AME 20213 - Measurements and Data Analysis

Spring Semester 2009 - Mid-Term Exam

NAME: KEY

This examination is 'closed-book'. Only one crib sheet and a calculator are allowed. You *must* turn in your crib sheet with your exam solutions. To receive credit on *any* problem, you must show all of your work in arriving at the answer.

Two possibly useful tables are attached to this exam: [1] a unit conversion table, and [2] Student's t-variable table.

DO NOT OPEN THIS EXAMINATION UNTIL YOU ARE TOLD TO DO SO.

This is a 70-minute exam. You will be given several 'last-minute warnings' during the last few minutes of the exam. Any exam not turned in on time will immediately loose 10 points as a late penalty.

This is Example 9.10 in the text.

Problem No. 1 (25 points): Determine the combined standard uncertainty in the density of air (in kg/m^3) assuming that air behaves as an ideal gas $p = \rho RT$. Assume negligible uncertainty in R ($R_{air} = 287.04$ J/kg·K). The nominal conditions are $T = 24$ °C and $p = 760$ mm Hg. The smallest divisions indicated on the thermometer and manometer are 1 K and 1 mm Hg, respectively. Express your final answer with the correct number of significant figures.

SOLUTION

The uncertainty in the density (a result) becomes

$$u_\rho = \sqrt{\left(\frac{\partial \rho}{\partial T} u_T\right)^2 + \left(\frac{\partial \rho}{\partial P} u_P\right)^2}$$
$$= \sqrt{\left(\frac{-P}{RT^2} u_T\right)^2 + \left(\frac{1}{RT} u_P\right)^2},$$

where

$$u_P = \frac{1}{2}(1 \text{ mm Hg}) = \frac{1}{2}\left(\frac{1.01 \times 10^5 \text{ Pa}}{760 \text{ mm Hg}} \times 1 \text{ mm Hg}\right)$$
$$= \frac{1}{2}(133 \text{ Pa}) = 67 \text{ Pa},$$

and

$$u_T = 0.5(1\,°\text{C}) = 0.5(1 \text{ K}) = 0.5 \text{ K}.$$

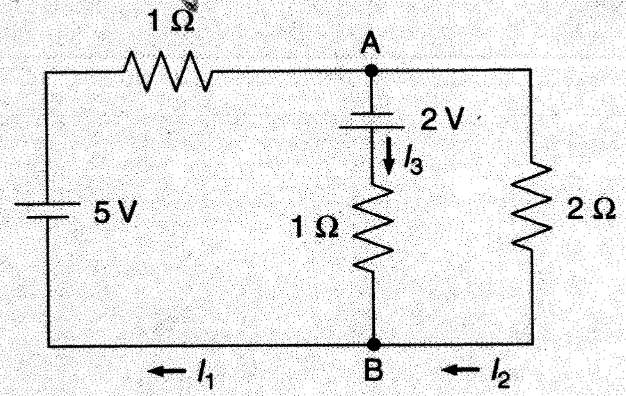
Thus,

$$u_\rho = \left\{\left[\frac{101\,325}{(287.04)(297)^2}(0.5)\right]^2 + \left[\frac{1}{(287.04)(297)}(67)\right]^2\right\}^{1/2}$$
$$= \left(4.00 \times 10^{-6} + 0.62 \times 10^{-6}\right)^{1/2},$$
$$= \boxed{2.15 \times 10^{-3} \text{ kg/m}^3.}$$

Because the stated 'smallest divisions' each have only ONE significant figure, the final answer would be

$$u_\rho = 2 \text{ g/m}^3$$

This is Example 4.3 in the text. See the 2nd Printing Errata for corrections.



Problem No. 2 (25 points): For the electrical circuit shown in the figure above, determine [a] the magnitude of the current, I_3, (in amperes) in the branch between nodes A and B, and [b] the direction of that current (either from A to B or from B to A). Indicate on the figure below any loops or nodes that you consider when applying Kirchoff's laws and clearly note the equations corresponding to these loops or nodes.

Left Loop (going clockwise):

① $5V - I_1 \cdot 1\Omega + 2V - I_3 \cdot 1\Omega = 0$

Right Loop (going clockwise):

② $-I_2 \cdot 2\Omega + I_3 \cdot 1\Omega - 2V = 0$

Node A OR B: $I_1 - I_2 - I_3 = 0 \Rightarrow I_1 = I_2 + I_3$ ③

③ into ①: $7 - I_2 - I_3 - I_3 = 0$

$\Rightarrow I_2 + 2I_3 = 7$

② $\Rightarrow -I_2 + \frac{1}{2} I_3 = 1$

addition $\Rightarrow 2.5 I_3 = 8 \Rightarrow I_3 = 3.2\text{ A}$

$(I_2 = 0.6\text{A}, I_1 = 3.8\text{ A})$

Because I_3 is positive, the I_3 direction is as drawn, that is, from node A to node B.

