Stability and Performance of Model-Based Networked Control Systems with Intermittent Feedback

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Abstract: In this paper, we apply the concept of Intermittent Feedback to a class of networked control systems known as Model-Based Networked Control Systems (MB-NCS). We begin by introducing the basic architecture for model-based control with intermittent feedback, then address the case with output feedback (through the use of a state observer), providing a full description of the state response of the system, as well as a necessary and sufficient condition for stability in each case. Extensions of our results to cases with nonlinear plants are also presented. We then shift our focus to performance and optimal control issues; in particular, as an initial benchmark, we focus on controller design for an LQ-based performance measure for a discrete-time MB-NCS. Finally, we propose future research directions.

1. INTRODUCTION

A networked control system (NCS) is a control system in which a data network is used as feedback media. NCS is an important area in control, see for example Nair [2000], Took [2002], and Walsh [1999]. The use of networks as media to interconnect the different components of an industrial system is rapidly increasing. However, the use of NCSs poses some challenges. One of the main problems to be addressed when considering an NCS is the size of the bandwidth required by each subsystem. Clearly, the bandwidth required by the communication network is a major concern. Recently, modeling, analysis and control of networked control systems with limited communication capability has emerged as a topic of significant interest to control community, see for example Wong [1999], Brockett [2000], Elia [2001], Zhang [2001], Ishii [2002], and recent special issue Antsaklis [2004]. An efficient way to address this is reducing the rate at which packets are transmitted.

A particular class of NCSs is model-based networked control systems (MB-NCS), introduced by Montestruque [2002]. The MB-NCS architecture makes explicit use of the knowledge of the plant dynamics to enhance the performance of the system, and it is an efficient way to address the issue of reducing packet rate. In this paper we extend the work done in MB-NCS by taking advantage of intermittent feedback. In the previous work done in MB-NCS, the state updates given to the model of the plant were for a time instant only, but with intermittent feedback the system remains in closed loop control for more extended intervals. This notion makes sense as it is motivated by human motor control observation. For example, when driving a car, when approaching a curve or hilly terrain, we pay attention to the road for a longer time, which is equivalent to staying in closed-loop mode, and we only reduce our attention -switch to open loop control- when the road is once again straight. While intermittent control is a very intuitive notion, its combination with the MB-NCS architecture allows for obtaining important results and opening new paths in controlling NCSs effectively.

With the finite bit-rate constraints, quantization has to be taken into consideration in NCSs. Therefore, quantization and limited bit rate issues have attracted many researchers' attention with the aim to identify the minimum bit rate required to stabilize a NCS, see for example Delchamps [1990], Brockett [2000], Elia [2001], Tatikonda [2004], Nair [2000]. In Delchamps [1990] it is shown that asymptotic stability cannot be achieved by (static) quantization. Recently, in Brockett [2000] Brockett and Liberzon proposed a dynamic quantization scheme, so called "zoom-in, zoom-out" approach, to asymptotically stabilize linear systems. The idea behind the "zoom-in, zoom-out" scheme is to provide more detailed information when the state come closer to the origin through finer quantization (zoom-in), while only coarser quantization (zoom-out) is sufficient for states farther away from the origin. As an interesting observation of a person's response in face of changing environment, one usually tends to pay longer attention to objects of concern, instead of paying closer attention. This motivates us to use intermittent feedback in NCSs.

The rest of the paper is organized as follows. In Section 2, we describe the problem formulation in detail. In Section 3, we derive the complete description of the output of such a system. In Section 4, we present a necessary and sufficient condition for the stability of the system. An example is provided in Section 5. Finally, in Section 6, we provide conclusions and propose future work.

2. MB-NCS WITH INTERMITTENT FEEDBACK: SETUP AND FORMULATION

Let us start by introducing model-based control with intermittent feedback, in its simplest setup. The problem formulation is as follows.

The basic setup for MB-NCS with intermittent feedback is essentially the same as that proposed in the literature for traditional MB-NCS. Please see references Montestruque [2002] for more results on MB-NCS.

