

International Macroeconomics and Finance.

Solutions to end-of-chapter problems.

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Chapter 3—The Monetary Model.

1. We are given $E_t(f_{t+k}) = \bar{f}_t + \rho^k z_t$. Using this formula in (3.12) gives the result.
2. The fundamentals in this case are $f_t = z_t$. Taking the variance of s_t in (3.28) gives the result. Notice that the factor or proportionality is less than 1. That's because when $f_t < 0$, $s_t < f_t$ and vice-versa. (Note: $E(f_t) = 0$). So $f_t - s_t > 0$ predicts $\Delta s_t > 0$ but this occurs when f_t lies below its mean.
3. Take first differences of (3.28) and note that ϵ_t and Δz_t are independent. Note that $(1 - \rho^2)\text{Var}(z_t) = \sigma_u^2$. Now $\Delta z_t = (\rho - 1)z_{t-1} + u_t \Rightarrow \text{Var}(\Delta z_t) = 2(1 - \rho)\text{Var}(z_t)$ and $\text{Var}(\Delta f_t) = \sigma_\epsilon^2 + 2(1 - \rho)\text{Var}(z_t)$ and $1 + \lambda(1 - \rho) > 1$ so $\text{Var}(\Delta s_t) < \text{Var}(\Delta f_t)$ and the variance of the depreciation still does not exceed the variance of fundamentals growth.

Chapter 4. The Lucas Model

1. The planner's problem is to maximize

$$\phi[\theta \ln c_x + (1 - \theta) \ln c_y] + (1 - \phi)[\theta \ln c_x^* + (1 - \theta) \ln c_y^*]$$

subject to

$$c_x + c_x^* = x, \quad \text{and} \quad c_y + c_y^* = y.$$

With $\phi = 1/2$, the efficient risk sharing conditions are,

$$(1 - \phi)c_{xt} = \phi c_{xt}^*, \quad (1 - \phi)c_{yt} = \phi c_{yt}^*.$$

If the home agent owns all of the x - firm, the foreign agent owns all of the y - firm, under zero capital mobility with goods trade, the home agent's problem

is to maximize

$$\theta \ln c_{xt} + (1 - \theta) \ln c_{yt}$$

subject to

$$c_{xt} + q_t c_{yt} = x_t.$$

Taking q_t as given, we get $q_t = [(1 - \theta)/\theta](c_{xt}/c_{yt})$. Substitute this into the budget constraint to get the demand for x , $c_{xt} = \theta x_t$. The foreign agent maximizes

$$\theta \ln c_{xt}^* + (1 - \theta) \ln c_{yt}^*$$

subject to

$$c_{xt}^* + q_t c_{yt}^* = q_t y_t.$$

The foreign demand for y is $c_{yt}^* = (1 - \theta)y_t$. The equilibrium under goods trade with zero capital mobility is therefore,

$$\begin{aligned} c_{xt} &= \theta x_t, & c_{xt}^* &= (1 - \theta)x_t \\ c_{yt} &= \theta y_t, & c_{yt}^* &= (1 - \theta)y_t \end{aligned}$$

We get,

	Marginal Utility	
	x_t	y_t
Home	θ/x_t	$(1 - \theta)/y_t$
Foreign	θ/x_t	$(1 - \theta)/y_t$

The equilibrium attains the efficient risk-sharing condition with $\phi = \theta$.

2. The efficient risk-sharing condition is $\phi/c_t = (1 - \phi)/c_t^*$. The implied allocations are $c_t = \phi(x_t + x_t^*)$ and $c_t^* = (1 - \phi)(x_t + x_t^*)$. Under zero capital mobility, no goods trade can occur. The equilibrium is that the agents consume their endowments and live in complete autarchy $c_t = x_t$ and $c_t^* = x_t^*$. If x_t and x_t^* are imperfectly correlated, the marginal utility of the home and foreign agents will also be imperfectly correlated.
3. The planner's problem is to maximize

$$\phi u(c_x, c_y, c_N) + (1 - \phi)u(c_x^*, c_y^*, c_N^*)$$

subject to

$$\begin{aligned}x &= c_x + c_x^* \\y &= c_y + c_y^* \\c_N &= N \\c_N^{**} &= N^*\end{aligned}$$

