Distributed Control with Integral Quadratic Constraints

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Abstract: In this paper, stability conditions for distributed system under general integral quadratic constraints (IQC) for interconnections are derived. These results take the form of coupled linear matrix inequalities (LMIs), and the multipliers are shaped by the underlying IQCs to model these interconnections. It is further shown that these results can be exploited for distributed controllers synthesis similar to the gain-scheduling controller design in the linear parameter varying (LPV) systems via elimination lemma.

Keywords: Distributed control, IQC, controller synthesis.

1. INTRODUCTION

Over the past few years, there has been renewed research interest in distributed control of large scale systems Cedric Langbort et al. [2004], G.E. Dullerud and R.D' Andrea [2004], Langbort et al. [2006], Ugrinovskii et al. [2000], Scorletti and Due [2001], Chen and Lall [2003]. Many of these systems are formed by the interconnection of multiple homogeneous or heterogeneous subsystems. These systems typically exhibit overall complex dynamical behavior dictated by their distributed nature and the dynamical interactions between the subsystems.

A great challenge that faces the control community is to deal with such systems that are physically distributed. The distributed nature of these system implies that the observation is highly distributed and thus has motivated the development of new research directions in control theory, namely control under communication constraints. In particular, researchers have considered control problems with non-ideal communication links such as limited bandwidth Tatikonda [2000], Nair and Evens [2000], delay, and packet dropout between sensors, actuator of these subsystems. Some new results have been reported in Antsaklis and Baillieul [2004]. Standard control design techniques for these systems often fail because of the high dimension of the global system and the forbidding communication and computation burdens needed to implement centralized control algorithms. In many simplified cases decentralized control schemes are deployed for large-scale applications. Successful synthesis methods have been proposed for existence of decentralized controllers guaranteeing performance of the closed-loop systems. However, these decentralized controllers are generally conservative and only apply to particular specific interconnections. It is often useful to utilize the structure of the systems for controller synthesis to balance between performance of these systems and complexity of the controllers.

Recently, a distributed control theory has been developed for spatially-invariant distributed systems Cedric Langbort et al. [2004]. It was shown that the controllers have 'identical' structure as the underlying subsystems. A linear matrix inequality(LMIs) based control synthesis algorithm for this class of interconnected systems was developed in Cedric Langbort et al. [2004], G.E. Dullerud and R.D' Andrea [2004] using a multidimensional system theory. These results were further extended in Cedric Langbort et al. [2004], Langbort et al. [2006], C. Langbort and R.D' Andrea [2003] to distributed system over an arbitrary graph under various connections. Specifically, the results take the form of a set of coupled linear matrix inequalities. The design variables for the LMIs are shaped by the interconnections.

In our view, this distributed control approach Cedric Langbort et al. [2004], Langbort et al. [2006] has been well-developed in the literature of gain-scheduling techniques for linear parameter varying(LPV) systems Scorletti and Ghaoui [1998], A. Packard [1994]. The stability results follow from an application of the S-procedure developed in Yakubovich [1977], Megretski and Treil [1993], Yakubovich [1992] when parameterizing the interconnections as a family of IQCs. Furthermore, the stability condition under perfect communication can be proved via the block S-procedure Scherer [2001], Iwasaki and Shibata [2001] if proper quadratic separator is chosen. Through this way, all the stability results can be interpreted from a graph separation point view (Safonov [1980], Moylan and Hill [1978] Iwasaki and Hara [1998]) following a similar proof as in Scorletti and Ghaoui [1998]. While the sufficiency of the stability results can be easily derived via an graph separation argument, the necessity part for some specific interconnections follows from the lossless (D, G) scaling theorem for LPV uncertainties Meinsma et al. [2000]. As mentioned in Cedric Langbort et al. [2004], there stability results can be explained in the general framework of dissipative theory Willems [1972]. They are well connected to the integral quadratic constraints Megretski and Rantzer [1997] analysis methods since the interconnections are generally modeled by IQCs. For stability, in this paper, we try to explore this connection so that we can unify all these stability results and treat systems with more general imperfect communication links in this framework. As for synthesis, based on a recently extended elimination lemma in Helmersson [1999], the synthesis inequalities turn out to be convex in all variables, including the scalings Scherer [2001] and controller parameters. However, these techniques could only be applied under certain inertia hypothesis on the multipliers which are unduly conservative. We further point out that instead of $n_{ij}^K = 3n_{ij}$ for the synthesis condition in Cedric Langbort et al. [2004] for the ideal interconnection case, if the dimension of the interconnected signals for the controller is equal to the associated interconnected signals for the plants, i.e, $n_{ij}^K = n_{ij}$, there exist distributed controllers to guarantee the global control performance.

The remainder of the paper is organized as follows. The interconnected system is introduced in Section 2, where each of the individual Linear Time Invariant (LTI) subsystems are represented in the state space forms, operators are introduced to model the interconnections. The performance and stability analysis theorem for the global systems with a general IQC for the interconnections is presented in Section 3, as applications of this theorem, stability conditions for several interconnections are also derived. In Section 4, based on the stability results for the specific interconnections and the elimination lemma, we presented distributed controller synthesis results.

1.0.0.1. Notation The notation is standard. Real and negative reals are denoted by $\mathbb{R}$, $\mathbb{R}^+$. $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices. The transpose (complex conjugate transpose) of matrix M is denoted by $M^T (M^*)$. Let $U \in \mathbb{R}^{r\times n}$ with $r < n$. $U_\perp$ denotes an orthogonal complement of U, i.e $UU_\perp = 0$ and $[U^T U_\perp]$ is of full rank. We use $\mathbb{R}^n_S$ to denote $n \times n$ real symmetric matrices. If $M \in \mathbb{R}^n_S$, then $M > 0 (M \geq 0)$ indicate M is positive definite (positive semidefinite) matrix, and $M < 0 (M \leq)$ denotes negative definite (negative semidefinite) matrix. For any matrix P, $\ker(P)$ stands for the null space of P. The inertia of a symmetric matrix A is the ordered triple $\text{in}(A) = (i_+(A), i_0(A), i_-(A))$ where $i_+(A), i_-(A), i_0(A)$ are the numbers of positive, negative and zero eigenvalues of A, all counting multiplicity.