Consider the control of a continuous linear plant where the state sensor is connected to a linear controller/actuator via a network. In this case, the controller uses an explicit model of the plant that approximates the plant dynamics and makes possible the stabilization of the plant even under slow network conditions.

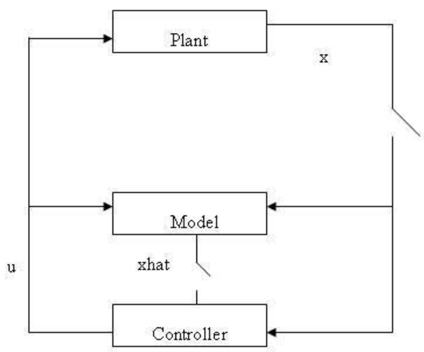

Fig. 1. MB-NCS with intermittent feedback - basic architecture

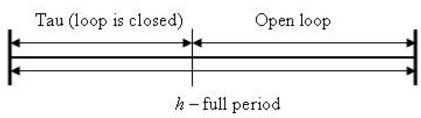

Fig. 2. Partition of the time interval into close and open loop intervals

The main idea here is to perform the feedback by updating the model's state using the actual state of the plant that is provided by the sensor. The rest of the time the control actions is based on a plant model that is incorporated in the controller/actuator and is running open loop for a period of h seconds.

As mentioned before, the main difference between model-based networked control systems as have been studied previously, and the case with intermittent feedback, which we are here introducing, is that in the literature, the loop is closed instantaneously, and the rest of the time the system is running open loop. Here, we part from the same basic idea, but the loop will remain closed for intervals of time which are different from zero. Intuitively, we should be able to achieve much better results the longer the loop is closed, as the level of degradation of the information increases the longer the system is running open loop, so intermittent feedback should yield better results than those for traditional MB-NCS.

In dealing with intermittent feedback, we have two key time parameters: how frequently we want to close the loop, which we shall denote by h, and how long we wish the loop to remain closed, which we shall denote by τ. Naturally, in the more general cases both h and τ can be time-varying. For the purposes of this paper, however, we will deal only with the case where both h and τ are fixed.

We consider then a system such that the loop is closed periodically, every h seconds, and where each time the loop is closed, it remains so for a time of τ seconds. The loop is closed at times t_k, for $k = 1, 2, ...$ Thus, there are two very clear modes of operation: closed loop and open loop. The system will be operating in closed loop mode for the intervals $[t_k, t_k + \tau)$ and in open loop for the intervals $[t_k + \tau, t_{k+1})$. When the loop is closed, the control decision is based directly on the information of the state of the plant, but we will keep track of the error nonetheless.

The plant is given by $\dot{x} = Ax + Bu$, the plant model by $\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u$, and the controller by $u = K\hat{x}$. The state error is defined as $e = x - \hat{x}$ and represents the difference between plant state and the model state. The modeling error matrices $\tilde{A} = A - \hat{A}$ and $\tilde{B} = B - \hat{B}$ represent the plant and the model. We also define the vector $z = [x\ e]^T$.

We derived the full state response of the system and a necessary and sufficient condition for stability in Estrada [2006]. For completeness, we summarize the results here.

Proposition 1. The system described above with initial conditions $z(t_0) = \begin{bmatrix} x(t_0) \\ 0 \end{bmatrix} = z_0$ has the following response:

$$z(t) = \begin{cases} e^{\Lambda_c(t-t_k)} \left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^k z_0 \\ \qquad \text{for } t \in [t_k, t_k + \tau) \\ e^{\Lambda_o(t-(t_k+\tau))} e^{\Lambda_c(\tau)} \left(\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \right)^k z_0 \\ \qquad \text{for } t \in [t_k + \tau, t_{k+1}) \end{cases} \tag{1}$$

where

$$\Sigma = e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)},$$

$$\Lambda_o = \begin{bmatrix} A + BK & -BK \\ \tilde{A} + \tilde{B}K & \hat{A} - \tilde{B}K \end{bmatrix},$$

$$\Lambda_c = \begin{bmatrix} A + BK & -BK \\ 0 & 0 \end{bmatrix},$$

and $t_{k+1} - t_k = h$.

