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Limitations to Teaching Children $2 + 2 = 4$:
Typical Arithmetic Problems Can Hinder Learning of Mathematical Equivalence

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Abstract

Do typical arithmetic problems hinder learning of mathematical equivalence? Second- and third-graders (7-9 years old, $N = 80$) received lessons on mathematical equivalence either with or without typical arithmetic problems (e.g., $15 + 13 = 28$ vs. $28 = 28$, respectively). Children then solved math equivalence problems (e.g., $3 + 9 + 5 = 6 + \underline{\quad}$), switched lesson conditions, and solved math equivalence problems again. Correct solutions were less common following instruction with typical arithmetic problems. In a supplemental experiment, fifth-graders (10-11 years old, $N = 19$) gave fewer correct solutions after a brief intervention on mathematical equivalence that included typical arithmetic problems. Results suggest that learning is hindered when lessons activate inappropriate existing knowledge.

Limitations to Teaching Children $2 + 2 = 4$:

Typical Arithmetic Problems Can Hinder Learning of Mathematical Equivalence

Some of the most well-known and widely studied phenomena in developmental psychology involve children's failures to learn in seemingly straightforward situations. For example, 3-year-old children continue to sort cards on the basis of one dimension (e.g., color), despite being told repeatedly that they should start sorting according to another dimension (e.g., shape; Zelazo, Frye, & Rapus, 1996), and 9-year-old children continue to hold their initial, incorrect hypotheses about factors that influence the speed of a racecar (e.g., presence of a muffler), even after observing the outcomes of several experiments that yield results to the contrary (Schauble, 1990). Why do children sometimes fail to learn in seemingly straightforward situations, despite substantial instruction? This question has broad theoretical and practical implications. Although it is tempting to attribute children's failures to general conceptual limitations in childhood, several researchers have noted the importance of domain knowledge and basic learning processes in children's failures (see Siegler, 2000).

Here we focus on children's failures to learn a seemingly straightforward topic in mathematics—mathematical equivalence. *Mathematical equivalence* is the relation between two quantities that are interchangeable. It can be expressed symbolically in the form of a math sentence (e.g., $3 + 4 = 7$) or equation (e.g., $x + 1 = 5$) containing two mathematical expressions separated by the equal sign. The symbolic form implies that the expression on the left side of the equal sign can be replaced by the expression on the right side and vice versa. Although there are nuances that distinguish equivalence from other relations such as equality and identity, these nuances are beyond the scope of the present study. The term “mathematical equivalence” will be used throughout this paper because it is the most inclusive and flexible (Kieran, 1981). The term will be used to refer specifically to mathematical equivalence in symbolic form. The key issue here is that we want children to be able to interpret math sentences and equations that contain the

equal sign *relationally* as statements of mathematical equivalence because a relational understanding of the equal sign is important for understanding algebra (Kieran, 1989; Knuth et al., 2007; Carpenter et al., 2003).

It is well documented that elementary school children (ages 7-11) in the United States have substantial difficulty learning to interpret the equal sign as a relational symbol of mathematical equivalence (Baroody & Ginsburg, 1983; Behr, Erlwanger, & Nichols, 1980; Kieran, 1981). Consider, for example, a task in which children are asked to check the “correctness” of math sentences written by a girl or boy who “attends another school.” Most children mark sentences such as $10 = 6 + 4$ as “incorrect” and change them to $6 + 4 = 10$, $4 + 6 = 10$, or even $10 + 6 = 4$ (Cobb, 1987; see also Behr et al., 1980, Baroody & Ginsburg, 1983; Rittle-Johnson & Alibali, 1999). Similar misconceptions are also apparent when children are asked to solve addition problems that have operations on both sides of the equal sign (e.g., $3 + 4 + 5 = 3 + \underline{\quad}$). These problems have been called “mathematical equivalence problems” in prior work (Perry, Church, & Goldin-Meadow, 1988; Perry, 1991). In the absence of instruction, approximately 82% of children (ages 7-11) solve mathematical equivalence problems incorrectly (McNeil, 2005), and even after receiving instruction, many children continue to have difficulty (Alibali, 1999; Perry, 1991; Rittle-Johnson & Alibali, 1999).

Why do children have trouble learning mathematical equivalence, and how can we help them overcome their difficulties? Contrary to long-established theories of children’s (mis)understanding of mathematical equivalence (e.g., Collins, 1974, as cited in Kieran, 1981; Piaget & Szeminska, 1941/1995), it does not appear as though children’s difficulties can be attributed to domain general conceptual limitations in childhood. Indeed, children in this age range succeed when asked to identify which pair of numbers (e.g., $3 + 4$, $7 + 1$, $5 + 6$) is equal to a given pair (e.g., $5 + 3$), suggesting that they can identify an equivalence relation between two addend pairs (Rittle-Johnson & Alibali, 1999). Children in this age range also are relatively good at solving math equivalence problems when they are presented in a semi-symbolic form in which faces of dice are used in place of the Arabic numerals, and the answer can be chosen from one of

four possible answers (Sherman, 2007). Moreover, a majority (~90%) of Chinese elementary school children solve math equivalence problems correctly, suggesting that age-related conceptual limitations are not the primary source of children's difficulties (Ding, Li, Capraro, & Capraro, 2007).