A block diagonal matrix with $X_k, \ldots, X_l$ is denoted $\mathbf{diag}_{k\leq i\leq l} X_i = \mathbf{diag}\{X_k, \ldots, X_l\}$, likewise, if $e1, \ldots, e_L$ are elements of sets $E_1, \ldots, E_L$, $\mathbf{cat}_{k\leq i\leq l} e_i$ will designate the elements $(e_k, \ldots, e_l) \in E_k \times \ldots E_l$ when $1 \leq k \leq l \leq L$. We will sometimes write $\mathbf{diag}_i$ and $\mathbf{cat}_i$ instead of $\mathbf{diag}_{1\leq i\leq L}$ and $\mathbf{cat}_{1\leq i\leq L}$.

The Euclidean norm of a vector $x \in \mathbb{R}^n$ is denoted by $\|x\| = (x^T x)^{1/2}$. The space of square integrable n-dimensional functions $f : (0, \infty) \to \mathbb{R}^n$ is denote by $\mathcal{L}_2^n$; this is abbreviated as $\mathcal{L}_2$ when n is clear from context or not relevant. The inner product between two signals in $\mathcal{L}_2$ is denoted by $< \cdot, \cdot >$. The Fourier transform of a $\mathcal{L}_2$ function f is denoted as $\hat{f}(j\omega)$. The norm of an $\mathcal{L}_2$ signal and the induced norm of an operator on $\mathcal{L}_2$ is denoted by $\|\cdot\|$, so for an operator $F : \mathcal{L}_2 \to \mathcal{L}_2$, $\|F\| = \sup_{u\in\mathcal{L}_2} \frac{\|Fu\|}{\|u\|}$. An operator $\Delta : \mathcal{L}_2^n \to \mathcal{L}_2^n$ is said to be contractive if $\|\Delta v\| < \|v\|, \forall v \in \mathcal{L}_2^n$. Lower case δ's always denote operators from $\mathcal{L}_2^1$ to $\mathcal{L}_2^1$. Then for $u, v \in \mathcal{L}_2^n$, the expression $v = \delta I_n u$ is defined to mean that u_k of u and v_k of v satisfy $u_k = \delta v_k$. An operator $\delta : \mathcal{L}_2 \to \mathcal{L}_2$ is called self-adjoint if $< u, \delta v >=< \delta u, v >, \forall u, v \in \mathcal{L}_2$. Note that all real-valued static linear time varying (LTV) operators are self-adjoint.

2. PROBLEM FORMULATION

2.1 Problem Formulation

In this paper, we are concerned with systems formulated in Cedric Langbort et al. [2004]. The global system consists of an assembly of L subsystem $G_i, i = 1, \ldots, L$, connected arbitrarily.

Each subsystem G_i is described by the following state-space equation:

$$\begin{bmatrix} \dot{x}_i(t) \\ w_i(t) \\ z_i(t) \end{bmatrix} = \begin{bmatrix} A^i_{TT} & A^i_{TS} & B^i_{Td} \\ A^i_{ST} & A^i_{SS} & B^i_{Sd} \\ C^i_{Tz} & C^i_{Sz} & D^i_{zd} \end{bmatrix} \begin{bmatrix} x_i(t) \\ v_i(t) \\ d_i(t) \end{bmatrix} \quad (2.1)$$

$$x_i(0) = x_i^0 \quad (2.2)$$

where $x_i(t) \in \mathbb{R}^{m_i}, d_i(t) \in \mathbb{R}^{p_i}, z_i(t) \in \mathbb{R}^{q_i}, v_i(t), w_i(t) \in \mathbb{R}^{n_i}$ for all $t \geq 0$. In (2.1), d_i is the disturbance and z_i is the performance associated with G_i, while v_i and w_i are the overall interconnection signals used by G_i. For each given i, v_i and w_i are further partitioned into v_{ij}, w_{ij} respectively, i.e., the n_{ij}-dimension signal that is shared by G_i and G_j. We model the interconnection via an operator Δ_{ij}, such that,

$$v_{ij} = \Delta_{ji} w_{ji}, \quad \forall i, j, 1 \leq i, j \leq L \quad (2.3)$$

For example, a simply case would be, $w_{ij} = v_{ji}$ which is called ideal/perfect interconnection. However, we generally model the interconnection signal subspace as $\mathcal{W}(\Delta_{ij})$, such that

$$\mathcal{W}(\Delta_{ji}) = \left\{ \begin{bmatrix} v_{ij} \\ w_{ji} \end{bmatrix} \in \mathcal{L}_2^{2n_{ji}} : v_{ij} = \Delta_{ji} w_{ji} \right\} \quad (2.4)$$

We denote $v = \mathbf{cat}_i v_i$, where each v_i can be further partioned as $v_i = \mathbf{cat}_j v_{ij}$. Note that the dimension of v_{ij}, v_i and v are n_{ij}, n_i and $\mathcal{N}$ where $n_i = \sum_{j=1}^L n_{ij}$, $\mathcal{N} = \sum_{i=1}^L n_i$. The global system signals, $x = \mathbf{cat}_i x_i, w = \mathbf{cat}_i w_i, z = \mathbf{cat}_i z_i, d = \mathbf{cat}_i d_i$ are similarly defined.

