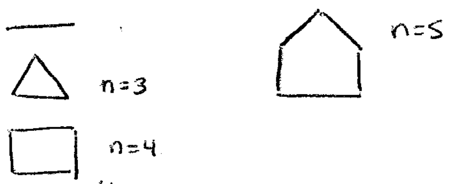
Meagan Pittuck

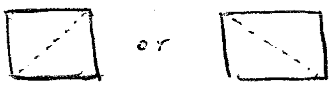
Catalan Number

Mon. 10/15/07

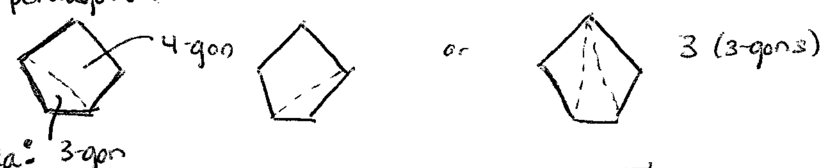
I) Suppose we draw a polygon → an n-gon



What is the number of ways we can triangularize the n-gon by non-intersecting diagonals? (useful in topology)

 = 1 way  = 2 ways

what about pentagons?



Recursive Formula:

$$\boxed{D_n = D_1 D_{n-1} + D_2 D_{n-2} + \cdots + D_{n-2} D_2 + D_{n-1} D_1}$$

remember Catalan recursive formula is $C_n = C_0 C_{n-1} + C_1 C_{n-2} + \cdots + C_{n-2} C_1 + C_{n-1} C_0$

it follows that $D_n = C_{n-1}$

* H.W. Problem #1 Check that the D_n formula is correct and is another way of defining the Catalan number. (Check pentagon and hexagon)

II) Multiplication Scheme (non-commutative)

suppose we have $a_1, a_2, \ldots, a_n$

If we just take $a_1 \longrightarrow (a_1)$ is the only option.

$a_1, a_2 \longrightarrow (a_1 \times a_2)$, $(a_2 \times a_1)$ [remember the a_i are noncommutative]

$a_1, a_2, a_3 \longrightarrow \{(a_1 \times (a_2 \times a_3)),\ ((a_1 \times a_2) \times a_3),\ ((a_1 \times a_3) \times a_2), \ldots\}$ ∃ 12 total

* H.W. Problem #2 Write down the multiplication scheme for ① a_1, a_2, a_3 and ② a_1, a_2, a_3, a_4.

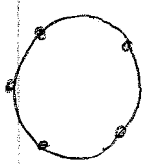
$(a_1(a_2a_3)a_4)$

multiplication number → $\boxed{m_n = n!\, C_{n-1}}$ $\quad \dfrac{m_n}{n!} = C_{n-1}$

(number of schemes)

* H.W. Problem #3 p. 317 #1, 2 (similar to above)

draw circle and place 2n dots on circle. Count number of ways of connecting these points with non-intersecting segments.



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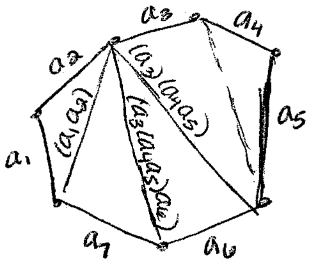
Suppose we have $a_1, a_2, a_3, \dots a_7$

Notation: dividing lines defined as in $\longrightarrow$



III [Stirling Numbers (Partition Numbers)]

1) $h_0, h_1, h_2, \dots h_n, \dots$ (all integers)

2) $\Delta h_0, \Delta h_1, \Delta h_2, \dots \Delta h_n \dots$ (difference between consecutive numbers)

$\searrow$ difference sequence i.e. $\Delta h_0 = h_1 - h_0$; $\Delta h_1 = h_2 - h_1$, ... $\Delta h_n = h_{n+1} - h_n$

3) $\Delta^2 h_0 \quad \Delta^2 h_1, \dots \Delta^2 h_n \rightarrow$ difference sequence of the difference sequence

$\Delta^2 h_0 = \Delta(\Delta h_0) \quad \Delta^2 h_1 = \Delta(\Delta h_1)$

Example $h_n = n^2$, 0-row $0, 1, 4, 9, 16, 25, \dots$

1-row $1, 3, 5, 7, 9 \dots = \Delta h_n$

2-row $2, 2, 2, 2, \dots = \Delta^2 h_n$

3-row $0, 0, 0, 0, \dots = \Delta^3 h_n$

Example $h_n = n^3 + 1$ 0-row: $1, 2, 9, 28, 65$.

1-row: $1, 7, 19, 37$ -

2-row: $6 \; 12 \; 18, \dots$

3-row: $6 \; 6, \dots$

4-row: $0, 0, \dots$

Thm: Let h_n be a polygon of degree m, then the $(m+1)^{th}$ row of the difference sequences consists only of zeros.

Proof By Induction $h_n = a_m n^m + a_{m-1} n^{m-1} + \dots$

1) deg $n=1$ $h_n - h_{n-1} = a_m n^m - a_m (n-1)^m +$ lower degree term

$= a_m n^m - a_m \left[n^m - \binom{m}{1} n^{m-1} + \dots \right] +$ lower degree term $\Big\}$

is a polynomial of degree at most $m-1$