The emerging consensus is that children's difficulties with mathematical equivalence are caused, at least in part, by specific knowledge that children construct from their early experience with arithmetic in school (e.g., Baroody & Ginsburg, 1983; Carpenter, Franke, & Levi, 2003; McNeil & Alibali, 2005b; Seo & Ginsburg, 2003). In the United States, elementary school mathematics is monopolized by arithmetic (Beaton et al., 1996), which is usually taught in a very procedural fashion with little or no reference to the equal sign or the concept of mathematical equivalence. Moreover, typical arithmetic problems are almost always presented with the operations to the left of the equal sign and the "answer blank" to the right (e.g., $3 + 4 = \underline{\quad}$, Seo & Ginsburg, 2003), a format that does not highlight the interchangeable nature of the two sides of the equation. This narrow experience with arithmetic leads children to construct overly narrow knowledge structures that do not generalize beyond typical arithmetic problems (McNeil & Alibali, 2005b). Reliance on these knowledge structures, in turn, contributes to difficulties with higher-order problems, including math equivalence problems. Note that the claim is not necessarily that experience with arithmetic *per se* contributes to the difficulties, but rather that experience with arithmetic *as typically taught in U.S. schools* contributes to the difficulties.

Prior work has identified at least three patterns that children learn from their experience with arithmetic in school. These patterns have been called *operational patterns* (McNeil & Alibali, 2005b). First, children learn a perceptual pattern related to the format of math problems, namely the "operations = answer" format in which the operations are to the left of the equal sign and the answer blank is to the right (McNeil & Alibali, 2004; Seo & Ginsburg, 2003). Second, they learn a concept of the equal sign, namely that it means "calculate the total" (Baroody & Ginsburg, 1983; Behr et al., 1980; McNeil & Alibali, 2005a; Kieran, 1981). Third, they learn a strategy for solving math problems, namely "perform all the given operations on all the given

numbers'' (McNeil & Alibali, 2005b; Perry et al., 1988). With continued practice, children's representations of these operational patterns increase in strength. Subsequently, when these representations are activated in the face of math problems, they guide attention and influence how information is encoded, interpreted, and ultimately applied (cf. Bruner, 1957; McGilly & Siegler, 1990; McNeil & Alibali, 2004). Although this process likely facilitates fast and accurate performance on typical arithmetic problems, it may interfere with performance on problems that deviate from the typical form, such as math equivalence problems.

In accordance with this view, studies have shown that children do, indeed, misapply their knowledge of typical arithmetic problems to math equivalence problems. For example, when asked to reconstruct the problem $3 + 4 + 5 = 3 + \underline{\quad}$ after viewing it briefly, children rely on their knowledge of the typical "operations = answer" problem format and write " $3 + 4 + 5 + 3 = \underline{\quad}$ " (McNeil & Alibali, 2004). When asked to define the equal sign in the problem, children provide answers such as "it means what all the numbers add up to" (McNeil & Alibali, 2005a). When asked to solve the problem, children add up all the numbers and put "15" in the blank. This "add-all" strategy is the most common strategy used by children who have not received instruction on mathematical equivalence (McNeil, 2005; Perry et al., 1988), and use of this strategy increases as the amount of experience with traditional arithmetic increases from first to third grade (McNeil, 2007). Moreover, a recent study has even suggested that children's reliance on their knowledge of these operational patterns may hinder their ability to benefit from a brief one-on-one intervention on math equivalence problems (McNeil & Alibali, 2005b).

Although much of the available evidence supports the hypothesis that children's narrow knowledge of arithmetic hinders learning of mathematical equivalence, experimental evidence is lacking. One exception is an experiment by McNeil and Alibali (2005b, Experiment 2) in which undergraduates' knowledge of arithmetic was activated, or was not activated in a control condition. Undergraduates whose knowledge of arithmetic was activated were less likely to perform well on math equivalence problems. This finding is important from a theoretical standpoint because it shows that the activation of arithmetic knowledge can interfere with

performance on math equivalence problems. However, the practical significance is limited. The finding indicates that activating *undergraduates'* knowledge of arithmetic hinders *performance* on math equivalence problems in a *laboratory* setting, but we primarily want to know how activating *children's* knowledge of arithmetic affects their *learning* in *classroom* settings. If similar processes affect children's learning of mathematical equivalence, then aspects of the environment that activate and reinforce children's overly narrow knowledge of arithmetic may make it more difficult for children to learn mathematical equivalence.

If the activation of children's knowledge of typical arithmetic problems does indeed hinder children's learning of mathematical equivalence, then teachers may want to avoid lesson contexts that contain typical arithmetic problems when trying to improve children's understanding of mathematical equivalence. Different contexts activate different representations (e.g., Barsalou, 1982), and the context of a typical arithmetic problem activates children's overly narrow representations of the operational patterns (McNeil & Alibali, 2005a; McNeil et al., 2006). Granted, elementary school children may activate these representations to some degree anytime they see a math sentence or equation containing the equal sign; however, the likelihood and magnitude of that activation will be greatest in the context of a typical arithmetic problem because the problem maps directly onto the operational patterns (McNeil & Alibali, 2004; McNeil et al., 2006). Once children's knowledge of the operational patterns is activated, children may be less open to information that deviates from these patterns. This perspective suggests a somewhat counterintuitive method of remediation. That is, instead of teaching mathematical equivalence by presenting familiar arithmetic problems such as " $2 + 2 = 4$ " and teaching children that "two plus two is the same amount as four," it might be better to teach mathematical equivalence in a context that does not contain typical arithmetic problems (e.g., present " $4 = 4$ " and teach "four is the same amount as four"). According to this perspective, manipulations of the environment that are designed to activate children's knowledge of typical arithmetic problems should hinder learning of mathematical equivalence.