Based on the state space representations of G_i, the state space representation of the global system can be described by the following state-space representation

$$\begin{bmatrix} \dot{x}(t) \\ w(t) \\ z(t) \end{bmatrix} = \left[\begin{array}{c|cc} A_{TT} & B_{TS} & B_{Td} \\ \hline A_{ST} & A_{SS} & B_{Sd} \\ C_{Tz} & C_{Sz} & D_{zd} \end{array}\right] \begin{bmatrix} x(t) \\ v(t) \\ d(t) \end{bmatrix} \quad (2.5)$$

$$v(t) = \Delta P_r w(t) \quad (2.6)$$

where Δ is a (causal) operator from $\mathcal{L}_2^{\mathcal{N}}$ to $\mathcal{L}_2^{\mathcal{N}}$ generated via Δ_{ij},

$$\Delta = \mathbf{diag}_i \mathbf{diag}_j \Delta_{ji} \quad (2.7)$$

and the permutation matrix P_r is chosen such that

$$\bar{w} = \mathbf{cat}_i \mathbf{cat}_j w_{ji} = P_r w = P \mathbf{cat}_i \mathbf{cat}_j w_{ij} \quad (2.8)$$

$A_{TT} = \mathbf{diag}_i A^i_{TT}$. All other matrices in (2.5) is similarly defined. The signals $w(t)$ and $v(t)$ are $\mathbb{R}^{\mathcal{N}}$-valued internal signals. The signal space for for v, w can be described as

$$\mathcal{W}(\Delta) = \left\{ \begin{bmatrix} v \\ w \end{bmatrix} \in \mathcal{L}_2^{2\mathcal{N}} : \begin{bmatrix} v_{ij} \\ w_{ji} \end{bmatrix} \in \mathcal{L}_2^{2n_{ji}}, v_{ij} = \Delta_{ji} w_{ji} \right\} \quad (2.9)$$

For the state-space representation of the global system (2.5), we represent its transfer function by

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \quad (2.10)$$

which has been partitioned to conform with the vector (v, d). In this paper, the interconnected system is called well-posed and stable if the system (2.5) is internal stable regardless of the uncertainty of the interconnection operator Δ_{ij} (2.9).

Definition 2.1. The interconnected system consisting of subsystems (2.5) and the interconnection constraints (2.9) is said to be well-posed and stable if the map $(I - \Delta P G_{11})$ has a bounded inverse on $\mathcal{L}_2$, for any Δ in a prescribed uncertainty set.

Finally, we will say that such a system (2.5) is contractive if it is stable and $\|z\| < \|d\|$ for all $d \in \mathcal{L}_2$ and all interconnection Δ_{ij} (2.9).

3. STABILITY ANALYSIS VIA IQC

The main idea here is to first use integral quadratic constraints to model the interconnection operator Δ_{ij}. The performance under the integral quadratic constraints (IQC) for the internal signal v, w can then be casted as an unconstrained quadratic optimization problem Megretski and Rantzer [1997] via S-procedure. For the LTI system, the stability results admit a LMI formulation. For this purpose, we need the following definitions of IQC and dissipative.

Definition 3.1. Megretski and Rantzer [1997] A class of signal $\mathcal{W}, \mathcal{W} \subset \{w : w \in \mathcal{L}_2^n\}$ is said to satisfy the IQC defined by $\Pi(\omega)$ if $\sigma(w, \Pi(\omega)) \geq 0, \forall w \in \mathcal{W}$, where σ is of the form

$$\sigma(w, \Pi(\omega)) = \int_{-\infty}^{\infty} \hat{w}(j\omega)^* \Pi(\omega) \hat{w}(j\omega) d\omega \quad (3.11)$$

$\hat{w}(j\omega)$ is the Fourier transform of w, and $\Pi(j\omega) = \Pi^*(j\omega)$ is a matrix function referred to as the multiplier of σ and assumed to be bounded on the imaginary axis. In the sequel, we will refer condition $\sigma(w, \Pi(\omega)) \geq 0$ (3.11) as an IQC with multiplier $\Pi(\omega)$.

Definition 3.2. Let $\mathcal{H} : \mathcal{L}_{2e} \to \mathcal{L}_{2e}$ be an operator, we say $\mathcal{H}$ is $\{X, Y, Z\}$-dissipative if there exist real matrices X, Y, Z such that

$$\Phi = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$$

is a full rank matrix and with $p(t) = \mathcal{H}(q(t)), p, q \in \mathcal{L}_{2e}$

$$\int_0^{\infty} \begin{bmatrix} p(t) \\ q(t) \end{bmatrix}^T \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \begin{bmatrix} p(t) \\ q(t) \end{bmatrix} dt \geq 0 \quad (3.12)$$

Note that, condition (3.12) can be easily represented in the frequency domain as IQC (3.11) if $\mathcal{H}$ is stable and time-invariant, in this case $\Pi(\omega)$ is restricted to be constant matrix, we often call (3.12) an IQC in the time-domain form.

Many important interconnections used in stability analysis can be characterized by IQC's with proper multiplier $\Pi(\omega)$. A collection of commonly used IQC's has been summarized in Megretski and Rantzer [1997]. Based on results on (D, G)-scaling, the following linear time varying (LTV) operators of fixed block and scalar operators can be equivalently represented by IQCs with proper constant multiplier Π Iwasaki and Hara [1998].

Lemma 3.1.

- Suppose $\tilde{\delta} : \mathcal{L}_n^2 \to \mathcal{L}_n^2$, if the LTV operator $\tilde{\delta}$ is self-adjoint and contractive, then for any $D \in \mathbb{R}_S^{n \times n}, D \geq 0$ and $G = -G^T$, $\tilde{\delta} I_n$ is $(-D, G, D)$-dissipative.
- Suppose $\delta : \mathcal{L}_n^2 \to \mathcal{L}_n^2$, if the LTV operator is contractive , then for any $D \in \mathbb{R}_S^{n \times n}, D \geq 0$, δI_n is $(-D, 0, D)$-dissipative.
- There is a contractive LTV operator, $\Delta : \mathcal{L}_n^2 \to \mathcal{L}_n^2$ such that $p = \Delta q$ if and only if Δ is $(-I, 0, I)$-dissipative.

