AME250: MEASUREMENTS
Technical Memo No. 00-1

Subject: Golf Ball Trajectory Analysis
Submitted by:
Performed with:
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Summary:

The golf ball trajectory analysis compared the predicted range to the actual measured range of a golf ball from a simple pendulum apparatus. The model created predicted a range considering the uncertainties in the experiment. The actual measured range from the pendulum apparatus exceeded the predicted range from the model. The range from the experiment measured 328 cm ±3 cm and the model predicted the range at 313 cm ± 4 cm. The fact that the angle the golf ball left the tee was assumed to be 0 in the model presented a discrepancy between the two ranges. Systematic errors in the pendulum apparatus also contributed to the two ranges not being consistent within errors.

Findings:

The model developed to predict the range the golf ball would travel consisted of a three part analyis. The first component of the model determined the initial speed the golf ball left the tee. The conservation of energy was used to find V_b, the speed of the block the instant before it hit the golf ball. The pendulum began with a potential energy equal to:

$$\pi_a = m_s g l + m_r g \frac{l_r}{2} \quad . \tag{1}$$

The pendulum system had no initial kinetic energy because it started at rest. However, at the bottom of the swing, the rod and block system had potential and kinetic Energy. Since the origin was set at the center of the pendulum, the final potential energy was equal to the negative of the initial potential energy. The final kinetic energy consisted of two components: the kinetic energy of the block and the kinetic energy of the rod. The formula for the final kinetic energy is shown below:

$$\tau_b = \frac{1}{2} m_s v_b^2 + \frac{1}{2} I_o \omega^2$$

To find V_b, the conservation of energy equation was used. Unfortunately, energy was not completely conserved in this system. Energy was lost due to friction in the bearing. The energy loss due to friction was equal to:

$$Friction \quad (1 - cos\theta)\pi_a \tag{3}$$

θ was found experimentally by an angle indicator attached to the bearing. The indicator measured the distance the pendulum came from reaching a full rotation. The angle found, in reality, equaled 2θ since only half a cycle was used in the experiment.

The initial potential energy, final potential and kinetic energies, and friction were placed in one equation to solve for V_b, the velocity of the block just before impact.

$$\pi_a = \pi_b + \tau_b + friction \tag{4}$$

The second part of the model consisted of the impact between the block and the golf ball. V_b, as solved for in equation 4, was the initial velocity of the block just before impact. Although energy was not completely conserved in this system, angular momentum was conserved. The velocity of the golf ball, V_g, was initially zero.

Just after impact the block developed a new velocity, V_b', and the golf ball gained a velocity, V_g'. Equation 6 was developed by setting the initial angular momentum of the block, rod, and golf ball equal to the final angular momentum of those components.

$$H_o^- = H_o^+ \tag{5}$$

$$m_s V_b l + I_o \omega = m_s V_b' l + I_o \omega' + m_g V_g' l \tag{6}$$

The equation for the coeffiecient of restitution was used in conjuction with equation 6 to solve for the velocity of the golf ball after impact, V_g'.

$$e = \frac{V_g' - V_b'}{V_b} \tag{7}$$

For the third component of the model, V_g' was used to predict a horizontal range, R, based on the following physics formulas for parabolic trajectory.

$$y \quad V_{o_y} t \quad \frac{1}{2} g t^2$$

$$R = V_{o_x} t$$

For the model, θ was estimated at 0 degrees. Therefore, V_g' was strictly in the horizontal direction. By subsitituting, V_g'=V_{o_x}, y=-b, and V_{o_y}=0, the predicted range the golf ball traveled was easily calculated.

MEASUREANDS	*SI*	*UNCERTAINTY*	*TECHNICAL ENGLISH*	*UNCERTAINTY*
Velocity of Block Before Impact	4.26 m/s	0.0085 m/s	14.0 ft/s	0.028 ft/s
Velocity of Block After Impact	3.66 m/s	0.0073 m/s	12.0 ft/s	0.024 ft/s
Velocity of Ball After Impact	7.49 m/s	0.011 m/s	24.6 ft/s	0.036 ft/s
Coefficient of Restitution	0.899	0.001	0.899	0.001
Range of Golf Ball	3.13 m	.0368 m	10.3 ft	.121 ft

Table 1: Measureand and Result Uncertainties(Calculations in Appendix B)

The predicted range (calculations and uncertainties found in Appendix A and B) equaled 313 cm ± 4 cm. The range found experimentally measured at 328 cm ± 3 cm (See Appendix C and Table for calculations and uncertainties).

Due to the fact that θ was assumed to be zero, the experimented range measured shorter than the predicted range. The golf ball in reality took off at an initial angle that was not accounted for in the predictions. An initial angle would increase the flight time, therefore increasing the distance traveled (See equation 9). The model did not account for this angle, so the predicted range came up short.

In the experiment, the golf ball did not take off in a perfectly straight line of motion. This fact altered the level of accuracy of the physics formulas for parabolic trajectory, equations 8 and 9, used to predict the range.