This is Example 9.6 in the text.

Problem No. 3 (25 points): An analog-to-digital (A/D) converter with the manufacturer's specifications listed below is to be used in an environment in which the A/D converter's temperature may change by $\pm 10\ °C$. Determine the converter's [a] quantization error (in mV), [b] instrument error (in mV), [c] combined standard uncertainty (in mV), and [d] overall uncertainty in the converted voltage (in mV) assuming 95 % confidence and using the 'large-scale approximation' for the coverage factor.

E_{FSR}	0 V to 10 V
M	12 bits
Linearity	± 3 bits/E_{FSR}
Temperature drift	1 bit/5 °C

SOLUTION

The instrument uncertainty is the combination of uncertainty due to quantization errors, e_Q, and to conversion errors, e_c,

$$(u_I)_E = \sqrt{e_Q^2 + e_I^2}.$$

The resolution of a 12-bit A/D converter with a full scale range of 0 V to 10 V is given by (see Chapter 6)

$$Q = \frac{E_{\text{FSR}}}{2^{12}} = \frac{10}{4096} = 2.4 \text{ mV/bit}.$$

The quantization error per bit is found to be

$$e_Q = 0.5Q = \boxed{1.2 \text{ mV.}}$$

The conversion error is affected by two elements:

$$\begin{aligned} \text{linearity error} = e_1 &= 3 \text{ bits} \times 2.4 \text{ mV/bit} \\ &= 7.2 \text{ mV} \\ \text{temperature error} = e_2 &= \frac{1 \text{ bit}}{5\ °\text{C}} \times 10\ °\text{C} \times 2.4 \text{ mV/bit} \\ &= 4.8 \text{ mV}. \end{aligned}$$

Thus, an estimate of the conversion error is

$$\begin{aligned} e_I &= \sqrt{e_1^2 + e_2^2} \\ &= \sqrt{(7.2 \text{ mV})^2 + (4.8 \text{ mV})^2} = \boxed{8.6 \text{ mV.}} \end{aligned}$$

The combined standard uncertainty in the digital representation of the analog value due to the quantization and conversion errors becomes

$$\begin{aligned} (u_I)_E &= \sqrt{(1.2 \text{ mV})^2 + (8.6 \text{ mV})^2} \\ &= \boxed{8.7 \text{ mV.}} \end{aligned}$$

$$[d]\ U_x = k u_x = (2)(8.7) = \boxed{17.4 \text{ mV}}$$

'large-scale approximation'

Problem No. 4 (25 points): [a] Prove that a least-squares linear regression fit of (x, y) data always goes through the point $(\bar{x}, \bar{y})$, where $\bar{x}$ and $\bar{y}$ are the mean values of the x and y data, respectively. [b] Develop the expressions (but do not solve them using the data) for the linearly intrinsic variables involving x, y, a, and b such that a least-squares linear regression analysis could be used to 'fit' the (x, y) data pairs (0,0.2),(1,1.3),(2,4.8), and (3,10.7) to the non-linear expression $y = axe^x + b$, where a and b are the 'best-fit' constants. Finally, [c] determine the value of b.

[a] see pp. 322 - 323

$$\frac{\partial D}{\partial a_0} = 0 = \frac{\partial}{\partial a_0}\left\{\sum\left[y_i - (a_0 + a_1 x_i)\right]^2\right\} \qquad \sum \text{ is } \sum_{i=1}^{N}$$

$$= -2\sum(y_i - a_0 - a_1 x_i)$$

$$\Rightarrow \sum y_i = a_0 N + a_1 \sum x_i$$

$$\Rightarrow \frac{1}{N}\sum y_i = a_0 + a_1 \frac{1}{N}\sum x_i \quad \text{OR} \quad \bar{y} = a_0 + a_1 \bar{x}$$

So, this first normal equation of the best fit contains (i.e., must go through) the point $(\bar{x}, \bar{y})$.

[b] $y = axe^x + b \Rightarrow \ln(y-b) = \ln a + \ln x + x$

let $\boxed{\ln(y-b) = y^*}$ and $\boxed{\ln x + x = x^*}$

$\Rightarrow y^* = \ln a + x^*$, which is linear

[c] when $x = 0$, $y = b$

Using the data pair $(0, 0.2)$ implies $\boxed{b = 0.2}$