Definition 3.3. A quadratic performance is a quadratic functional $\sigma_p(z, d)$ defined as

$$\sigma_p(z, d) = \int_0^{\infty} \begin{bmatrix} d(t) \\ z(t) \end{bmatrix}^T \begin{bmatrix} \Pi_{p1} & \Pi_{p2} \\ \Pi_{p2}^T & \Pi_{p3} \end{bmatrix} \begin{bmatrix} d(t) \\ z(t) \end{bmatrix} dt \quad (3.13)$$

A system satisfies σ_p-performance criterion over a set of disturbance $\mathcal{W}$ if the system is well-posed, internally stable and its performance measurement z satisfies $\sigma_z(z, d) < 0$.

The following proposition gives a sufficient condition for the system that satisfies the performance criterion $\sigma_p < 0$ over a class of signals $\mathcal{W}$ which can be characterized by IQCs. If he operator Δ used to model the interconnection $v = \Delta w$ can be characterized by several IQCs, $\sigma_{w1}, \sigma_{w2}, \ldots, \sigma_{wn}$, then the performance can be formulated as a convex feasibility problem over the set of IQCs via the S-procedures,

$$\sigma_p(z, d) + \sum_{i=1}^{n} \lambda_i \sigma_{wi}(w) < 0, \ \forall w \in \mathcal{L}_2. \quad (3.14)$$

The following proposition is a direct application of the S-procedure to the interconnected system (2.5) and (2.6).

Proposition 1. Scorletti and Ghaoui [1998] Suppose the operator ΔP in (2.6) is $\{X, Y, Z\}$-dissipative, then the interconnected systems (2.5), (2.6) satisfies $\sigma_p(z, d)$ performance (3.13), if there exists symmetric matrix $X_T \in \mathbb{R}_S^{m \times m}, X_T > 0$, such that the following LMI holds true.

$$M^T P M < 0 \quad (3.15)$$

where

$$M = \left[\begin{array}{ccc} I & 0 & 0 \\ A_{TT} & B_{TS} & B_{Td} \\ \hline 0 & I & 0 \\ A_{ST} & A_{SS} & B_{Sd} \\ \hline 0 & 0 & I \\ C_{Tz} & C_{Sz} & D_{zd} \end{array} \right]$$

$$P = \left[\begin{array}{cc|cc|cc} 0 & X_T & 0 & 0 & 0 & 0 \\ X_T & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & X & Y & 0 & 0 \\ 0 & 0 & Y^T & Z & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & \Pi_{p1} & \Pi_{p2} \\ 0 & 0 & 0 & 0 & \Pi_{p2}^T & \Pi_{p3} \end{array} \right]$$

Note here, we are only use the sufficient part of the S-procedure to derive the above sufficient performance conditions. From the lossless (D, G) scaling theorem for linear time invariant(LTI) systems with LPV uncertainties, we know that for the contractive operators ($\widetilde{\delta}, \delta$ and Δ) considered in lemma 3.1, the above results are both necessary and sufficient Meinsma et al. [2000] with proper multiplier X, Y, Z, and they are referred to (D, G)-scalings for such LTV operators. Generally speaking, the sufficient part of Proposition 1 can be easily proved via an separation of graph argument, and the inner matrix in equation (3.15) can be interpreted as a hyperplane to separate the graph of the linear time invariant system and the operator to model the time-varying interconnections. The necessary part follows the idea proposed in Shamma [1994] for the full block uncertainty LTV Δ to construct a causal destabilizing operator when strict separation of the two graph is violated, the scalar case $\delta, \widetilde{\delta}$ has been proved in Megretski and Treil [1993],Meinsma et al. [2000] respectively. For the contractive operators list in lemma 3.1, the above proposition is a LMI reformulation of the necessary and sufficient condition presented in Meinsma et al. [2000] via an application of the celebrated KYP lemma to the LTI system (2.5) with scaling matrices X, Y, Z.

IQC for the interconnections We introduce the following IQC to model the global interconnection, $v = \Delta P w$. For each $i = 1, \ldots, L$, let us introduce the quadratic form on $\mathbb{R}^{n_i} \times \mathbb{R}^{n_i}$, such that

$$\mathcal{P}_{ij}(v_{ij}, w_{ij}) = \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix}^T X_{ij} \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix} \tag{3.16}$$

The scaling matrix X_{ij} is further partitioned into four $n_{ij} \times n_{ij}$ blocks as

$$X_{ij} = \begin{bmatrix} X_{ij}^{11} & X_{ij}^{12} \\ (X_{ij}^{12})^T & X_{ij}^{22} \end{bmatrix} \tag{3.17}$$

We are now able to state our first analysis conditions, the proof of Theorem 2 follows from Proposition 1 by utilizing the diagonal structure of the global system (2.5).

Theorem 2. The interconnected system (2.5), (2.6) is well-posed, stable and contractive if there exist symmetric matrices, $X_T^i \in \mathbb{R}_S^{m_i \times m_i}$ and $X_{ij} \in \mathbb{R}_S^{2n_{ij} \times 2n_{ij}}$, $X_T^i > 0$ such that

$$M_i^T P_i M_i < 0 \tag{3.18}$$

for all $i = 1, \ldots, L$. where

$$M_i = \left[\begin{array}{ccc} I & 0 & 0 \\ A_{TT}^i & A_{TS}^i & B_{Td}^i \\ \hline 0 & I & 0 \\ A_{ST}^i & A_{SS}^i & B_{Sd}^i \\ \hline 0 & 0 & I \\ C_{Tz}^i & C_{Sz}^i & D_{zd}^i \end{array}\right] \tag{3.19}$$

$$P_i = \left[\begin{array}{cc|cc|cc} 0 & X_T^i & 0 & 0 & 0 & 0 \\ X_T^i & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & P_i^{11} & P_i^{12} & 0 & 0 \\ 0 & 0 & (P_i^{12})^* & P_i^{22} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & -I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{array}\right] \tag{3.20}$$

$$P_i^{11} = \mathbf{diag}_{1 \le j \le L} X_{ij}^{11} \tag{3.21}$$

$$P_i^{22} = \mathbf{diag}_{1 \le j \le L} X_{ij}^{22} \tag{3.22}$$

$$P_i^{12} = \mathbf{diag}_{1 \le j \le L} X_{ij}^{12} \tag{3.23}$$

and

$$\begin{aligned} \sigma(P_X) &= \int_0^\infty \begin{bmatrix} v \\ w \end{bmatrix}^T P_X \begin{bmatrix} v \\ w \end{bmatrix} dt \\ &= \sum_{1 \le i,j \le L} \int_0^\infty \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix}^T \begin{bmatrix} X_{ij}^{11} & X_{ij}^{12} \\ (X_{ij}^{12})^T & X_{ij}^{22} \end{bmatrix} \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix} dt \\ &\ge 0 \end{aligned} \tag{3.24}$$

As applications of Theorem 2, it is of interest to use the above stability results to model different interconnections.