Theorem 2. The system described above is globally exponentially stable around the origin if and only if the eigenvalues of $\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ are strictly inside the unit circle, where $\Sigma = e^{\Lambda_o(h-\tau)} e^{\Lambda_c(\tau)}$.

While this theorem is restricted to the case where the time parameters remain constant and full information of the state is known, we believe it is a very valuable first step in understanding more general situations. As we will see in the next section, the case with state observers is dealt with in a very similar fashion.

3. OUTPUT FEEDBACK CASE (STATE OBSERVER)

When the state of the plant is not directly measurable, we must resort to a state observer. In this section we derive results for this situation.

As in the architecture used in Montestruque [2002] for instantaneous model-based feedback, we assume that the state observer is collocated with the sensor. We use the plant model to design the state observer. See Figure 3. Our configuration is based on the analogous setup for model-based control with output feedback, proposed by Montestruque.

In Montestruque [2002] it is justified that the sensor carry the computational load of an observer by the fact that, typically, sensors that can be connected to a network have an embedded processor (usually in charge of performing the sampling, filtering, etc.) inside.

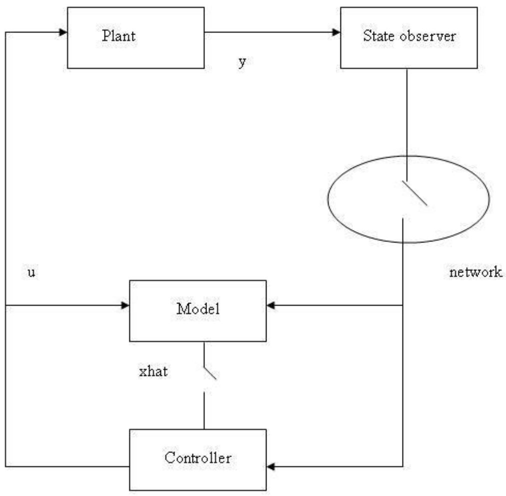

Fig. 3. MB-NCS with intermittent feedback - state observer

The observer has the form of a standard state observer with gain L. It makes use of the plant model.

In summary, the system equations are the following:

Plant: $\dot{x} = Ax + Bu, \; y = Cx + Du$

Model: $\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u, \; y = \hat{C}x + \hat{D}u$

Controller: $u = K\hat{x}$

Observer: $\dot{\bar{x}} = (\hat{A} - L\hat{C})\bar{x} + \left[\hat{B} - L\hat{D} \;\; L \right] \begin{bmatrix} u \\ y \end{bmatrix}$

Controller model state: $\hat{x}$

Observer's estimate: $\bar{x}$

When loop is closed: $e = 0$

Error matrices: $\tilde{A} = A - \hat{A}, \; \tilde{B} = B - \hat{B}, \; \tilde{C} = C - \hat{C}, \; \tilde{D} = D - \hat{D}$

The state response of the system is summarized in the following proposition.

Proposition 3. The system described above has a state response:

$$z(t) = \begin{cases} e^{\Lambda_c (t-t_k)} \Sigma^k z_0 \;, \; t \in [t_k, t_k + \tau) \\ e^{\Lambda_o (t-(t_k+\tau))} e^{\Lambda_c(\tau)} \Sigma^k z_0 \;, \; t \in [t_k + \tau, t_{k+1}) \end{cases} \quad (2)$$

where $\Sigma = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} e^{\Lambda_o (h-\tau)} e^{\Lambda_c(\tau)} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}$, and

$$\Lambda_o = \begin{bmatrix} A & BK & -BK \\ LC & \hat{A} - L\hat{C} + \hat{B}K + L\tilde{D}K & -\hat{B}K - L\tilde{D}K \\ LC & L\tilde{D}K - L\hat{C} & A - L\tilde{D}K \end{bmatrix},$$

$$\Lambda_c = \begin{bmatrix} A & BK & -BK \\ LC & \hat{A} - L\hat{C} + \hat{B}K + L\tilde{D}K & -\hat{B}K - L\tilde{D}K \\ 0 & 0 & 0 \end{bmatrix}.$$

The following gives a necessary and sufficient condition for stability.