Although this prediction follows logically from the account that has been advanced thus far, there are also reasons to recommend the opposite. Indeed, intuition suggests that it might be easier for children to learn a new concept in the context of familiar content. Consistent with this view, many educators in the United States hold a review-based instruction philosophy, which assumes that previously learned material provides a supportive scaffold for new learning. This philosophy is evident in math classrooms in the United States where educators often implement a spiral mathematics curriculum in which old information is reintroduced year after year (see Jensen, 1990 for harms associated with the spiral mathematics curriculum). Similarly, several lines of research suggest that the most effective math and science instructions are the ones that “bridge” from children’s well-established knowledge to new material (e.g., Bryce & MacMillan, 2005; Clement, Brown, & Zeitsman, 1989; Koedinger & Nathan, 2004; Nathan, Stephens, Masarik, Alibali, & Koedinger, 2002). Both the review-based philosophy and the bridging approach suggest that it may be beneficial for teachers to design lessons that use children’s well-established knowledge (e.g., typical arithmetic) as support when introducing a new concept (e.g., mathematical equivalence).

Another reason to expect the presence of typical arithmetic problems to facilitate learning of mathematical equivalence is that it may provide an opportunity for cognitive conflict. Several theories suggest that cognitive conflict promotes learning and cognitive development (e.g., Limon, 2001; Piaget, 1957; VanLehn, 1996). These theories suggest that children will benefit from lessons that purposefully activate their incorrect, operational view of the equal sign and then contradict it. Previous research has shown that typical arithmetic problems such as “ $3 + 4 = \underline{\quad}$ ” activate the operational view (McNeil & Alibali, 2005a; McNeil et al., 2006). Thus, when an educator or parent teaches the relational meaning of the equal sign in the context of a typical arithmetic problem, it may push children into a state of cognitive conflict, which in turn may prompt them to reevaluate their operational view (cf. Limon, 2001).

The goal of the present study was to examine whether the presence of typical arithmetic problems undermines or enhances children’s learning from lessons on mathematical equivalence.

Children received an intervention that was designed to teach that the mathematical expression on the right side of the equal sign has to be the same amount as (equal to) the mathematical expression on the left side of the equal sign. Elementary school children understand what it means for two amounts to be equal (Rittle-Johnson & Alibali, 1999), so there was potential for them to benefit from this intervention. The intervention was given either in an *arithmetic condition* with typical arithmetic problems, or in a *control condition* without typical arithmetic problems. After the intervention, children solved math equivalence problems. If activating children's knowledge of typical arithmetic problems interferes with learning of mathematical equivalence (as predicted), then children should benefit less from the intervention in the arithmetic condition than from the intervention in the control condition. That is, children should solve fewer math equivalence problems correctly after receiving lessons in the arithmetic versus control condition.

Experiment 1

Method

Participants

The study was conducted at a public school in a small town near Raleigh, North Carolina. The racial/ethnic makeup of the school was 24% African American, 1% Asian, 6% Hispanic, and 69% White. Approximately 37% of children received free or reduced-price lunch. Ninety-three second- and third-grade children participated. Eleven were excluded because they were absent on one or more days. Two were excluded because their performance was three standard deviations away from the mean. The final sample contained eighty children (38 boys, 42 girls).

Materials

Scripted lessons on mathematical equivalence in symbolic form. Each daily lesson followed the same basic script, regardless of condition. The script had four main steps. First, teachers read the following instructions: "Today you are going to solve some problems. You may not talk to your neighbor, and you may not look at your neighbor's paper. Don't worry though! It's not a test, so you won't be graded. This is just to see what you think about the problems. If a

problem seems too hard for you to solve, then it's okay for you to guess." Second, children were presented with a math problem and asked to solve it. Third, children were presented with a correctly solved version of the problem, and the concept of mathematical equivalence was used to justify the correct answer. Fourth, children were presented with three incorrectly solved versions of the problem, and the concept of mathematical equivalence was used to explain why the answers were wrong. Steps 2-4 were then repeated with a second problem. Children and teachers received booklets to follow along with each daily lesson. The teachers' booklets served as their script for the lessons. Figure 1 shows the script for one problem ($15 + 13 = \underline{28}$) as it was presented in the children's booklets. Given that children worked with different problems across conditions within the same classroom (see procedure), the actual problems were not written in the teachers' booklets. Instead, all problems were replaced with the words "problem is here in students' booklet." With that exception, the teachers' booklets were identical to the children's.

Equation-solving test. Children were given unlimited time to solve twelve math equivalence problems, including six "right-blank" problems (e.g., $5 + 4 + 7 = 5 + \underline{\quad}$, $3 + 9 + 5 = 6 + \underline{\quad}$) and six "left-blank" problems (e.g., $9 + 5 + 7 = \underline{\quad} + 7$, $6 + 4 + 8 = \underline{\quad} + 3$). Problems of this type have been used extensively in previous research on children's understanding of mathematical equivalence (e.g., Alibali, 1999; McNeil & Alibali, 2005b; Perry et al., 1988). In order to solve these problems correctly without direct instruction on a correct procedure, children must understand that the equal sign is a symbol used to indicate that the mathematical expressions on either side of it are (or are set to be) equivalent. Children also must notice where the equal sign is in the problem and understand the rules for how numbers and operations can be manipulated in order to come up with a solution that makes both sides of the problem have the same value. Children's performance solving math equivalence problems correlates well with other measures designed to assess children's understanding of mathematical equivalence, such as defining the equal sign (Rittle-Johnson & Alibali, 1999) and encoding the structure of equations (McNeil & Alibali, 2004).

Prior to administering the equation-solving test, teachers read the following instructions: “Today we’re going to see how well you can solve some problems. Don’t worry! It’s not a test, so you won’t be graded. I just want you to do your best. After you finish solving the problems, close your booklet and sit quietly until everyone has finished. If a problem seems too hard for you to solve, then don’t worry, you can just give it your best guess.”

