Corrections to Linear Systems – January 2002

p. xvi: line 2-3
should read: "...should also prove valuable to researchers and practitioners for self-study. Many simple examples..."

p. 15: Figure 1.3
eliminate line segment crossing capacitor C_1.

p. 24: Figure 1.9(a)
move (t_0, x_0) closer to the center point.

p. 39: line after monkowski's Inequality
it should read $p = q = 2$.

p. 46: line 8
reduce space between $|x_i - x_{i0}| \leq b_i$ and $i = 1, \ldots, n$.

p. 69: line 21 - p. 70: line 4
eliminate paragraphs beginning with "We will view $y \in Y$ and $u \in U$ as system..." on p. 69, and ending with "...called the *unit pulse response* (or the *unit impulse response*) of a linear discrete-time system." Replace with:

We will view $y \in Y$ and $u \in U$ as system outputs and system inputs, respectively, and we let $T : U \to Y$ denote a linear transformation that relates u to y. We first consider the case when $u(k) = 0$ for $k < k_0, k, k_0 \in Z$. Also, we assume that for $k > n \geq k_0$, the inputs $u(k)$ do not contribute to the system output at time n (i.e., the system is *causal*). Under these assumptions, and in view of the linearity of T, and by invoking the representation of signals by (16.8), we obtain for $y = \{y(n)\}, n \in Z$, the expression $y(n) = T(\sum_{k=-\infty}^{\infty} u(k)\delta(n-k)) = T(\sum_{k=k_0}^{n} u(k)\delta(n-k)) = \sum_{k=k_0}^{n} u(k)T(\delta(n-k)) = \sum_{k=k_0}^{n} h(n,k)u(k), n \geq k_0$, and $y(n) = 0, n < k_0$, where $T(\delta(n-k)) \triangleq (T\delta)(n-k) \triangleq h(n,k)$ represents the response of T to a unit pulse (resp.,discrete-time impulse or unit sample) occurring at $n = k$.

When the assumptions in the preceding discussion are no longer valid, then a different argument than the one given above needs to be used to arrive at the system representation. Indeed, for *infinite sums,* the interchanging of the order of the summation operation $\sum$ with the linear transformation T is no longer valid. We refer the reader to a paper by I. W. Sandberg ("A Representation Theorem for Linear Discrete-Space Systems",*IEEE Transactions on Circuits and Systems - I*, Vol. 46, No. 5, pp. 578-580, May 1998) for a derivation of the representation of general linear discrete-time systems. In that paper it is shown that an extra term needs to be added to the right-hand side of equation (16.9), even in the representation of *general*, linear, time-invariant, causal, discrete-time systems. (In the proof, the Hahn-Banach Theorem (which is concerned with the extension of bounded linear functionals) is employed and the extra required term is given by $\lim_{l\to\infty} T(\sum_{k=-\infty}^{-c_l-1} u(k)\delta(n-k) + \sum_{k=c_l+1}^{\infty} u(k)\delta(n-k))$ with $c_l \to \infty$ as $l \to \infty$. For a statement and proof of the Hahn-Banach theorem, refer, e.g., to reference [12, pp. 367-370] given at the end of this chapter.) In that paper it is also pointed out, however, that cases with such extra *non-zero* terms are not necessa rily of importance in applications. In particular, if inputs and outputs are defined (to be non-zero) on just the non-negative integers, then for causal systems no additional term is needed (or more specifically, the extra term is zero), as seen in our earlier argument. In any event, *throughout the present book we will concern ourselves with linear discrete -time systems which can be represented by equation (16.9)* for the single-input/single-output case (and appropriate generalizations for multi-input/multi-output cases).

p. 71: line 4 from bottom of page
should read: "A discrete-time system described by (16.15) is said to be..."

p. 74: line 17-20
delete sentence beginning with "To put it another way..." and ending with "...sequence of functions is not even defined."

p. 112: table
change j_i to j_1.

p.125: line 7 from bottom of page
should read "...we note that condition (2.102) written as $(f-g)^l(\lambda_i)=0$ implies that..."

p. 129: line 3 from bottom of page
change 1 in column 3, row 3 of A matrix to 2.

p. 130: line 13
change Subsection G to Subsection O.

p. 134: line 14
should read: "...*primary decomposition theorem* presented in Subsection M..."

p. 139: line 6 from bottom of page
change coefficient α to α_i (insert subscript *i*).

p. 140: line 2
change Subsection 3.1B to Subsection 2.2B.

p.146: line 6
should read: "Substituting $\phi(t_0)=\Phi(t_0,t_0)\alpha+\phi_p(t_0)$, we obtain $\alpha=\phi(t_0)-\phi_p(t_0)=x_0-\phi_p(t_0)$."