3.0.1.1. Ideal Interconnections Here we assume that $\Delta_{i,j=I_{n_{ij}}}$, $\forall i, j = 1, \ldots, L$, i.e., at anytime t

$$v_{ij}(t) = w_{ji}(t), \quad \forall i, j, t \ge 0 \tag{3.25}$$

In this case, suppose we choose for all $1 \le i, j \le L$

$$X_{ij}^{11} + X_{ji}^{22} = 0$$
$$X_{ij}^{12} + (X_{ji}^{12})^T = 0$$

then

$$\begin{aligned} \sigma(P_{X_{ideal}}) &= \sum_{1 \le i,j \le L} \int_0^\infty \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix}^T X_{ij} \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix} dt \\ &= 0 \end{aligned}$$

The family of multiplier X_{ideal} can thus be characterized by the two sets of matrices,

$$\left\{ X_{ij}^{11} \in \mathbb{R}_S^{n_{ij} \times n_{ij}}, i, j = 1, \ldots, L \right\}$$

and

$$\left\{ X_{ij}^{12} \in \mathbb{R}_S^{n_{ij} \times n_{ij}} : X_{ii}^{12} \text{skew-symmetric}, 1 \le j \le i \le L \right\}$$

Once we have identified the IQC for such perfect interconnection $\Delta_{ij} = I_{n_{ij}}$, as an application of Theorem 2, we have the following proposition.

Proposition 3. The interconnected system (2.5), (2.6) is well-posed, stable and contractive for all $\Delta_{ij} = I_{n_{ij}}$if there exist symmetric matrices, $X_T^i \in \mathbb{R}_S^{m_i \times m_i}$ and $X_{ij}^{11} \in \mathbb{R}_S^{n_{ij} \times n_{ij}}$ for all $i, j = 1, \ldots, L$, and matrices $X_{ij}^{12} \in \mathbb{R}_S^{n_{ij} \times n_{ij}}$ for all $i \ge j$ with X_{ii}^{12} skew-symmetric, such that $X_T^i > 0$ and the LMIs(3.18) hold true for all $i = 1, \ldots, L$ with

$$P_i^{11} = \mathbf{diag}_{1 \le j \le L} X_{ij}^{11}$$
$$P_i^{22} = \mathbf{diag}_{1 \le j \le L} - X_{ji}^{11}$$
$$P_i^{12} = \mathbf{diag} \left(\mathbf{diag}_{1 \le j \le i} X_{ij}^{12}, \mathbf{diag}_{i \le j \le L} - (X_{ji}^{12})^T \right)$$

3.0.1.2. Directed interconnection with $\Delta_{ij} = \delta I_{n_{ij}}$, $\|\delta\| \le 1$. Let us now consider the new class of interconnected systems with $\Delta_{ij} = \delta_{ij} I_{n_{ij}}$ and $\|\delta_{ij}\| \le 1$. We are seeking a new IQC to model such interconnections.

Following similar derivation, we can parameterize the multipliers X_{ij} by the following sets of matrices

$$\left\{X_{ij}^{11} \in \mathbb{R}_S^{n_{ij}\times n_{ij}} : X_{ij}^{11} < 0, i, j = 1, \ldots, L\right\}$$

Besides, we require $X_{ji}^{22} = X_{ij}^{11}$ and $X_{ij}^{12} = 0$ for all $i, j = 1 \ldots, L$.

In this case, it is easy to verify

$$\begin{aligned}\sigma(P_{X_\delta}) = \frac{1}{2} \sum_{1\le i,j\le L} & \left\langle \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix}, \begin{bmatrix} X_{ij}^{11} & 0 \\ 0 & -X_{ji}^{11} \end{bmatrix} \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix} \right\rangle \\ &+ \left\langle \begin{bmatrix} v_{ji} \\ w_{ji} \end{bmatrix}, \begin{bmatrix} X_{ji}^{11} & 0 \\ 0 & -X_{ij}^{11} \end{bmatrix} \begin{bmatrix} v_{ji} \\ w_{ji} \end{bmatrix} \right\rangle \\ &= \sum_{1\le j\le i\le L} < v_{ij}, X_{ij}^{11} v_{ij} > - < w_{ji}, X_{ij}^{11} w_{ji} > \\ &\ge 0\end{aligned}$$

Once we have find the IQC to model the interconnection δ_{ij}, it's straightforward to apply Theorem 2 to get the sufficient part of the following proposition, and the necessary part follows from the lossless-(D,G)-scaling theorem for LTV uncertainties, we omit the details for reason of space.

Proposition 4. The interconnected system (2.5), (2.6) is well-posed, stable and contractive for all $\Delta_{ij} = I_{n_{ij}}\delta, \|\delta\| \le 1$if and only if there exist symmetric matrices, $X_T^i \in \mathbb{R}_S^{m_i\times m_i}$ and $d_{ij} \in \mathbb{R}$ for all $i, j = 1, \ldots, L$, such that $X_T^i > 0, d_{ij} < 0, X_{ij}^{11} = d_{ij} I_{n_{ij}}$ and LMI (3.18) are satisfied for all $i = 1 \ldots, L$, with $P_i^{11} = \mathbf{diag}_j(X_{ij}^{11}), P_i^{22} = \mathbf{diag}_j(-X_{ji}^{11})$ and $P_i^{12} = 0$.

