# Supporting Text

## Determining the Phase Boundary

As pointed out in the main paper, in order to construct a two-dimensional phase boundary, we must obtain the extrema (the maximum and the minimum) of $m_{11}$ given $m_{10}$, or vice versa. Equivalent, but more convenient, is to extremize $m_{11}$ or $m_{10}$ given the degree sum $K_1 = 2m_{11} + m_{10}$ of the $n_1$ nodes in the group. Writing as $K_{\max}$ and $K_{\min}$ the largest and the smallest possible degree sums for $n_1$ nodes in the network, we partition the range $[K_{\min}, K_{\max}]$ into $S$ segments of equal length $\Delta_K \equiv (K_{\max} - K_{\min})/S$. Then we run an optimization algorithm such as the Fiduccia-Mattheyses method (1-4) to obtain the extrema of $m_{11}$ or $m_{10}$ while limiting $K_1$ to be within the range

$$K_{\min} + (i \pm \frac{1}{2})\Delta_K \tag{1}$$

for $0 \leq i \leq S$. This procedure is visualized in Fig. 4.

## Stationarity of the Phase Boundary in ERG Networks

Unlike the Erdös-Rényi and the scale-free networks shown in Fig. 2 in the paper, the power-law networks generated by the exponential random graph method (5) show a slow convergence to a limiting value near the $2m_{11}+m_{10} = K_{\max}$ line for small $a$ (Fig. 5). We show, however, that this is a finite-size effect, and it disappears as the degree distribution $p_k$ approaches a true power law as $N \to \infty$. We first note that the farthest right-upper point of the phase diagram corresponds to $K_{\max}$. It is well known that in a finite power-law network, the largest node degree is $N^{1/(\tau-1)}$ (6), i.e., shows a sublinear scaling with respect to $N$. Therefore, $a$ can be written as ($C$ is a constant)

$$a \;=\; C\int_{k_0}^{N^{1/(\tau-1)}} k^{-\tau} = \frac{C}{\tau-1}\left(k_0^{-(\tau-1)} - \frac{1}{N}\right), \tag{2}$$

from which we get the appropriate lower bound of the integral

$$k_0 = (\frac{a(\tau-1)}{C} + \frac{1}{N})^{1/(-\tau+1)}. \tag{3}$$

Finally, this gives

$$\begin{aligned} \frac{K_{\max}}{N} &= C\int_{k_0}^{N^{1/(\tau-1)}} k\ k^{-\tau} = \frac{C}{\tau-2}\big(k_0^{-(\tau-2)} - N^{\frac{2-\tau}{1-\tau}}\big) \\ &= \frac{C}{\tau-2}\Big[(\frac{a(\tau-1)}{C} + \frac{1}{N})^{\frac{2-\tau}{1-\tau}} - (\frac{1}{N})^{\frac{2-\tau}{1-\tau}}\Big], \end{aligned} \tag{4}$$

which is an increasing function of $N$ that converges to a constant value as $N \to \infty$ for $2 < \tau < 3$, with a smaller rate of convergence for smaller $a$. This and the excellent agreement in other parts of the phase boundary are very strong indications of the stationarity of the phase diagram for this class of networks models. We also note that, when compared with the ER network, the phase boundary of the power-law networks is elongated into the upper right side. This is a natural consequence of the fat-tailed degree distribution: since $(m_{11}, m_{10})$ always satisfy $2m_{11}+m_{10} = K_1$ (the degree sum of $n_1$ nodes) which may be much larger in the SF network, more pairs $(m_{11}, m_{10})$ in the upper right side are available.

## Protein Functional Classes and Mobile Services

Tables 1 and 2 list the details of functional classes in *S. cerevisiae*, and the most dyadic mobile services in the mobile communication network.

# References


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