p.155: Table 4.2
the fourth line of the second column should read $f(t-a)p(t-a), a>0$.

p. 168: line 9-10
should read: "...we take the Laplace transform of both sides of (6.12), we obtain..."
line 5 from bottom of page
should read: "...obtained (6.20) directly by taking the Laplace transform of $H(t)$ given in (6.12)."

p. 172: line 2
transfer function equation numerator should be $-5s-1$ instead of $-3s-1$.

p. 179: Table 7.2
in column 2, table heading should be $\{f(k)\}, k \geq 0$.

p. 184: line 4
should read: "$x(t_{k+1}) = \Phi(t_{k+1}, t_k)x(t_k) + ...$"

p. 194: line 9-10
should read: "...representations of a vector v^1..."

p. 198: line 20-22 (in Exercise 2.43)
should read: Plot the components of the solution ϕ. For different initial conditions $x(0) = (a,b)^T$..."

p. 199 : line 17
$= v_i \tilde{v}$ should be $v_i \tilde{v}_i$.

p. 200: line 19 (in Exercise 2.53)
replace parentheses with square brackets on column vectors in (a). It should read: "For $x(0) = [1,1,1,1]^T$ and for $u(t) = [1,1]^T$ $t \geq 0$,..."

p. 201: Exercise 2.58
replace in system 1 $[\ 1,\quad 0\ ]u(k)$ by $u(k)$.
delete in system 2 $[\ 0,\quad 1\ ]u(k)$.

p. 203: line 6
in part (b), (ii) should read: "$H(s) = \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$,".

p. 205: line 22 (in Problem 2.66(b))
should read: "...where $X \in R^{p \times n}$ and $Y \in R^{n \times m}$..."

p. 208: line 1
should be x_2 instead of x_3.

p. 208: Exercise 2.75
in entry $(4,5)$ should be k_3 instead of k_2.

line 4
in column 1, row 4 of matrix, replace $\frac{k_1}{m_1}$ with $\frac{k_1}{m_2}$.

p. 209: line 4 (in (a))
last element of vector should be $x_5 - x_3$ instead of $x_5 - x_1$.

p. 214: line 6
should read: "...will now be studied at length."

p. 230: line 12-17 from bottom of page
delete sentences beginning with "This implies that the null space of …" at line 17, ending with "… so $x_2^T x_1 \neq 0$", on line 12. Replace with:

This implies that the null space of $W_r(t_0,t_1)$ is nonempty. W_r is symmetric, and so the range of W_r is the orthogonal complement of its null space (prove this). Thus for any $u \in \mathcal{R}(W_r)$ and $v \in \mathcal{N}(W_r)$, $u^T v = 0$. Also, we may write $x_1 = x_1' + x_1''$ with $x_1' \in \mathcal{R}(W_r)$ and $x_1'' \in \mathcal{N}(W_r)$ ($x_1'' \neq 0$ since $x_1 \notin \mathcal{R}(W_r)$). Then there exists $x_2 \in \mathcal{N}(W_r)$ such that $x_2^T x_1'' \neq 0$, which implies $x_2^T x_1 \neq 0$.

p.236: line 13-18 from bottom of page
delete sentences beginning with "Indeed, this assumption implies that the null space of W_r is nonempty, …" at line 18, ending with "Therefore $x_2^T x_1 \neq 0$" on line 13. Replace with:

This implies that the null space of W_r is nonempty. W_r is symmetric, and so the range of W_r is the orthogonal complement of its null space (prove this). Thus for any $u \in \mathcal{R}(W_r)$ and $v \in \mathcal{N}(W_r)$, $u^T v = 0$. Also, we may write $x_1 = x_1' + x_1''$ with $x_1' \in \mathcal{R}(W_r)$ and $x_1'' \in \mathcal{N}(W_r)$ ($x_1'' \neq 0$ since $x_1 \notin \mathcal{R}(W_r)$). Then there exists $x_2 \in \mathcal{N}(W_r)$ such that $x_2^T x_1'' \neq 0$, which implies $x_2^T x_1 \neq 0$.

p. 241: line 5 from the bottom
should be $s_i I - A$ instead of $sI - A$.

p. 253: line 10 from bottom
should read: "...for every finite t. Therefore, in view of (3.22)..."

p. 254: line 9
should read: "...(3.24) is satisfied. In view of Lemma 3.6, (3.25) follows..."

p. 259: line 4
replace R_m by R^{pn}.

p. 261: line 17
should read: "...implies that $R_{\bar{cn}} = \{0\}$ or that the system is constructible."

