Kumaraswamy Velupillai, **Computable Economics: the Arne Ryde Memorial Lecture Series,** Oxford: Oxford University Press, 2000, xiii, 222, index.

Professor Velupillai has been striving to get the economics profession to take the mathematics of formal computation theory more seriously for over a decade, so one looks forward with pleasure to a synthetic statement of his position occasioned by the Arne Ryde lectures. He has been one of the very few to realize that neoclassical economics has not so much driven by the smooth development of generic mathematical 'tools' as the historical fact it took a concerted turn immediately following WWII in what might characterize as a 'Bourbakist' direction, to the neglect of an alternative path marked out by recursive function theory and effectively calculable mathematics. As usual, when being reminded of the road not taken, the audience looks for a combination of historical narrative which tries to contextualize the wrong turning, along with some weighing up of the analytical pros and cons of the two paths. Unfortunately, perhaps due to the lecture format, that is not what we get here. Instead, what we are proffered is a list of precursors, the sketch of an argument that preference orderings are not generally Turing computable, a brief description of neural nets as a model of adaptive behavior, some observations on machine learning and induction, a series of reflections on Michael Rabin's 1957 paper on effective play of arithmetical games, and some disjoint observations on some scattered selected economic models. The threads that connect these chapters (beyond repeated appeal to computation) are frequently left for the reader to work out for herself. None of these are presented in sufficiently elementary form to serve as a primer for someone unfamiliar with the literatures cited, nor are proofs fully written out, although the book does come equipped with a brief explanation of Turing computability in an appendix.

The lectures certainly start out by asking the right questions: Is there a computability property distinctly characteristic of the economy *per se*? Should we treat all knowledge as algorithmic? What, if any, is the formal relationship between rationality and computation? Is there some special role for mechanical induction in the economy? But more often than not, Velupillai treats their answers as obvious to all right-thinking economists, even while admitting that much of his approach will be rejected out of hand by the orthodoxy. The incongruity of this veil drawn over what one by all rights would expect would be an impassioned expression of faith in the computational approach derives, I suspect, from a fundamental ambivalence which lies at the heart of the book: If indeed neoclassical theory is uncomputable 'all the way down' to the primitive concept of the preference ordering (a result dating back to Gerald Kramer and Alain Lewis), and back upwards through optimal choice, rational expectations, the calculation of equilibria, effective play of games (and so on), then what should be the role of the theory of computation in economic theory? Furthermore, what could the most major transformation in the neoclassical program in the 20th century – namely, the shift away from dependence upon static allocation models and towards treating the economic agent as information processor—portend, if it were predicated upon a severe misunderstanding of the imperatives of real computers? Some (and John von Neumann was among them) would respond: so much the worse for neoclassical theory; start over with the computer as the theoretical template for economic organizations such as the market. Herbert Simon (someone often misleadingly treated as precursor by Velupillai) would also say ditch the neoclassical model, but replace it with computer simulations of various modular algorithms characteristic of specialized human problem solving or simulations of organizational structures which resemble program flow-charts; fundamentally, he would reject the quest for a general architecture of the mind.

Velupillai does not even entertain these options; rather, possibly with a view to maintain peace with the orthodoxy, he opts for an entirely different principle: "I think it is imperative that we model the agent in its full complexity, i.e. as a Turing machine. It is the collective [of agents] that is simplistic and may fail to have the capacity of universal computation" (p.64). Although he nowhere states it directly, the argument seems to be that since the neoclassical orthodoxy persists in treating the agent in a sloppy and unrigorous way as a utility computer, why not go the whole nine yards and make the agent a formal abstract computer? The promissory note, never fully cashed out in the lectures, is that the orthodoxy might then be able to do much, if not all, of everything it already does, now admirably bound by the discipline of true Turing computability (e.g., p.100, 114, 179), just like Ulysses and the Sirens.

I think the reader may be forgiven if she wonders whether many of the formal results cited in the book do not so much support this position as undermine it. For instance, the non-computability of preference orderings might be circumvented by rendering the commodities and their power sets finite, but at the expense of losing many of the most vaunted results of postwar neoclassical theory, such as the existence proofs, the welfare theorems, and so on (see recent articles by Marcel Richter and Kam Chan Wong). If market pricing is rendered computationally effective, then one rapidly discovers that the conviction that there is a unique generic Walrasian market process looks very odd indeed. If one opts for the Kolmogorov definition of randomness, then one is brought up starkly against the consequence that the attendant probabilities may themselves be noncomputable. So it is not at all clear what parts of neoclassical economics, if anything, are left standing tall and proud after the computational critique. But there is a deeper problem with the lectures: Velupillai never justifies *why* we should commit to the model of the agent as Turing machine. This is a subject of protracted and heated controversy amongst philosophers (Searle's Chinese Room argument is an old chestnut in this literature); and economists have been notoriously fickle about commitment to any specific school of actual human psychology (the modern 'behavioral' movement included). Very few cognitive scientists are willing to subscribe to the principle that the Turing machine is an adequate model of the brain, which renders Velupillai's reference to neural networks all the more awkward.

Much of this goes to show that, when contemplating large-scale intellectual trends in the sciences like the impact of the computer on economics (just as on physics, or biology, or psychology), mathematics plays an important subsidiary role, but cannot explain *why* certain positions are adopted and other, beyond all logical imperative, are left to languish. Indeed, the pressure for a self-conscious computational economics will continue to be felt with increasing urgency by economists, but the forms their reactions will assume will not be a simple function of the mathematical theorems one finds being taught in departments of computer science. The question remains: Is there a computability property *uniquely characteristic* of the economy?