Following similar argument, we have the following Propositions 5, the sufficient part can be similarly proved with proper chosen multipliers X_{ij}, the necessary part follows from the lossless-(D,G)-scaling theorem for these LTV interconnection operators Meinsma et al. [2000].

Proposition 5. The interconnected system (2.5), (2.6) is well-posed, stable and contractive for all LTV $\Delta_{ij}, \|\Delta_{ij}\| \le 1$, if and only if there exist symmetric matrices, $X_T^i \in \mathbb{R}_S^{m_i\times m_i}$, , $X_T^i > 0$ and for all $i, j = 1, \ldots, L$, $d_{ij} < 0$, $X_{ij}^{11} = d_{ij} I_{n_{ij}}$ and the LMIs (3.18) are satisfied for all $i = 1 \ldots, L$, with $P_i^{11} = \mathbf{diag}_j(X_{ij}^{11}), P_i^{22} = \mathbf{diag}_j(-X_{ji}^{11})$ and $P_i^{12} = 0$.

The necessary part of the following proposition has been proved in Cedric Langbort et al. [2004] as an extension of the standard S-procedure, and the sufficient part can be similarly derived via Theorem 2.

Proposition 6. The interconnected system (2.5) , (2.6) is well-posed, stable and contractive for all LTV unitary operator $\delta_{ij}, 1 \le j \le i \le L$ with $\Delta_{ij} = I_{n_{ij}}\delta_{ij}$ and $\delta_{ji} = \delta_{ij}^{-1}$ for $i \ge j$ if and only if there exist symmetric matrices, $X_T^i \in \mathbb{R}_S^{m_i\times m_i}$ and $X_{ij}^{11} \in \mathbb{R}_S^{n_{ij}\times n_{ij}}$ for all $i, j = 1, \ldots, L$, and matrices $X_{ij}^{12} \in \mathbb{R}_S^{n_{ij}\times n_{ij}}$ for all $i \ge j$ with X_{ii}^{12} skew-symmetric, such that $X_T^i > 0$ and the LMIs(3.18) hold true for all $i = 1, \ldots, L$.

Before we apply the stability analysis results to synthesis, we want to comment here.

Remark 3.1. Theorem 2 unifies the stability result for different interconnections which can be modeled as integral quadratic constraints. This theorem renders the performance specification based on the interconnected implicit uncertain systems to an explicit expression through S-procedure with multipliers X_{ij}, which are shaped by the structure and properties of the interconnection operator Δ_{ij}. Generally speaking, Theorem 2 reflects the simple idea of topological separation of the graph generated via the LTI plant and the LTV uncertainty. Although sufficient stability conditions can be derived in this framework easily, the necessity part are not trivial, they can only be established in those special cases in Cedric Langbort et al. [2004], Megretski and Treil [1993], Meinsma et al. [2000], Shamma [1994] .

4. SYNTHESIS VIA THE ELIMINATION LEMMA

The synthesis part of this paper follows the same line of the derivation presented in Cedric Langbort et al. [2004], which is based on the extended elimination lemma Helmersson [1999]. We want to point out that for the synthesis condition that needed for Theorem 2 in Cedric Langbort et al. [2004], $n_{ij}^K = n_{ij}$ is enough, since the inertia constraints are automatically satisfied if the associated LMIs are feasible and the multipliers are nonsingular.

Now let us consider each of subsystem G_i with control input u_i and a measured output y_i, in addition to the signals given in (2.1), such that

$$\begin{bmatrix} \dot{x}_i(t) \\ w_i(t) \\ z_i(t) \\ y_i(t) \end{bmatrix} = \begin{bmatrix} A_{TT}^i & A_{TS}^i & B_{Td}^i & B_{Tu}^i \\ A_{ST}^i & A_{SS}^i & B_S^i & B_{Su}^i \\ C_T^i & C_S^i & D^i & D_{zu}^i \\ C_{Ty}^i & C_{Sy}^i & D_{yd}^i & D_{yu}^i \end{bmatrix} \begin{bmatrix} x_i(t) \\ v_i(t) \\ d_i(t) \\ u_i(t) \end{bmatrix}$$
$$v_{ij} = \Delta_{ji} w_{ji}$$

for all $t \ge 0$ and $i = 1, \ldots, L$, here Δ_{ji} is an operator used to model the interconnection. In the rest of this paper, without loss of generality, we assume that $D_{yu}^i = 0$ for all i. Similar to the controller considered in the LPV literature, we are seeking controller with 'similar' structure as the plant: another interconnected system K with subsystems $K_i, i = 1, \ldots, L$ given by

$$\begin{bmatrix} \dot{x}_i^K(t) \\ w_i^K(t) \\ u_i(t) \end{bmatrix} \begin{bmatrix} (A_{TT}^i)_K & (A_{TS}^i)_K & (B_T^i)_K \\ (A_{ST}^i)_K & (A_{SS}^i)_K & (B_S^i)_K \\ (C_T^i)_K & (C_S^i)_K & D_K^i \end{bmatrix} \begin{bmatrix} x_i^K(t) \\ v_i^K(t) \\ y_i(t) \end{bmatrix} \tag{4.26}$$

such that the closed loop system is well-posed, stable and contractive. In addition, we require $n_{ij}^K = 0$ whenever $n_{ij} = 0$, which means if there is no interaction between G_i and G_j, the controllers K_i and K_j will not communicate with each other either.