Theorem 4. The system described above is globally exponentially stable around the solution $z = \begin{bmatrix} x \\ \bar{x} \\ e \end{bmatrix} = \mathbf{0}$ if and only if the eigenvalues of Σ are strictly inside the unit circle, where where $\Sigma = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} e^{\Lambda_o (h-\tau)} e^{\Lambda_c(\tau)} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}$, and Λ_o, Λ_c as before.

This result is useful for situations when the full state of the plant is unavailable. An extension of our results to nonlinear plants is presented in the next section.

4. NONLINEAR PLANTS

In the previous sections we have restricted our study to the cases where the plant is linear. Let us now lift this restriction and seek to find the corresponding stability properties for nonlinear plants with intermittent feedback.

The setup and procedure that follows closely mirrors that proposed by Montestruque [2003] for traditional MB-NCS. The sufficient conditions obtained relate the stability of the nonlinear MB-NCS with the value of a function that depends on the Lipschitz constants of the plant and model as well as the stability properties of the compensated non-networked model. The results are obtained by studying the worst-case behavior of the norm of the plant state and the error, thus leading to conservative results.

Let the plant be given by:

$$\dot{x} = f(x) + g(u) \quad (3)$$

We use a model on the actuator side of the plant to estimate the actual state of the plant. The controller will be assumed to be a nonlinear state feedback controller. The control signal u is generated by taking into account the plant model state . The plant state sensor will send through the network the real value of the plant state to the model (that is, the loop will be closed) every h seconds, and the loop will remain closed for τ seconds during each cycle. During these times, the state of the model is set to be the same as that of the plant. We will assume the plant model dynamics are given by:

$$\dot{\hat{x}} = \hat{f}(x) + \hat{g}(u) \tag{4}$$

And the controller has the following form:

$$u = \hat{h}(\hat{x}) \tag{5}$$

We define as the error between the plant state and the plant model state, $e = x - \hat{x}$. Combining the above, we obtain:

$$\begin{aligned} \dot{x} &= f(x) + g\left(\hat{h}(\hat{x})\right) = f(x) + m(\hat{x}) \\ \dot{\hat{x}} &= \hat{f}(x) + \hat{g}\left(\hat{h}(\hat{x})\right) = f(x) + \hat{m}(\hat{x}) \end{aligned} \tag{6}$$

Assume also that the plant model dynamics differ from the actual plant dynamics in an additive fashion:

$$\begin{aligned} \hat{f}(\zeta) &= f(\zeta) + \delta_f(\zeta) \\ \hat{m}(\zeta) &= m(\zeta) + \delta_m(\zeta) \end{aligned} \tag{7}$$

Thus:

$$\begin{aligned} \dot{x} &= f(x) + m(\hat{x}) \\ \dot{\hat{x}} &= f(x) + \hat{m}(\hat{x}) + \delta_f(\hat{x}) + \delta_m(\hat{x}) \end{aligned} \tag{8}$$

Assume that f and δ satisfy the following local Lipschitz conditions for with $x, y \in B_L$, a ball centered on the origin:

$$\begin{aligned} \|f(x) - f(y)\| &\leq K_f \|x - y\| \\ \|\delta(x) - \delta(y)\| &\leq K_\delta \|x - y\| \end{aligned} \tag{9}$$

It is to be noted that if the plant model is accurate the Lipschitz constant K_δ will be small.