p. 262: line 18
should read: "...$[\tilde{y}^T(-n), \ldots, \tilde{y}^T(-1)]^T$. Equation (3.56) must be solved for..."

p. 270: line 11-12 from bottom of page
should read: ”*Proof.* For details of the proof, refer to [8] and to R.E. Kalman, ”On the Computation of the Reachable/Observable Canonical Form,” SIAM J. Control and Optimization, vol. 20, no. 2, pp. 258-260, 1982, where further clarifications to [8] and an updated method of selecting Q are given.”

p. 280: line 6 from bottom of page
insert space or comma between $qA^{i-1}B = 0$ and $i = 1, \ldots, n-1$.

p. 302: line 4
replace both occurrences of $\lambda(s)$ with $\Lambda(s)$.

p. 313: Figure 3.6
add distance y demarcation to right of figure, extending from top to bottom of drawing.

p. 335: line 7
add filled box at end of line after ”...which is Ackermann's formula, (2.21).”

p. 336: line 14
change subscript of A from 12 to 22.

p. 338: line 20
should read: ”with $\begin{bmatrix} M_j \\ -D_j \end{bmatrix}$ a...”.

p. 376: line 26-28 (in Exercise 4.15)
remove indentations before ”Remark,” as it pertains to all four parts of the Exercise.

p. 379: Exercise 4.26(b)
should be $C = [\ 1, \ \ 0, \ \ 1\ ]$ instead of $C = [\ 1, \ \ 1\ ]$

p. 396: line 3-4
column 3, row 3 of the first and second matrices should be modified to be $CA^{2n-2}B$ and $\bar{C}\bar{A}^{2n-2}\bar{B}$, respectively.

p.405: line 7
replace $+x_1^{(n)}$ by $-x_1^{(n)} + x_1^{(n)}$

p.406: line 3
should read: ”$H(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$”.

p. 409: line 19

row 2, column 2 of the $D(s)$ matrix should be "s" instead of 2.

p. 409: line 16 from bottom
entry $(2,2)$ in $D(s)$ should be s instead of 2.

p. 419: last line on page
in right hand side of equation (4.47), replace $\delta_{m_H(s)}$ with $m_H(s)$,

p. 430: line 8 from bottom of page (in Exercise 5.22)
eliminate parentheses from denominator of equation for transfer function $H(s)$.
line 5 from bottom of page
replace $\dot{\hat{x}}$ with $\dot{x}$.

p. 431: Figure 5.10
add S to upper right hand corner of block diagram.
line 7
part (b) should read: "Determine a state-space representation for the *entire* system S, using your answer in (a)."

p.478: line 22
equation (8.11) should read: "$||\phi(t)|| \le (\frac{c_2}{c_1})^{1/2}||x_0||e^{1/2(\gamma/c_2)(t-t_0)}, \ t \ge t_0 \ge 0$,"

p. 507: line 5 from bottom of page
should read: "If we now assume that (10.23) is controllable (from-the-origin) and observable..."

p.536: line 15
should read: "Alternatively, it can be shown that any crd $G^*{}_R(s)$ of $P_1(s)$ and $P_2(s)$ [or a cld $G^*{}_L(s)$..."

p. 547: line 8 from bottom of page
should read: "The Eliminant Matrix..."

p. 554: line 7
should read: "at $t = 0$. (This is true if and only if $P^{-1}(q)$..."

p. 563: line 3
should read: "...the equilibrium $x = 0$ of the free system $\dot{x} = Ax$ is asymptotically stable."

p. 610: Figure 7.8
in caption, move $r = 0$ to end of previous line.

p. 613: line 12 from bottom of page
remove minus sign so to read $(X'_1, Y'_1) = (X'_0, Y'_0)$.

p. 621: line 5 from bottom
should be $\tilde{D}_2'^{-1}$ instead of $\tilde{D}_2^{-1}$ in H_2

p. 625: line 22
should be Theorem 4.21 instead of Theorem 4.20.

p. 625: line 5, equation (4.160)
should be

$$C = (X_1' - K'\tilde{N}')^{-1}[-(X_2' + K'\tilde{D}'), X'],$$

p. 625: line 6
should be ... Also, $U'U'^{-1} =$

p. 633: line 9 from bottom of page
should be Exercises 7.23, 7.26 instead of Exercises 7.23, 7.2b.

p. 637: line 9
should read: "...in the Fractional Approach to Design," *Int. J. Control*..."

p.639: line 11 (in Exercise 7.3(d))
should read: "(d) If $U(s)$ and $V(s)$ (in (c)) are unimodular..."
line 29-30 (in Exercise 7.6)
indent second and third lines of part (a).
line 34 (in Exercise 7.7)
indent second line of part (a).