 0$. Find the center of mass of the region R.

(i) Sketch the region R.

(ii) What is the area of the region R?

(iii) Find the center of mass of R.

16. Evaluate the integral:

$$\int \frac{1}{\sqrt{4x - x^2}}\, dx.$$

(i) Fill in the blank.

$$(2 - x)^2 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

(ii) Express $4x - x^2$ as $a^2 - u^2$ by completing the square.

(iii) Evaluate the integral via substitution by using (ii).

17. Let R be the right-angled triangle with vertices $(0,0),(h,0)$ and (h,r). Find the volume of the solid obtained by revolving the region R about the x-axis.

(i) Sketch the region.

(ii) Indicate clearly on the sketch the radius of rotation r and write down the integral representing the volume of the solid. The limits of integration and the variable of integration (i.e. dx or dy) should be clearly indicated.

(iii) Find the volume of the solid by evaluating the integral in (ii).