# 56th Midwest PDE Seminar – Short Talks

**1. Marian Bocea,** University of Utah: *"Partial Differential Equations Related to Dielectric Breakdown and Polycrystal Plasticity"*

**Abstract:** We discuss several examples of PDEs and systems of PDEs arising from a Gamma-convergence analysis for a general class of "power law" functionals in the context of dielectric breakdown and Polycrystal Plasticity. One such example is the anisotropic version of the infinity Laplace equation. Joint work with Vincenzo Nesi (University of Rome).

**2. Jun Chen,** University of Wisconsin at Madison: *"Transonic Flows in a 2-D nozzle governed by Full Euler Equations"*

**Abstract:** We study small perturbations from a constant flow from supersonic to subsonic with a straight shock in between. Given a perturbed supersonic flow at upstream, we find the subsonic flow at downstream, as well as a shock between the two flows. The shock and the subsonic flow are also small perturbations from the constant states. We also have the uniqueness of the solutions.

**3. Huseyin Coskun,** University of Iowa: *"An inverse problem formulation for ameboid cell movements"*

**Abstract:** In this article a 1-D mathematical model for ameboid cell movements using linear viscoelastic fluid dynamics with free boundary formulation is introduced. The model is a system of six nonlinear parabolic, hyperbolic, and elliptic partial differential equations with a free boundary formulation. Based on the model, the inverse problem can be posed: depending on the constitutive relations and governing equations, what kind of characteristic properties must the model parameters and unknowns have in order to reproduce a given movement of the cell, provided that the velocity field at any point is given? Primarily, this inverse problem is formulated and discussed in this paper. The inverse problem provides the model parameters that give some insight, principally into the mechanical aspect, but also, through qualitative reasoning, into chemical and biophysical aspects of the cell. Some numerical analysis and results of the inverse problem is also discussed.

**4. Dan-Andrei Geba,** MSRI and University of California, Berkeley: *"New results in nonlinear wave equations"*

**Abstract:** These results are joint work with Professor Daniel Tataru. For hyperbolic operators with rough coefficients, we prove dispersive estimates and sharp local well-posedness for the corresponding semilinear problem in 4+1 dimensions.

**5. Peter Hinow,** Vanderbilt University: *"Tumor suppressor p53's DNA binding obeys reaction-diffusion kinetics"*

**Abstract:** The tumor suppressor protein p53 plays a key role in guarding genomic stability of mammalian cells and preventing malignant transformation. In this study we investigated the intracellular diffusion of a p53–GFP fusion protein using confocal fluorescence recovery after photobleaching (FRAP). We show that the diffusion of p53-GFP within the nucleus is well described by

a mathematical model for diffusion of particles that bind temporarily to a spatially homogeneous immobile structure with binding and release rates $k_1$ and $k_2$, respectively. These data are consistent with a model in which oligomeric p53 continuously scans DNA under steady state conditions. The emphasis in this talk will be on the close interplay between mathematical modeling, statistical analysis and designing photobleaching experiments. This is joint work with Carl Rogers, Christopher Barbieri, Jennifer A. Pietenpol, Anne K. Kenworthy, and Emmanuele DiBenedetto.

**6. Gautam Iyer**, University of Chicago: *"A stochastic Lagrangian representation of the 3-dimensional Navier-Stokes equations"*

**Abstract:** In this talk I will derive a representation for the 3-dimensional Navier-Stokes equations based on stochastic Lagrangian paths. The particle trajectories obey a SDE with drift $u$ (the velocity field of the fluid) and diffusion coefficient $\sqrt{2\nu}$. We use the inviscid Webber formula to recover the velocity from the stochastic flow map. This method admits a self contained local existence proof, and can be extended to formulate representations of semi-linear reaction diffusion equations and related hydrodynamic-type equations, including the viscous Burgers and Camassa-Holm equations.

**7. Katarina Jegdic,** University of Houston: *"Analysis of transonic regular reflection for the nonlinear wave system"*

**Abstract:** We consider a Riemann problem for the nonlinear wave system which gives rise to regular shock reflection with the subsonic state immediately behind the reflected shock. When written in self-similar coordinates, the system changes type - it is hyperbolic away from the origin and of mixed type (hyperbolic-elliptic) in a region near the origin. Using the hyperbolic techniques we resolve the wave interactions in the supersonic region and obtain a free boundary problem for the subsonic solution and the position of the reflected shock. We show existence of solution in a neighborhood of the reflection point using Schauder estimates and fixed point theory.

**8. Stephen Pankavich,** Indiana University: *"The Vlasov Poisson System with Infinite Mass and Energy"*

**Abstract:** The existence of a unique, global in time, classical solution to the Vlasov-Poisson system with fixed background is shown. As opposed to the traditional Vlasov-Poisson problem, the total charge and energy are infinite, and we must consider solutions which tend to the fixed background rather than zero, as $x$ →∞. Thus, energy conservation (which is an essential component of global existence for the traditional problem) is unavailable. Instead, a previously known conserved quantity related to the energy, has been shown to be bounded, and is of similar use. The global existence proof combines energy and decay estimates, and a crucial argument which bounds the velocity support of the number density function.

**9. Misha Perepelitsa,** Nothwestern University: "The existence of weak, small energy solutions for the equations of motion of 3D compressible, viscous fluid flows with the no-slip boundary conditions"

**Abstract:** We consider the equations of motion of a compressible, viscous, isentropic fluid in a bounded domain of $R^3$ or $R^2$ with the no-slip boundary conditions. Given a constant, equilibrium state we construct a global in time, regular weak solution, provided that initial data are close to the equilibrium when measured by weak norms and discontinuities in the initial density decay near the boundary.

**10. Roman Shvydkoy,** University of Illinois at Chicago: *"Inherent instability of fluid flows"*

**Abstract:** The inherent instability conjecture states that any 3D fluid flow, except the isochronous flow, is unstable with respect to the enstrophy norm. We give a partial answer to this conjecture by showing that any non-isochronous flow is at least linearly unstable in the Sobolev space $H^s$ for velocities, with $s > 1/2$.

**11. Feride Tiglay,** University of New Orleans: *"The periodic Cauchy problem of an Euler-Poisson equation"*

**Abstract:** We discuss the well-posedness of the periodic Cauchy problem for a bihamiltonian system of equations.

**12. Jesenko Vukadinovic,** University of Chicago: *"The Doi Model for Liquid Crystalline Polymers"*

**Abstract:** The Doi model for liquid crystalline polymers has been successful in predicting many interesting rheological features of these materials. Despite an abundance of experimental, numerical and semi-analytical studies, rigorous mathematical results remain scarce. In this short talk, I will give a review of the model and recent rigorous mathematical results concerning the isotropic-nematic phase transition.

**13. Chunshan Zhao,** The University of Iowa: *"Locating the peak(s) of least-energy solutions to a quasilinear elliptic Neumann problem"*

**Abstract:** We will study the shape of least-energy solutions to the quasilinear problem $\varepsilon^m \Delta_m u - u^m + f(u) = 0$ with homogeneous Neuman boundary condition. We use an intrinsic variational method to show that as $\varepsilon$ goes to zero, the point where least-energy solutions achieves maximum goes to a point where the mean curvature of the boundary achieves its maximum. We also give a complete proof of exponential decay of least energy solutions. Even for the case $m = 2$ our proof is an extension of earlier ones in that the non-degeneracy of the ground state is not required in our work.