Procedure

The study was conducted over a two-week period. Children’s teachers administered the scripted lessons and collected the measures in the classroom setting. Teachers first taught two days of scripted lessons on mathematical equivalence. Lessons were on Tuesday and Thursday, and they lasted approximately 15-20 minutes each day. Then, children completed the 12 math equivalence problems on the equation-solving test on Friday. The following week, children again received scripted lessons on a Tuesday and Thursday in whichever condition they had not already received (e.g., children who already received lessons in the arithmetic condition now received lessons in the non-arithmetic condition). Finally, children completed the 12 problems on the equation-solving test again on Friday.

All children received the same *spoken* lessons because lessons were scripted. The lessons taught that the amount on the right side of the equal sign has to be the same as (equal to) the amount on the left side of the equal sign. However, children differed according to what they were looking at during the lessons. Children were given booklets to follow along with the lessons, and children were randomly assigned to lesson conditions through the use of these booklets. In the *arithmetic condition*, booklets contained typical arithmetic problems (e.g., $15 + 13 = \underline{28}$). In the *non-arithmetic condition*, booklets contained problems with the same solution designed to *not* activate knowledge of arithmetic (e.g., $28 = \underline{28}$). Table 1 presents the problems used in each condition. As can be seen in the table, math equivalence problems were not presented in either lesson condition. Thus, children never received direct instruction on a procedure for solving math equivalence problems. Figure 2 presents a brief excerpt from each of the lesson conditions as it was presented in students’ booklets.

The booklets enabled children *within the same classrooms* to be randomly assigned to conditions. This method of random assignment within classrooms is unique. Typically, teachers are randomly assigned to conditions, and children are nested within teachers. Randomly assigning children within classrooms offers several advantages, including higher power with fewer children and greater assurance that children across conditions are comparable. The development of this new methodology is an important contribution of the present study.

The final sample included 34 children who received lessons in the arithmetic condition first and 46 children who received lessons in the non-arithmetic condition first. No order effects were observed. That is, time point (first or second) did not have significant main effect on performance, and it did not interact with lesson condition to affect performance on equivalence problems (all p -values $> .10$). Nonetheless, to ensure the reliability of the reported results, three additional analyses were performed in which 12 of the 46 children who received the non-arithmetic condition first were omitted randomly (to simulate equal n). Conclusions based on the “equal- n ” analyses were the same as those reported here.

Coding

Children’s performance on the math equivalence problems on the equation-solving test was coded using a system developed by Perry et al. (1988). Children’s use of a correct strategy on a given problem was based on an objective criterion—the solution children wrote in the blank. As in past work, children were given credit for using a correct strategy as long as they were within ± 1 of the correct answer. For example, on the problem $4 + 8 + 9 = 4 + \underline{\quad}$ students would be given credit for using a correct strategy if they put 17, 16, or 18 in the blank.

Results and Discussion

Performance on the equation-solving test was poor. Even after receiving several lessons on mathematical equivalence from their classroom teachers, children solved only 0.73 ($SD = 0.98$) math equivalence problems correctly (out of 12). This poor performance is consistent with prior work (e.g., Alibali, 1999; McNeil & Alibali, 2005b, Experiment 1; Perry et al., 1988), and it highlights just how difficult it is for children to learn and apply mathematical equivalence.

Poor performance was characteristic of both second-grade children ($M = 0.69$, $SD = 0.52$) and third-grade children ($M = 0.74$, $SD = 0.83$).

A repeated-measures ANOVA was performed with lesson condition (arithmetic or non-arithmetic) as the independent variable and number correct on the equation-solving test (out of 12) as the dependent measure. There was a significant main effect of lesson condition, $F(1, 79) = 5.40$, $p = .023$, $\eta_p^2 = .06$. As predicted, children solved fewer problems correctly after receiving lessons in the arithmetic condition ($M = 0.56$, $SD = 0.90$) than after receiving lesson in the non-arithmetic condition ($M = 0.89$, $SD = 1.03$). However, it is important to note that children's performance on the math equivalence problems was very poor overall.

Given the potential floor effects, a nonparametric analysis is preferred. Therefore, in accordance with previous studies (e.g., McNeil & Alibali, 2005b; McNeil, 2007), we reanalyzed the data in terms of whether children solved at least one math equivalence problem correctly. Across conditions, 65% of children solved at least one math equivalence problem correctly. Previous studies have shown a much lower percentage (around 10-20%) for children in second and third grade who have not received lessons on mathematical equivalence, so this result suggests that the lessons did improve understanding of the symbolic form of mathematical equivalence, albeit weakly. Note that it is not a trivial matter for children to solve one math equivalence problem correctly. Children were free to write any number in the blank. Thus, the probability that children were able to generate the correct answer by guessing is low. Consistent with McNeil (2007), a greater percentage of second grade children (74%) than third grade children (60%) solved at least one math equivalence problem correctly.

We examined the effect of lesson condition on performance using a nonparametric test appropriate for repeated measures data. The McNemar test was used to compare children's likelihood of solving at least one math equivalence problem correctly after receiving lessons in the arithmetic condition versus the non-arithmetic condition. Results were consistent with the ANOVA. Out of the 46 children who did not solve at least one math equivalence problem correctly after receiving lessons in the arithmetic condition, 18 (39%) solved at least one math

equivalence problem correctly after receiving lessons in the non-arithmetic condition; however, only 4 (12%) of the 32 children who did not solve at least one math equivalence problem correctly after receiving lessons in the non-arithmetic condition solved at least one math equivalence problem correctly after receiving lessons in the arithmetic condition, $\chi^2(1, N = 80) = 8.91, p = .003$.