Here superscripts K, C are introduced to denote the controller signals and closed-loop signals respectively. The state variable for the subsystem x_i^C has dimension $m_i^C = m_i + m_i^K$,

$$x_i^C = \begin{bmatrix} x_i \\ x_i^K \end{bmatrix}.$$

The interconnection signal w_{ij}^C, v_{ij}^C has dimension $n_{ij}^C = n_{ij} + n_{ij}^K$,

$$w_{ij}^C = \begin{bmatrix} w_{ij} \\ w_{ij}^K \end{bmatrix} \tag{4.27}$$

$$v_{ij}^C = \begin{bmatrix} v_{ij} \\ v_{ij}^K \end{bmatrix} \tag{4.28}$$

Besides, since the controller K and the plant G share the same interconnection operator Δ_{ij} between each subsystem, we further require

$$w_{ij}^C = \Delta_{ji} v_{ji}^C \tag{4.29}$$

.

We are now ready to apply the analysis result to the close-loop systems.

Proposition 7. The closed-loop system is well-posed, stable and contractive if there exist symmetric matrices $(X_T^i)^C \in R_S^{m_i^C \times m_i^C}$ and $X_{ij}^{11} \in R_S^{n_{ij}^C \times n_{ij}^C}$ for all $i, j = 1, \ldots, L$, and $(X_{ij}^{12})_C \in R^{n_{ij}^C \times n_{ij}^C}$ for all $i \geq j$, with $(X_{ii}^{12})_C$ skew symmetric , such that $(X_T^i)_C > 0$ and

$$(M_i^C)^T P_i^C M_i^C < 0 \tag{4.30}$$

with

$$M_i^C = \left[\begin{array}{ccc} I & 0 & 0 \\ (A_{TT}^i)_C & (A_{TS}^i)_C & (B_T^i)_C \\ \hline 0 & I & 0 \\ (A_{ST}^i)_C & (A_{SS}^i)_C & (B_S^i)_C \\ \hline 0 & 0 & I \\ (C_T^i)_C & (C_S^i)_C & (D^i)_C \end{array}\right] \tag{4.31}$$

$$P_i^C = \left[\begin{array}{cc|cc|cc} 0 & (X_T^i)_C & 0 & 0 & 0 & 0 \\ (X_T^i)_C & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & (Z_i^{11})_C & (Z_i^{12})_C & 0 & 0 \\ 0 & 0 & (Z_i^{12})_C^* & (Z_i^{22})_C & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & -I \end{array}\right] \tag{4.32}$$

for all $i = 1, \ldots, L$. and

$$(Z_i^{11})_C = \mathbf{diag}_j (X_{ij}^{11})_C$$
$$(Z_i^{22})_C = \mathbf{diag}_j - (X_{ji}^{11})_C$$
$$(Z_i^{12})_C = \mathbf{diag}(\mathbf{diag}_{1 \leq j \leq i} (X_{ij^{12}})_C, \mathbf{diag}_{i<j\leq L}((X_{ji}^{12})^T)_C)$$

The following synthesis result can be derived if we use the elimination lemma to eliminate controller parameters for the above closed-loop performance conditions.

Proposition 8. There exist distributed controllers with state representation (4.26) with $n_{ij}^K = n_{ij}$ and interconnection $\Delta_{ij} = I$ such that such that the closed-loop system conditions (4.30) are satisfied if and only if there exist symmetric matrices $(X_T^i)_G, (Y_T^i)_G \in \mathbb{R}_S^{m_i \times m_i}$ and $(X_{ij}^{11})_G^T, (Y_{ij}^{11})_G^T \in \mathbb{R}_S^{n_{ij} \times n_{ij}}$ for all $i, j = 1, \ldots, L$, and matrices $(X_{ij}^{12})_G, (Y_{ij}^{12})_G \in \mathbb{R}^{n_{ij} \times n_{ij}}$ for $i \geq j$, with $(X_{ii}^{12})_G, (Y_{ii}^{12})_G$ skew-symmetric such that $(X_T^i)_G > 0, (Y_T^i)_G > 0$ and (4.36),(4.37),(4.38) are satisfied, where Ψ^i, Φ^i, M_i, N_i are defined as (4.33), (4.34), (3.19), (4.35), respectively.

$$\Psi^i = \ker \left[\, C_{Ty}^i \;\; C_{Sy}^i \;\; D_{yd}^i \,\right] \tag{4.33}$$

$$\Phi^i = \ker \left[\, (B_{Tu}^i)^T \;\; (B_{Su}^i)^T \;\; (D_{zu}^i)^T \,\right] \tag{4.34}$$

and

$$(Z_i^{11}) = \mathbf{diag}_{1\leq j \leq L} (X_{ij}^{11})_G$$
$$(Z_i^{22}) = -\mathbf{diag}_{1\leq j \leq L} (X_{ji}^{11})_G$$
$$(Z_i^{12}) = \mathbf{diag}\left\{\mathbf{diag}_{1\leq j\leq i}(X_{ij}^{12})_G, -\mathbf{diag}_{i<j\leq L}(X_{ji}^{12})_G^*\right\}$$
$$(\widetilde{Z}_i^{11}) = \mathbf{diag}_{1\leq j \leq L} (Y_{ij}^{11})_G$$
$$(\widetilde{Z}_i^{22}) = -\mathbf{diag}_{1\leq j \leq L} (Y_{ji}^{11})_G$$
$$(\widetilde{Z}_i^{12}) = \mathbf{diag}\left\{\mathbf{diag}_{1\leq j\leq i}(Y_{ij}^{12})_G, -\mathbf{diag}_{i<j\leq L}(Y_{ji}^{12})_G^*\right\}$$

$$Z_i = \left[\begin{array}{cc|cc|cc} 0 & (X_T^i)_G & 0 & 0 & 0 & 0 \\ (X_T^i)_G & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & (Z_i^{11})_G & (Z_i^{12})_G & 0 & 0 \\ 0 & 0 & (Z_i^{12})_G^* & (Z_i^{22})_G & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & -I \end{array}\right]$$

$$\widetilde{Z}_i = \left[\begin{array}{cc|cc|cc} 0 & (\widetilde{X}_T^i)_G & 0 & 0 & 0 & 0 \\ (\widetilde{X}_T^i)_G & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & (\widetilde{Z}_i^{11})_G & (\widetilde{Z}_i^{12})_G & 0 & 0 \\ 0 & 0 & (\widetilde{Z}_i^{12})_G^* & (\widetilde{Z}_i^{22})_G & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & -I \end{array}\right]$$