Assume that the non-networked compensated plant model is exponentially stable when $\hat{x}(t_0) \in B_S$, $\hat{x}(t) \in B_\tau$, for $t \in [t_0, t_0+\tau)$ with B_S and B_τ balls centered on the origin.

$$\|\hat{x}(t)\| \leq \alpha \|\hat{x}(t_0)\| e^{-\beta(t-t_0)} \text{ with } \alpha, \beta > 0\,. \tag{10}$$

Theorem 5. The non-linear MB-NCS with dynamics described above, and that satisfies the Lipschitz conditions described and with exponentially stable compensated plant model satisfying is asymptotically stable if:

$$\left(1 - \alpha\left(e^{-\beta(h-\tau)} + \left(e^{K_f(h-\tau)} - e^{-\beta(h-\tau)}\right)\left(\frac{K_\delta}{K_f + \delta}\right)\right)\right) > 0 \tag{11}$$

Due to the nature of the derivation of this theorem, the results are conservative, and the condition is sufficient. Finding tighter bounds for nonlinear plants in model-based networked control systems remains an open problem. Let us now turn our attention to the performance of model-based networked control systems.

5. CONTROLLER DESIGN FOR AN LQ-BASED PERFORMANCE MEASURE FOR MB-NCS

Having considered the advantages that intermittent feedback of MB-NCS provides in term of stability, the next step is to gauge the effects in terms of performance. As a first approach, we will consider a system with a discrete-time plant, where the full information of the state of the plant is only available with probability p. When the state of the plant is not available, the model is used to compute the control action. Our main objective is to find a controller that will optimize an LQ-based performance criterion for this system and to compute this optimal cost as well. This result will serve as a comparison point for the cases where access to the plant state is of an intermittent nature.

The setup is as follows. We will consider the discrete-time plant governed by the following equations:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ \hat{x}_{k+1} &= \hat{A}\hat{x}_k + \hat{B}u_k \\ u_k &= K(x_k + v_k e_k) \end{aligned} \tag{12}$$

where the probability of having access to the full state of the plant is governed by the stochastic variable ν_k, with

$$\nu_k = \begin{cases} 1\,, \text{ with probability } p \\ 0\,, \text{ with probability } 1-p \end{cases} \tag{13}$$

and where A, B are the plant matrices, $\hat{A}, \hat{B}$, are the model matrices, u_k is the input, x is the state of the plant, x is the state of the model, and K is the controller. Also, $e_k = x_k - \hat{x}_k$ denotes the error between the plant state and model state.

We use the expected total cost as our performance index:

$$J_\infty = E\left[\sum_{k=0}^{\infty} x_k^T Q x_k + u_k^T R u_k\right] \tag{14}$$

We will design an optimal controller K to optimize this performance index. We compute the optimal cost J_∞ as well.

Our procedure follows that of Schenato [2007] in that we define a cost-to-go function and calculate it iteratively. To begin, observe that the equations of the system can be rewritten as:

$$\begin{aligned} x_{k+1} &= [A + BK(1+v_k)]x_k - BKv_k\hat{x}_k \\ \hat{x}_{k+1} &= [\hat{A} + \hat{B}K(1+v_k)]x_k - \hat{B}Kv_k\hat{x}_k \\ u_k &= K(1+v_k)x_k - Kv_k\hat{x}_k \end{aligned} \tag{15}$$

Let us define $z = \left[x_k^T \; \hat{x}_k^T\right]^T$. We can thus write the equations concerning the plant and model state as $z_{k+1} = F(\nu_k)\, z$, where:

$$F(\nu_k) = \begin{bmatrix} [A + BK(1+v_k)] & -BKv_k \\ [\hat{A} + \hat{B}K(1+v_k)] & -\hat{B}Kv_k \end{bmatrix} \tag{16}$$