In summary, the presence of typical arithmetic problems hindered children's learning from lessons on mathematical equivalence. Given the same verbal lessons, children solved fewer math equivalence problems correctly on average after receiving lessons in the arithmetic condition than after receiving lessons in the non-arithmetic condition, and a smaller percentage of children solved at least one math equivalence problem correctly after receiving lessons in the arithmetic versus non-arithmetic condition. We suspect that the negative effects of the arithmetic condition are due to the overly narrow knowledge structures activated by typical arithmetic problems, specifically knowledge of the "operations = answer" problem format, the "calculate the total" interpretation of the equal sign, and the "perform all the given operations" problem-solving strategy. When these representations are activated, they have the potential to hinder learning of information that does not correspond to the typical form.

Although these results support the hypothesis that the activation of knowledge of typical arithmetic problems hinders learning of mathematical equivalence, there are at least two reasons to be cautious when interpreting the results. First, the experiment compared lessons presented in an arithmetic condition to lessons presented in a non-arithmetic condition. The non-arithmetic condition exposed children to nonstandard equation types (e.g., $28 = 28$), which are rarely presented in elementary mathematics instruction (Seo & Ginsburg, 2003). Thus, although we found evidence in support of the hypothesis that the arithmetic condition hinders learning of mathematical equivalence, another plausible interpretation of the results is that the non-arithmetic condition facilitates learning. To rule out this alternative hypothesis, the arithmetic condition would need to be compared to a context-free, control condition. Thus, we tested this comparison in a supplemental experiment (Experiment 2).

The second reason to be cautious is that performance on the math equivalence problems was very poor overall. Children in this age range perform well on typical arithmetic problems in which the goal is to determine the numerical value of several numbers and operations presented on the left side of the equal sign (e.g., $3 + 4 + 5 + 3 = \underline{\quad}$, McNeil & Alibali, 2004). They also perform well when asked to choose which addend pair (e.g., $1 + 6$, $3 + 5$, $2 + 4$) is equal to a given addend pair (e.g., $4 + 4$, Rittle-Johnson & Alibali, 1999). However, they struggle to solve math equivalence problems. The present experiment was no exception. Although the difference between the two lesson conditions in terms of use of a correct strategy was statistically significant and did not depend on the method of analysis, neither condition was very effective in increasing children's use of correct strategies on math equivalence problems. This limitation may be minor from a theoretical standpoint because results supported the hypothesis. Nonetheless, from a practical standpoint it would be more convincing if the effect could be replicated with a group of children who could reap more benefits from the intervention.

Unfortunately, finding such children is not a trivial task. It is a delicate balance. We need to find children for whom the operational way of thinking about equations is dominant prior to instruction, but who, at the same time, are likely to benefit from a brief intervention on mathematical equivalence. One possible solution is to screen older, more experienced children. Children who have more experience with a variety of mathematics topics (e.g., equivalent fractions, magnitude comparison problems) are less entrenched in the operational way of thinking (McNeil, 2007). They have the potential to exhibit both operational and relational ideas about equations depending on the context (McNeil & Alibali, 2005a; McNeil et al., 2006), so they should be more open to a brief intervention on mathematical equivalence. However, there may also be drawbacks to working with older children. Namely, it may be difficult to find older, more experienced children for whom the operational way of thinking about equations is dominant. Although older children continue to exhibit operational ideas about equations that hinder understanding of higher-order math topics in middle school and beyond (Kieran, 1981; Knuth, Stephens, McNeil, & Alibali, 2007; McNeil & Alibali, 2005b; McNeil et al., 2006), they

also exhibit relational ideas about equations in certain contexts (McNeil & Alibali, 2005a; McNeil et al., 2006) and solve math equivalence problems correctly (McNeil, 2007). It is crucial to work with children who still maintain the operational way of thinking about equations as their dominant view, so we can examine which conditions encourage (or discourage) children from moving beyond that view. At the same time, we need to work with children who have more experience with a broader array of mathematics topics, so we can increase the likelihood that they will be more open to an intervention than were the children in Experiment 1. Thus, we screened for such children in Experiment 2.

Experiment 2

Experiment 2 was designed to address two specific goals. First, we sought to compare the arithmetic condition to a control condition that did not include nonstandard equations. Second, we sought to test a group of older children who still exhibited the operational way of thinking about equations as their dominant view. We expected these children to be more likely than the second- and third-grade children to use correct strategies on math equivalence problems after a brief intervention.

Method

Participants

The study was conducted at a suburban public school in Connecticut. The racial/ethnic makeup of the school was 3% African American, <1% Asian, 2% Hispanic, and 94% White. Approximately 10% of children received free or reduced-price lunch. The children were selected from fifty-seven fifth-grade children who received consent to participate. Of those children, nineteen (11 boys and 8 girls) met criteria for participation because they interpreted the equal sign as an arithmetic operation on the pretest (see coding section for details). Of the thirty-eight children who did not meet criteria, thirty-one had other incorrect interpretations of the equal sign, and seven had a relational interpretation of the equal sign. The average reported math grades of children who met criteria for inclusion did not differ significantly from the average reported math grades of children who did not meet criteria, $F(1, 55) = 0.80, p = .37$.

Measures

Pretest measure of equal sign understanding. The measure of equal sign understanding was modeled after measures used in previous work (e.g., Knuth et al., 2006; McNeil & Alibali, 2005a; Rittle-Johnson & Alibali, 1999). There was an arrow pointing to the equal sign followed by two questions: (1) “The arrow above points to a symbol. What is the name of the symbol?” and (2) “What does the symbol mean?” This measure has good convergent validity (e.g., McNeil & Alibali, 2005a) and good concurrent validity (e.g., Knuth et al., 2006).