We have,

$$\begin{aligned}u_1 &= \theta_1 \frac{C^{1-\gamma}}{c_x}, & u_1^* &= \theta_1 \frac{C^{*(1-\gamma)}}{c_x^*} \\u_2 &= \theta_2 \frac{C^{1-\gamma}}{c_y}, & u_2^* &= \theta_2 \frac{C^{*(1-\gamma)}}{c_y^*} \\u_3 &= \theta_3 \frac{C^{1-\gamma}}{c_N}, & u_3^* &= \theta_3 \frac{C^{*(1-\gamma)}}{c_N^{**}}\end{aligned}$$

Efficient risk-sharing requires,

$$\begin{aligned}\phi u_1 &= (1 - \phi) u_1^* \\ \phi u_2 &= (1 - \phi) u_2^* \\ C_N &= N \\ C_N^{**} &= N^*\end{aligned}$$

The first two conditions for this utility function give,

$$\begin{aligned}\phi c_x^{\theta_1(1-\gamma)-1} c_y^{\theta_2(1-\gamma)} N^{\theta_3(1-\gamma)} &= (1 - \phi) [x - c_x]^{\theta_1(1-\gamma)-1} [y - c_y]^{\theta_2(1-\gamma)} N^{*\theta_3(1-\gamma)} \\ \phi c_x^{\theta_1(1-\gamma)} c_y^{\theta_2(1-\gamma)-1} N^{\theta_3(1-\gamma)} &= (1 - \phi) [x - c_x]^{\theta_1(1-\gamma)} [y - c_y]^{\theta_2(1-\gamma)-1} N^{*\theta_3(1-\gamma)}\end{aligned}$$

The planner's allocate rule of x and y to the domestic and foreign agents is contingent on N and N^* .

In the zero-capital mobility but trade in goods problem, domestic person maximizes $u(c_x, c_y, c_N)$ subject to

$$c_x + q_y c_y + q_N c_N = x + q_N N$$

where q_y is the relative price of y in terms of x and q_N is the relative price of N in terms of x . From the first-order conditions, we get,

$$\begin{aligned} q_y &= \frac{\theta_2 c_x}{\theta_1 c_y} \\ q_N &= \frac{\theta_3 c_x}{\theta_1 N} \end{aligned}$$

Substitute the Euler equations into the budget constraint to get the domestic demand function

$$c_x = \frac{\theta_1}{\theta_1 + \theta_2} x, \quad c_y = \frac{\theta_2 x}{(\theta_1 + \theta_2) q_y}, \quad c_N = N$$

Foreign person maximizes $u(c_x^*, c_y^*, c_N^*)$ subject to

$$c_x^* + q_y c_y^* + q_{N^*}^* c_{N^*}^* = q_y y + q_{N^*}^* N^*$$

where $q_{N^*}^*$ is the relative price of N^* in terms of x , which results in

$$c_x^* = \frac{\theta_1 q_y y}{(\theta_1 + \theta_2)}, \quad c_y^* = \frac{\theta_2 y}{(\theta_1 + \theta_2)}, \quad c_{N^*}^* = N^*$$

In equilibrium, $q_y = \frac{\theta_2 x}{\theta_1 y}$. It follows that

$$c_x = \frac{\theta_1 x}{(\theta_1 + \theta_2)}, \quad c_y = \frac{\theta_1 y}{(\theta_1 + \theta_2)}, \quad c_N = N$$

and

$$c_x^* = \frac{\theta_2 x}{(\theta_2 + \theta_2)}, \quad c_y^* = \frac{\theta_1 y}{(\theta_1 + \theta_2)}, \quad c_{N^*}^* = N$$

It should be clear that the efficient-risk sharing conditions cannot be satisfied with these equilibrium consumption rules due to the presence of nontraded goods under portfolio autarchy.

4. You should get the same exchange rate solution. Solutions for equity prices are different though.