$$N_i = \left[\begin{array}{ccc} -(A_{TT}^i)^T & -(A_{ST}^i)^T & -(C_{Tz}^i)^T \\ I & 0 & 0 \\ \hline -(A_{TS}^i)^T & -(A_{SS}^i)^T & -(C_{Sz}^i)^T \\ 0 & I & 0 \\ \hline -(B_{Td}^i)^T & -(B_{Sd}^i)^T & -(D_{zd}^i)^T \\ 0 & 0 & I \end{array}\right] \tag{4.35}$$

$$(\Psi^i)^* M_i^* Z_i M_i \Psi_i < 0 \tag{4.36}$$

$$(\Phi^i)^* N_i^* \widetilde{Z}_i N_i \Phi_i > 0 \tag{4.37}$$

$$\begin{bmatrix} (X_T^i)_G & I \\ I & (Y_T^i)_G \end{bmatrix} > 0 \tag{4.38}$$

Proof 4.1. Notice that, the closed loop system for the individual subsystem with the controller described by (4.26) is linear in the controller's parameter Θ_i with

$$\Theta_i = \begin{bmatrix} (A_{TT}^i)_K & (A_{TS}^i)_K & (B_T^i)_K \\ (A_{ST}^i)_K & (A_{SS}^i)_K & (B_S^i)_K \\ (C_T^i)_K & (C_S^i)_K & D_K^i \end{bmatrix} \tag{4.39}$$

then apply the elimination lemma from Helmersson [1999](see appendix) to each individual stability LMI(3.18) condition derived in Proposition 3 for the closed-loop system, the necessity part follows instantly. The sufficient part follows from similar techniques in Scherer [2001] to construct the extended multiplier for the overall interconnection w_{ij}^C, v_{ji}^C. Conditions (4.36), (4.37), (4.38) are sufficient to construct the extended multipliers and controller parameters.

The following synthesis conditions corresponding to Proposition 4, Proposition 5 can be proved similarly to Proposition 8 via elimination lemma.

Proposition 9. There exist distributed controllers with state space representation (4.26) with $n_{ij}^K = n_{ij}$ and interconnection $\Delta_{ij} = \delta I, \|\delta\| \leq 1$ such that the closed-loop system is well-posed, stable and contractive if and only if there exist symmetric matrices $(X_T^i)_G, (Y_T^i)_G \in \mathbb{R}_S^{m_i \times m_i}$ and matrices $(X_{ij}^{11})_G, (Y_{ij}^{11})_G \in \mathbb{R}_S^{n_{ij} \times n_{ij}}, (X_{ij}^{11})_G < 0, (Y_{ij}^{11})_G < 0$ for all $i, j = 1, \ldots, L$ such that $(X_T^i)_G >$

$0, (Y_T^i)_G > 0$ and (4.36),(4.36),(4.36) are satisfied for all i with $(Z_{12}^i)_G = (\widetilde{Z}_{12}^i)_G = 0$, and

$$\begin{bmatrix} (X_{ij}^{11})_G & -I \\ -I & (Y_{ij}^{11})_G \end{bmatrix} \leq 0 \quad \text{for all } i, j \tag{4.40}$$

Proposition 10. There exist distributed controllers with state space representation (4.26) with $n_{ij}^K = n_{ij}$ and interconnection $\Delta_{ij}, \|\Delta_{ij}\| \leq 1$ such that the closed-loop system is well-posed, stable and contractive if and only if there exist symmetric matrices $(X_T^i)_G, (Y_T^i)_G \in \mathbb{R}_S^{m_i \times m_i}$, and $x_{ij}, y_{ij} \in \mathbb{R}$ such that $x_{ij} < 0, y_{ij} < 0, (X_{ij}^{11})_G = x_{ij} I_{n_{ij}}, (Y_{ij}^{11})_G = y_{ij} I_{n_{ij}}$ for all $i, j = 1, \ldots, L$ such that $(X_T^i)_G > 0, (Y_T^i)_G > 0$ and (4.36),(4.37),(4.38) (4.40) are satisfied for all i with $(Z_{12}^i)_G = (\widetilde{Z}_{12}^i)_G = 0$.

5. CONCLUSION

In this paper, we derived stability conditions for distributed systems with various IQC constraints for the interconnection. Technically, the stability results follow from an application of the S-procedure and can be proved via a graph separation argument. Our stability theorem renders the global performance with implicit uncertainty interconnections to one explicit conditions with design multipliers parameterized by the uncertainty. Specifically, our theorem generalize the stability results presented in Cedric Langbort et al. [2004]. Besides, these results can be used for distributed controller synthesis based on the gain-scheduling techniques in linear parameter varying systems.

6. APPENDIX

The following result is basically an extension of the well-known elimination lemma A. Packard [1994] to a quadratic matrix inequality. It is convenient for elimination of controller parameters from the synthesis conditions.

Lemma 6.1. (Elimination Lemma Helmersson [1999]) Let P be a symmmetric matrix with inertia $\text{in}(P) = (m, 0, n)$ and $C \in \mathbb{R}^{n \times m}$. The quadratic matrix inequality,

$$\begin{bmatrix} I \\ A^T X B + C \end{bmatrix}^T P \begin{bmatrix} I \\ A^T X B + C \end{bmatrix} < 0 \tag{6.41}$$

in the unstructured unknown matrix variable X has a solution if and only if

$$B_\perp^T \begin{bmatrix} I \\ C \end{bmatrix}^T P \begin{bmatrix} I \\ C \end{bmatrix} B_\perp < 0 \tag{6.42}$$

$$A_\perp^T \begin{bmatrix} -C^T \\ I \end{bmatrix}^T P^{-1} \begin{bmatrix} -C^T \\ I \end{bmatrix} A_\perp > 0 \tag{6.43}$$

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