The cost-to-go function C_k is defined as follows:

$$C_k^N(z_k) = E\left[\sum_{k=0}^{\infty} x_k^T Q x_k + u_k^T R u_k\right] \tag{17}$$

where $Q_k = Q$ and $R_k = R$ except for the terminal cost $R_N = 0$. Following the standard procedure in these cases, we make the claim that this function can be written as

$$C_k^N(z_k) = E\left[z_k S_k z_k | z_k\right]\,, \tag{18}$$

which is clearly true for $k = N$ and $S_N = Q$. Then, by induction, we show this is true for all k. Suppose it is true for $k+1$, then:

$$
\begin{aligned}
C_k^N(z_k) &= E\left[\sum_{h=k}^{N} x_k^T Q x_k + u_k^T R u_k | z_k\right] \qquad (19)\\
&= E\left[x_k^T Q x_k + u_k^T R u_k + C_{k+1}^N | z_k\right]\\
&= E\,[x_k^T Q x_k + \left(K(1+v_k)x_k - K v_k \hat{x}_k\right)^T R \left(K(1+v_k)x_k - K v_k \hat{x}_k\right) + C_{k+1}^N | z_k]\\
&= E\,[z_k^T \begin{bmatrix} Q+(1+v_k^2)K^T R K & -(1+\nu_k)\nu_k K^T R K \\ -(1+\nu_k)\nu_k K^T R K & \nu_k^2 K^T R K \end{bmatrix} z_k + z_k^T F^T(\nu_k) S_{k+1} F(\nu_k) z_k | z_k]
\end{aligned}
$$

Therefore, the above claim is true. Moreover, we can write that

$$
\begin{aligned}
S_k &= \begin{bmatrix} Q+(1+v_k^2)K^T R K & -(1+\nu_k)\nu_k K^T R K \\ -(1+\nu_k)\nu_k K^T R K & \nu_k^2 K^T R K \end{bmatrix} \qquad (20)\\
&+ F^T(\nu_k) S_{k+1} F(\nu_k)\\
&= \mathcal{F}(S_{k+1}, K)\ ,
\end{aligned}
$$

where the operator $\mathcal{F}(S_{k+1}, K)$ is affine in S for fixed K, and quadratic in K for fixed S.

To obtain the infinite horizon cost, we take the limit as time goes to infinity of the cost-to-go function.

$$
J_\infty(K) = \lim_{N\to\infty} C_0^N(x_0) = x_0^T S_\infty x_0 \qquad (21)
$$

where S_∞ is the solution of the Lyapunov-like equation $S_\infty = \mathcal{F}(S_\infty, K)$, if the solution exists. The optimal gain K^* is defined as the minimizer of the infinite horizon cost $K^* = \arg\min_K x_0^T S_\infty x_0$.

Theorem 6. Consider the system defined by (12) and the infinite horizon cost defined in (14). Assume that the pairs (A, B) and $(A^T, Q^{1/2})$ are stabilizable. Then the optimal infinite horizon cost $J_\infty = \min_L J_\infty(K)$ is given by $J_\infty = x_0^T S_\infty x_0$ where S_∞ is the unique strictly positive solution of the Riccati-like equation:

$$
\begin{aligned}
S_\infty &= \begin{bmatrix} Q+(1+v_k^2)K^T R K & -(1+\nu_k)\nu_k K^T R K \\ -(1+\nu_k)\nu_k K^T R K & \nu_k^2 K^T R K \end{bmatrix} \qquad (22)\\
&+ F^T(\nu_k) S_\infty F(\nu_k)
\end{aligned}
$$

and $F(\nu_k)$ is defined as in (16) and the optimal gain is given by $K^* = \arg\min_K x_0^T S_\infty x_0$.

As we mentioned at the start of this section, this result should provide a benchmark for the performance of model-based networked control systems, against which to compare the case where the feedback is done intermittently.

6. CONCLUDING REMARKS AND FUTURE WORK

In this paper, we have provided a set of results for model-based networked control systems with intermittent feedback. We focused first on deriving stability results and provided necessary and sufficient conditions for the basic setup as well as the case with state observers. We also provided sufficient conditions for nonlinear systems. We then turned our attention to the performance aspect. By focusing on the discrete-time case with Bernoulli distribution probability of having closed loop control, we take a first step towards the understanding of performance for our setup.

The area of performance of networked control systems, both under the model-based architecture and otherwise, remains a vast and relatively unexplored ground for research. We expect to provide more concrete results on performance of model-based networked control systems with intermittent feedback in the future, and will consider other issues, such as robustness, filtering, and improving control as time elapses, as well.

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