Physics 10262 - Chapter 3
Homework 4 - Solution

1. Suppose you have a 10g carbon sample of 10000 years of age. Calculate the present activity of the sample using as original isotopic abundance ratio $^{14}C/^{12}C=1.3\cdot 10^{-12}$. How many counts do you expect from your measurement when you count for 1 day, 10 days, and for 100 days. What are the respective statistical uncertainties of your measurements?

$$N(^{12}C) = \frac{10}{12}\cdot 6.023\cdot 10^{23} = 5.02\cdot 10^{23}\ \text{[particles]}$$

$$N(^{14}C) = 1.3\cdot 10^{-12}\cdot 5.02\cdot 10^{23} = 6.53\cdot 10^{11}\ \text{[particles]}$$

$$N(^{14}C)_{t=10000y} = N(^{14}C)\cdot e^{-\lambda\cdot 10000y} = 1.95\cdot 10^{11}\ \text{[particles]}$$

$$A(^{14}C) = \lambda\cdot N(^{14}C) = \frac{ln 2}{5730}\cdot 1.95\cdot 10^{11} = 2.4\cdot 10^{7}\ \text{[decays/y]} = 6.46\cdot 10^{4}\ \text{[decays/day]}$$

$$\text{number of counts: N} = A(^{14}C)\cdot t,\ \text{uncertainty:}\sqrt{\text{N}}$$

$$\text{1 day:}\quad \text{N} = 6.46\cdot 10^{4} \pm 2.5\cdot 10^{2}\ \text{[decays]}\quad (0.2\%)$$

$$\text{10 days:}\quad \text{N} = 6.46\cdot 10^{5} \pm 8.0\cdot 102\ \text{[decays]}\quad (0.07\%)$$

$$\text{100 days:}\quad \text{N} = 6.46\cdot 10^{6} \pm 2.5\cdot 10^{3}\ \text{[decays]}\quad (0.04\%)$$

2. Recent radiocarbon dating of a supposedly ancient mummy from 600 BC gave an age to only 6 years. This would indicate that the mummy is a historical fake. How much less ^{14}C activity would you have expected for the mummy if the claimed age would have been correct?

$$A(6y) = A_0\cdot e^{-\lambda\cdot 6};\quad A(2600y) = A_0\cdot e^{-\lambda\cdot 2600}$$

$$\frac{A(2600y)}{A(6y)} = \frac{A_0\cdot e^{-\lambda\cdot 2600}}{A_0\cdot e^{-\lambda\cdot 6}} = e^{\lambda\cdot(6-2600)} = e^{-\frac{ln 2}{5730}\cdot 2554} = 0.734$$

3. You want to do radiocarbon dating using the AMS method. Suppose you have two tandem accelerators available, the first one provides you with 1MV terminal potential and the second one with 10 MV. With the first one you can produce carbon beams of charge state $q=2^+$, with the second one you produce carbon beams with $q=4^+$. Calculate the radii of the ^{12}C, ^{13}C, and ^{14}C trajectories in a separation magnet of B=0.1T field strength for both accelerators.

$$r = 0.144 \cdot \frac{\sqrt{(q+1)\cdot A \cdot V}}{q \cdot B}$$

r=	A=12	A=13	A=14
q=2+, 1MV	4.32 m	4.50 m	4.67 m
q=4+, 10MV	8.82 m	9.17 m	9.52 m

4. Determine the minimum amount of carbon material (in g) necessary to achieve a counting statistics of 3000±3 accuracy with AMS radiocarbon dating. How much carbon material would you need to achieve the same level of accuracy with the traditional LSC method?

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5. Calculate the age of radiocarbon sample with an initial $^{14}C/^{12}C$ ratio of $R=1.3\cdot 10^{-12}$ and a present ratio $3\cdot 10^{-13}$. What would be the error in age determination if your sample would have a 10% impurity of modern carbon material ($R=1.3\cdot 10^{-12}$)?

$$\frac{^{14}C}{^{12}C}(t) = \frac{^{14}C}{^{12}C}(t_o)\cdot e^{-\lambda\cdot(t-t_o)}; \quad \frac{3\cdot 10^{-13}}{1.3\cdot 10^{-12}} = 0.23 = e^{-\lambda\cdot(t-t_o)}$$

$$\ln(0.23) = -\lambda\cdot(t-t_o); \quad (t-t_o) = -\frac{5730\,[\mathrm{y}]\cdot \ln 0.23}{\ln 2} = 12149\,[\mathrm{y}]$$

$$\Delta t = 8266.64\cdot\left(\ln\frac{\frac{^{14}C}{^{12}C}(t_o)}{\frac{^{14}C}{^{12}C}(t) + 0.1\cdot\frac{^{14}C}{^{12}C}(t_o)} - \ln\frac{\frac{^{14}C}{^{12}C}(t_o)}{\frac{^{14}C}{^{12}C}(t)}\right)$$

$$\Delta t = 8266.64\cdot\left(\ln\frac{1.3\cdot 10^{-12}}{3\cdot 10^{-13} + 0.1\cdot 1.3\cdot 10^{-12}} - \ln\frac{1.3\cdot 10^{-12}}{3\cdot 10^{-13}}\right) = -2976\,[\mathrm{y}]$$

6. Calculate the age of a sample which you have dated using the potassium argon technique getting a $^{40}Ar/^{40}K$ ratio of $8.73 \cdot 10^{-6}$.

$$\frac{^{40}Ar}{^{40}K} = 0.11 \cdot (e^{\lambda \cdot t} - 1) = 8.73 \cdot 10^{-6}; \quad \lambda = \frac{ln\,2}{1.28 \cdot 10^{9}\ y} = 5.41 \cdot 10^{-10}\ [y^{-1}]$$

$$\frac{8.73 \cdot 10^{-6}}{0.11} + 1 = e^{\lambda \cdot t} = 1.000079364; \quad t = \frac{ln(1.000079364)}{\lambda} = \frac{0.000079360}{5.41 \cdot 10^{-10}\ [y^{-1}]} = 146692\,[y]$$

$$\text{simple linear approximation method: } t = \frac{\frac{^{40}Ar}{^{40}K}}{5.82 \cdot 10^{-11}} = \frac{8.73 \cdot 10^{-6}}{5.82 \cdot 10^{-11}} = 150000\ [y]$$

7. Explain how you can determine if you have ^{40}Ar diffusion from atmospheric ^{40}Ar into your sample. How can you correct for it? What is the origin of atmospheric ^{40}Ar?

Atmospheric argon has a fixed $^{40}Ar/^{36}Ar$ ratio, if ^{36}Ar is found in the sample, the ^{40}Ar content needs to be corrected using the atmospheric ratio since ^{36}Ar has diffused into material together with ^{40}Ar. The origin of all observed ^{40}Ar is due to the decay of the pre-solar ^{40}K.

8. Using the potassium argon method, the footprints of early hominoids in the volcanic ashes of eastern Africa have been dated to be 3,500,000 years old. Calculate and compare the ratio of ^{40}Ar and ^{40}K abundances using both, first the exact mother daughter relation for radioactive nuclei and second the linear approximation.

$$\frac{^{40}Ar}{^{40}K} = 0.11 \cdot (e^{\lambda \cdot t} - 1) = 0.11 \cdot (e^{5.41 \cdot 10^{-10}\ [y^{-1}] \cdot 3500000} - 1) = 2.084 \cdot 10^{-4}$$

$$\frac{^{40}Ar}{^{40}K} = 5.82 \cdot 10^{-11} \cdot t = 5.82 \cdot 10^{-11} \cdot 3500000 = 2.04 \cdot 10^{-4}$$