Equation-solving posttest. Children were given unlimited time to solve four right-blank math equivalence problems: $5 + 4 + 8 = 5 + _$, $4 + 8 + 9 = 4 + _$, $3 + 9 + 7 = 6 + _$, $9 + 4 + 6 = 3 + _$. We used four problems instead of 12 in favor of efficiency because past work has shown similar performance regardless of whether children solve three (e.g., McNeil & Alibali, 2000), four (e.g., Rittle-Johnson & Alibali, 1999), or greater than four math equivalence problems (e.g., Experiment 1, Alibali, 1999; Perry et al., 1988; McNeil & Alibali, 2004; Perry, 1991). We used only right-blank problems because several studies have used only right-blank problems (e.g., McNeil & Alibali, 2000; Rittle-Johnson & Alibali, 1999), and children in prior studies have been equally likely to use incorrect strategies on right- and left-blank problems (e.g., McNeil & Alibali, 2004). Indeed, children’s use of a correct problem-solving strategy does not seem to depend on (a) the location of the missing addend, (b) the symbol used in place of the missing addend ($_$ versus \square), or (c) the number of addends on the left side of the problem (see Carpenter et al., 2003; Ding et al., 2007). Children are just as likely to use an incorrect strategy on the problem $5 + 4 + 8 = 5 + _$ as they are on the problem $\square + 8 = 12 + 5$.

Procedure

The experiment was held in school during regular school hours. Children were given the pretest measure, the intervention, and the posttest measure as paper-and-pencil worksheets in a large-group setting. First, children completed the pretest measure of equal sign understanding. Then, they were randomly assigned to one of two brief interventions on the meaning of the equal sign. Figure 3 displays the two interventions as children saw them. As shown in the figure, the

interventions taught the same information about the equal sign—the amounts on both sides have to be the same value. Both interventions contained this conceptual information about the meaning of the equal sign. The factor that was manipulated between the interventions was the context in which the equal sign was presented. In the *arithmetic context*, the equal sign was presented in the context of a typical arithmetic problem (e.g., $3 + 4 = \underline{7}$). In the *alone context*, the equal sign was presented alone, without any associated math context (e.g., $=$). The alone context was used instead of the nonstandard equation types that were used in Experiment 1 (e.g., $28 = \underline{28}$) to ensure that any differences between conditions could be attributed to the negative effect of the typical arithmetic problems and not to the beneficial effect of nonstandard equation types.

Coding

Equal sign understanding. Children's responses on the pretest measure of equal sign understanding were coded based on the system used by McNeil and Alibali (2005b). This system differs from previous coding systems (e.g., McNeil & Alibali, 2005a; Rittle-Johnson & Alibali, 2001) because it distinguishes between different types of incorrect definitions of the equal sign. Specifically, it distinguishes children who mention arithmetic operations and/or arithmetic symbols in their definitions (e.g., "it means you need to add, subtract, multiply, or divide") from children who do not (e.g., "it means what the question is"). We know that children who mention arithmetic operations and/or arithmetic symbols in their definitions are more deeply entrenched in the operational view of equations than are children who provide other incorrect definitions of the equal sign (McNeil & Alibali, 2005b). Because our goal in the present experiment was to work with older children who are still entrenched in the operational view, we specifically screened for children who mentioned arithmetic operations and/or arithmetic symbols in their definitions of the equal sign. Reliability was established by having a second coder evaluate the interpretations of a randomly selected 20% sample ($N = 11$). Agreement between coders was 100%.

Equation-solving performance. Coding of equation-solving performance was identical to that in Experiment 1.

Results and Discussion

The fifth-grade children's performance on the equation-solving test was relatively good compared to the performance of second- and third-grade children in Experiment 1. After receiving the brief intervention on the meaning of the equal sign, children solved 2.10 ($SD = 1.97$) math equivalence problems correctly (out of 4). This finding is consistent with recent work showing that performance on math equivalence problems is better after age 9 (McNeil, 2007). An ANOVA was performed with intervention context as the independent variable and number correct on the equation-solving measure (out of 4) as the dependent measure. Children who received the intervention in the typical arithmetic context solved fewer problems correctly than did children who received the intervention in the alone context ($M = 1.00$, $SD = 0.50$ versus $M = 3.10$, $SD = 0.42$), $F(1, 17) = 7.26$, $p = .01$, $\eta_p^2 = .30$. Thus, there was a large effect of intervention context on performance (as indicated by the effect size). Another way to analyze the data is to use a nonparametric test to examine whether condition affects whether or not children ever solved at least one math equivalence problem correctly. Only three of nine children (33%) solved at least one math equivalence problem correctly after receiving the intervention in the arithmetic context, whereas eight of ten children (80%) solved at least one math equivalence problem correctly after receiving the intervention in the alone context, $\chi^2(1, N = 19) = 4.23$, $p = .04$.

Taken together with the results of Experiment 1, these findings support the hypothesis that the activation of children's knowledge of typical arithmetic problems hinders their learning of mathematical equivalence. When a typical arithmetic problem is present in the environment, it activates children's representations of the operational patterns. The activation of these representations, in turn, hinders learning of information that deviates from the typical arithmetic form. Findings suggest that learning of mathematical equivalence is hindered when overly narrow knowledge of typical arithmetic is activated. The potential processes involved in these effects and the implications are discussed in the next section.

General Discussion

Over twenty years of research suggests that children's difficulties with mathematical equivalence are caused, at least in part, by specific knowledge that children construct from their narrow experience with typical arithmetic problems in school (e.g., Baroody & Ginsburg, 1983; Carpenter et al., 2003; McNeil, 2007). According to this view, children should have more difficulty learning mathematical equivalence when their narrow knowledge is activated than when it is not activated. The present study was the first to test this hypothesis experimentally. In Experiment 1, second- and third-grade children were less likely to solve math equivalence problems (e.g., $3 + 9 + 5 = 6 + \underline{\quad}$) correctly after receiving lessons in the presence of typical arithmetic problems than after receiving lessons in the presence of other math problems. In Experiment 2, fifth-grade children who still interpreted the equal sign as an arithmetic operation solved fewer math equivalence problems correctly after receiving an intervention in the presence of typical arithmetic problems than after receiving an intervention in the presence of the equal sign alone. Both experiments support the hypothesis that the presence of typical arithmetic problems hinders learning of mathematical equivalence.