The experiment also had human errors. For example, the pendulum did not begin perfectly vertical and could have received a slight push from the experimenter's hand. This fact altered the accuracy of the conservation of energy equations. The golf ball was also not completely stationary on the tee. Movement between trials introduced error in the measured range.

The measured range and predicted range from the model were not consistant within errors. However, considering the plausible errors in performing both experiments, the two ranges were fairly comparable.

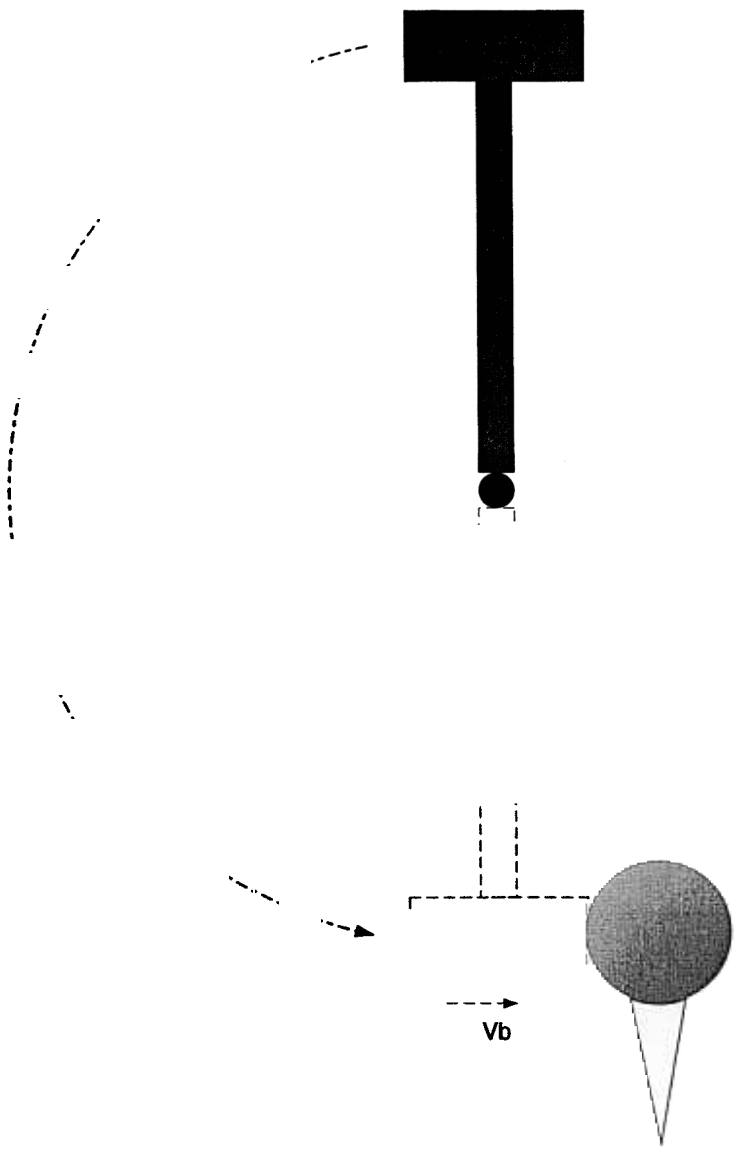

Figure 1: Pendulum Apparatus

MODEL INPUTS (Parameters)	*SI*	*UNCERTAINTY*	*TECHNICAL ENGLISH*	*UNCERTAINTY*
Mass of Block	.5288 kg	.0005 kg	.03623 slugs	.00003 slugs
Mass of Rod	.0898 kg	.0005 kg	.00615 slugs	.00003 slugs
Mass of Golf Ball	.04553 kg	.00001 kg	.003120 slugs	.000001 slugs
Length of Rod	.4660 m	.0005 m	1.529 slugs	.002 slugs
Length to Center of Block	.452 m	.001 m	.0310 slugs	0003 slugs
Height from Floor to Golf Ball	.86 m	.02 m	2.8 ft	.1 ft
Initial Height (for e calculation)	1.00 m	.001 m	3.28 ft	.003 ft
Final Height (for e calculation)	0.808 m	.001 m	2.65 ft	.003 ft
Theta	2.6 degrees	.25 degrees	2.6 degrees	.25 degrees

Table 2: SI and Technical English Units of Model Inputs

TRIAL #	DISTANCE (cm)
1	10.5
2	9.9
3	10.3
4	13.5
5	14.4
6	16.3
7	12.3
8	19.8
9	13.3
10	16.2
AVERAGE	13.7

Table 3: Average Measured Range

References:

1. Dunn, Patrick F., *Fundamentals of Measurements and Data Analysis*, Copyright January 2000(4th Edition)

2. Dunn, Patrick F., uncerthgt.m: Matlab m-file, Copyright 2000