

Physics 10262 - Chapter 2 – Homework

1. Calculate the x-ray K absorption edge for lead (Z=82) and mercury (Z=80) and compare it with the absorption edge for carbon (Z=6) and aluminum (Z=13).

$$E_K = (Z - 1)^2 \cdot 13.6 \quad [eV]$$

$$E_K(Pb) = 89.23keV; \quad E_K(Hg) = 84.88keV;$$

$$E_K(C) = 0.34keV; \quad E_K(Al) = 1.96keV;$$

15pts

2. You want to perform a radiography analysis of a Byzantine icon, which has a gold layer of $d=0.3$ mm thickness. The attenuation coefficient of X-rays in gold is $\mu_{Au}=19.5cm^{-1}$. Determine the fraction of the transmitted x-ray intensity.

$$I(d) = I_o \cdot e^{-\mu \cdot d}; \quad \frac{I(d)}{I_o} = e^{-\mu \cdot d} = e^{-19.5cm^{-1} \cdot 0.03cm} = 0.56 \equiv 56\%$$

10pts

3. In a radiograph of an oil painting with a $d_1=0.2$ mm layer of oil paint (carbon), a $d_2=0.05$ mm underground of white lead paint and a wood backing of $d_3=2$ cm thickness the initial x-ray intensity is reduced by a significant fraction. Calculate the total absorption and the absorbed (not transmitted) fraction of X-ray intensity for each layer.

$$I(d) = I_o \cdot e^{-\mu_C \cdot d_C} \cdot e^{-\mu_{Pb} \cdot d_{Pb}} \cdot e^{-\mu_W \cdot d_W} = I_o \cdot e^{-\mu_C \cdot d_C - \mu_{Pb} \cdot d_{Pb} - \mu_W \cdot d_W} =$$

$$= I_o \cdot e^{-0.21cm^{-1} \cdot 0.02cm - 22.6cm^{-1} \cdot 0.005cm - 0.21cm^{-1} \cdot 2cm} = 0.584; \quad \text{with } \mu_C = \mu_W$$

$$\frac{I_{abs}}{I_o} = 1 - \frac{I(d)}{I_o} = 1 - 0.584 = 0.416 \equiv 41.6\%$$

20pts

4. Calculate the wavelength and the energy of the characteristic L ($n_i=3 \Rightarrow n=2$) and K ($n_i=2,3, \Rightarrow n=1$) X-rays transitions for iron Fe (Z=26), copper Cu Z=29, silver Ag (Z=47), gold Au (Z=79), mercury Hg (Z=80), and lead Pb (Z=82). Assume an effective charge reduction term $\sigma=1$ for K-transitions and $\sigma=9$ for L-transitions. Compare the results with the observed characteristic x-ray energies that can be found at the web: <http://www.csrii.iit.edu/mucal.html>!

$$E_K = (Z - 1)^2 \cdot 13.6 \left(1 - \frac{1}{n_i^2} \right) \text{ [eV]}$$

$$E_L = (Z - 9)^2 \cdot 13.6 \left(\frac{1}{4} - \frac{1}{n_i^2} \right) \text{ [eV]}$$

Element	Z	transition energies in [keV]					
		$E_{K(2 \rightarrow 1)}$	web	$E_{K(3 \rightarrow 1)}$	web	$E_{L(3 \rightarrow 2)}$	web
Fe	26	6.38	6.40	7.56	7.06	0.55	0.00
Cu	29	8.00	8.05	9.48	8.90	0.76	0.00
Ag	47	21.58	22.16	25.58	24.94	2.73	2.98
Au	79	62.06	68.78	73.55	77.97	9.26	9.71
Hg	80	63.66	70.82	75.45	80.26	9.52	9.98
Pb	82	66.92	74.96	79.32	84.92	10.07	10.55

15pts

5. What distinguishes the red color vermilion from minium? What characteristic x-ray lines would you expect for these pigments in x-ray fluorescence analysis?

Vermilion is HgS would expect Hg L-line at ~10 keV; minium is Pb₃O₄ you expect Pb L-lines at ~10.5 keV

10pts

6. Calculate the range of 3 MeV protons for a PIXE analysis of a gold layered iron cup. Your energy loss of the protons in gold is about 290MeV/mm. If the gold layer is 0.005 mm thin, how deep will the protons penetrate into the Iron.

$$\frac{\Delta E}{\Delta x} = \frac{290 \text{ MeV}}{1 \text{ mm}} \Rightarrow \Delta E = 290 \text{ MeV} \cdot 0.005 = 1.45 \text{ MeV}$$

$$R_A = 3.2 \cdot 10^{-4} \cdot \frac{\sqrt{A_A}}{\rho_A} \cdot R_{Air};$$

$$\rho_{Fe} = 7.86 \text{ g/cm}^3; \quad A_{Fe} = 56; \quad R_{Air}(1.55 \text{ MeV}) = 10 \text{ cm}$$

$$R_{Fe} = 3.05 \cdot 10^{-3} \text{ cm} = 30.5 \mu\text{m} = 0.0305 \text{ mm}$$

The protons lose 1.5 MeV of their initial energy in the thin layer of gold and penetrate 0.0305 mm into the iron cup.

15pts

7. What characteristic x-ray transition do you expect to show in a PIXE analysis of Gall-ink? What x-ray lines would you see in a PIXE analysis of the color pigments azurite, ultramarine, yellow ochre, and cerulean blue?

- Gall ink has Fe component, Fe line at ~ 7 keV
- Azurite has Cu component with line at ~8.9 keV
- Ultramarine has only light element components up to S, difficult to see because of low x-ray energies, highest line is the one from S at ~2.4 keV
- Yellow ochre has Fe component with line at ~7 keV
- Cerulean blue has Co and Sn components with lines at ~8.4 keV and 3,4 keV & 25.7 keV respectively

15pts