Given the theoretical and practical implications of these findings, it is important to understand the processes that may account for the observed effects. The explanation that has been advanced throughout this paper is that the presence of typical arithmetic problems hinders learning because typical arithmetic problems activate overly narrow representations that do not generalize broadly. Specifically, typical arithmetic problems are thought to activate children's representations of: (a) the "operations = answer" problem format, (b) the "calculate the total" interpretation of the equal sign, and (c) the "perform all the given operations" problem-solving strategy (McNeil & Alibali, 2005b). When these representations are activated, they have the potential to influence how information is encoded, interpreted, and used (cf. Bruner, 1957; McGilly & Siegler, 1990), so information that does not map onto the patterns may not be learned as well as it could be in the absence of those activated representations. Although the results of the present study support this account, they do not rule out the possibility that other processes may have played a role in the observed effects.

For example, in addition to activating children's representations of specific patterns that interfere with learning (e.g., "operations = answer" problem format), the presence of typical arithmetic problems may trigger a more general cognitive bias, or mindset, that leads children to be inattentive to instructional input. Specifically, typical arithmetic problems may promote *mindlessness*, which has been defined as being committed to a "single, rigid perspective and... oblivious to alternative ways of knowing" (Langer, 2000). Mindlessness is a general state of mind that reduces individuals' sensitivity to the environment. Instead of noticing novel aspects of the environment and actively constructing strategies that apply to the current situation, mindless problem solvers are content to rely on the strategies they have used many times in the past. In the current study, the presence of typical arithmetic problems may have promoted mindlessness because children have seen and solved typical arithmetic problems many times in the past. If this is the case, then children may have been less attentive to the lessons in the arithmetic condition than in the control condition in both experiments.

It is also possible that some other systematic difference between the arithmetic and control conditions (other than the presence of typical arithmetic problems per se) contributed to the observed effects. In Experiment 1, the control condition included nonstandard math sentences such as $28 = 28$. These nonstandard math sentences may facilitate learning of mathematical equivalence. If this is the case, then the findings of Experiment 1 could be attributed to the beneficial effect of nonstandard math sentences, rather than to the negative effect of typical arithmetic problems. To rule out this alternative explanation, the control condition in Experiment 2 did not include nonstandard math sentences. Results were consistent with Experiment 1; however, the possibility remains that some other factor varied systematically between the arithmetic and control conditions across experiments.

For example, the stimuli in the arithmetic and control conditions differed in terms of novelty. Children rarely see the equal sign presented by itself or in the context of nonstandard math sentences (Seo & Ginsburg, 2003), so the stimuli presented in the control conditions were less familiar to children than were the stimuli presented in the arithmetic conditions. This novelty

alone may have improved children's performance by increasing children's attention to the equal sign either during the lessons, or during the equation-solving test, or both. Increasing children's attention to the equal sign has been shown to affect children's strategy choices on math equivalence problems (Alibali, McNeil, & Perrott, 1998). However, increased attention to the equal sign cannot, by itself, account for better performance in the control conditions because prior work has shown no evidence that it affects use of *correct* strategies (Alibali et al., 1998). Another factor that varied between the two conditions was the amount of exposure to numbers. There were more numbers in the arithmetic conditions than in the control conditions. It is possible that the preponderance of numbers increased children's attention to the numbers at the expense of other important features of the lessons. This increased attention to the numbers also may have hurt performance by carrying over to the equation-solving test. If children attended only to the numbers and ignored the location of the equal sign, then they would not have been able to solve the math equivalence problems correctly. Obviously, there is much work to be done before we fully understand the processes involved in the observed effects. Nonetheless, the current study is valuable because it provides the first evidence that children benefit less from a lesson on mathematical equivalence in the presence of typical arithmetic problems than in the presence of either nonstandard math sentences, or the equal sign alone. This finding has potential implications for mathematics classrooms.

Educational Implications

The current study adds to the debate about the most effective ways to teach children new information that diverges from their well-established knowledge. Nathan and colleagues (2002) argue that the most effective instructions are ones that "bridge" from children's well-established knowledge to new material (see also Bryce & MacMillan, 2005; Clement et al., 1989; Koedinger & Nathan, 2004), and some researchers suggest that it is especially beneficial to activate children's incorrect knowledge during instruction because it provides an opportunity for cognitive conflict (Limon, 2001; Piaget, 1957; VanLehn, 1996). However, the present results suggest that it may not always be a good idea to activate children's well-established knowledge.

In some cases, the activation of well-established knowledge may hinder children's learning of new information. Teachers may want to avoid activating well-established knowledge in these cases. Specifically, teachers may want to avoid activating children's well-established knowledge of typical arithmetic problems when teaching higher-order topics such as mathematical equivalence.

If the activation of children's knowledge of arithmetic contributes to children's difficulties learning mathematical equivalence, then it may be the case that children who are most proficient with typical arithmetic problems are slowest at developing an understanding of algebraic concepts. Consistent with this view, children with the strongest representations of the typical arithmetic patterns have the most difficulties learning from a one-on-one lesson on mathematical equivalence (McNeil & Alibali, 2005b). This hypothesis conflicts with intuition and traditional mathematics practices. Intuition suggests that children who are best at one math topic should be best at another math topic. And indeed, most schools assign, or track, children to algebra based on their performance in arithmetic. This policy is based on theories that suggest that children need to be proficient with "basic" arithmetic facts to be successful at algebra (e.g., Haverty, 1999; Haverty, Koedinger, Klahr, & Alibali, 2000; McKeown, 2002; Kaye, 1986). However, this policy might not be the best approach if children's overly narrow knowledge of arithmetic hinders learning of algebra. Future research in our lab will examine how proficiency with typical arithmetic problems affects understanding of algebra.

Do results of the present experiments imply that teachers should stop teaching arithmetic? Certainly not, but they do suggest that teachers may want to consider alternative approaches to teaching arithmetic. For example, instead of reintroducing arithmetic procedures every year and restricting algebra to a course in eighth or ninth grade, educators could integrate more algebraic ways of thinking into elementary school mathematics, starting in first grade as recommended by many math education researchers (e.g., Blanton & Kaput, 2003; Carpenter & Levi, 1999; NCTM, 2000). One strategy for doing this is to focus instruction on mathematical equivalence (Carpenter et al., 2003). For example, instead of simply practicing typical arithmetic "facts" such

as “ $3 + 4 = 7$,” children can learn “ $7 = 3 + 4$,” “ $3 + 4 = 5 + 2$,” and “ $7 = 7$.” Approaches such as this may prevent the entrenchment of overly narrow knowledge of arithmetic in the first place and facilitate the notoriously difficult transition from arithmetic to algebra. We are currently testing the benefits of practice with nonstandard arithmetic problems.

Critics may argue that the entrenchment of overly narrow knowledge of arithmetic is not an important educational problem because performance on math equivalence problems tends to improve after age nine (McNeil, 2007). In line with this view, the fifth-grade children in Experiment 2 performed reasonably well on math equivalence problems after only receiving a brief intervention. However, it may be possible for older children to improve their performance on math equivalence problems without developing a sophisticated understanding of mathematical equivalence. Indeed, children can learn to perform well on problems despite substantial limitations in their conceptual knowledge (Sophian, 1997). We know that representations of the overly narrow operational patterns continue to co-exist along side correct understandings of mathematical equivalence in middle school and beyond (McNeil & Alibali, 2005a; McNeil & Alibali, 2005b, Experiment 2). The activation of these overly narrow representations may contribute to many of the difficulties that older students have when working with symbolic expressions and equations in algebra (Kieran, 1981; Carpenter et al., 2003; Knuth et al., 2006). For example, failure to interpret the equal sign relationally may contribute to middle-school students' difficulties solving algebraic equations such as $4m + 10 = 70$ (Knuth et al.). Similarly, commitment to the “operations = answer” format may contribute to high-school students' difficulty understanding algebraic expressions that do not contain an equal sign such as $3a$, $a + 3$, and $3a + 5a$ (Kieran, 1981). For these reasons among others, it is important to study the best ways to help children overcome their overly narrow views of arithmetic.

Methodological Advancement

One of the primary contributions of the present study is a new methodology for studying factors that affect children's learning. Since the Education Sciences Reform Act of 2002, the establishment of the Institute of Education Sciences, and the publication of *Scientific Research in*

Education (National Research Council, 2002), researchers have increased their efforts to apply experimental methods to the study of educational problems. However, it is difficult to perform rigorously controlled experiments in schools. At the same time, educators are highly skeptical of rigorously controlled experiments that occur outside of schools (in laboratories) because they are often small in scale and do not involve the complexities encountered in real-life educational settings. The design used in Experiment 1 provides a way to implement the rigorous control of a laboratory experiment within the context of a classroom.

In Experiment 1, lesson booklets were used to randomize children within classrooms to conditions, and teachers were given a script that applied to both conditions. This method allowed researchers to have a high degree of control over what children saw and heard during the study (similar to the laboratory setting). At the same time, teachers administered the lessons and collected all measures in a typical classroom setting. Of course, critics of classroom-based *experiments* may argue that the booklet format provides a shallow imitation of the rich lessons that teachers should be implementing in the classroom, and critics of *classroom-based* research may argue that the experiment was not as systematic and controlled as it would have been if it were performed one-on-one in the laboratory. Given the large gulf between these two camps, the booklet methodology seems to offer a good compromise. Researchers who are interested in children's learning may want to consider including within-classroom randomization in their methodological toolkit.

Concluding Remarks

The present findings contribute to our understanding of the processes involved in children's failures to learn in seemingly straightforward situations. Such failures historically have been attributed to general conceptual limitations in childhood. However, the current study adds to a growing literature that emphasizes the role of domain knowledge and basic learning processes in children's failures. Results suggest that children's learning failures may be caused, at least in part, by the activation of well-established knowledge. Because many studies focus on how knowledge helps learning (e.g., Rittle-Johnson, Siegler, & Alibali, 2001), situations in

which knowledge hinders learning provide a unique window into how the mind works. Future research efforts should be devoted to developing a general framework for understanding when knowledge helps versus hinders learning. By studying how existing knowledge plays a role in the acquisition of new information, researchers can inform both our understanding of how humans acquire knowledge, and our efforts to modify the external environment to enhance children's learning and development.

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Table 1

Problems Used in Each Lesson Condition

Arithmetic condition	Non-arithmetic condition
$15 + 13 = \underline{28}$	$28 = \underline{28}$
$19 - 7 = \underline{12}$	1 foot = <u>12</u> inches
$6 + 5 - 8 = \underline{3}$	$\bullet\bullet\bullet = \underline{3}$
$91 + 4 = \underline{95}$	$95 = \underline{95}$

Figure Captions

Figure 1. Script for one problem as it appeared in the students' booklets. Each panel represents one page.

Figure 2. Brief excerpt from the students' lesson booklets in the arithmetic context (left panel) and non-arithmetic context (right panel).

Figure 3. The equal sign intervention in the arithmetic context (top panel) and alone context (bottom panel).