Models for Measuring and Hedging Risks in a Network Plan

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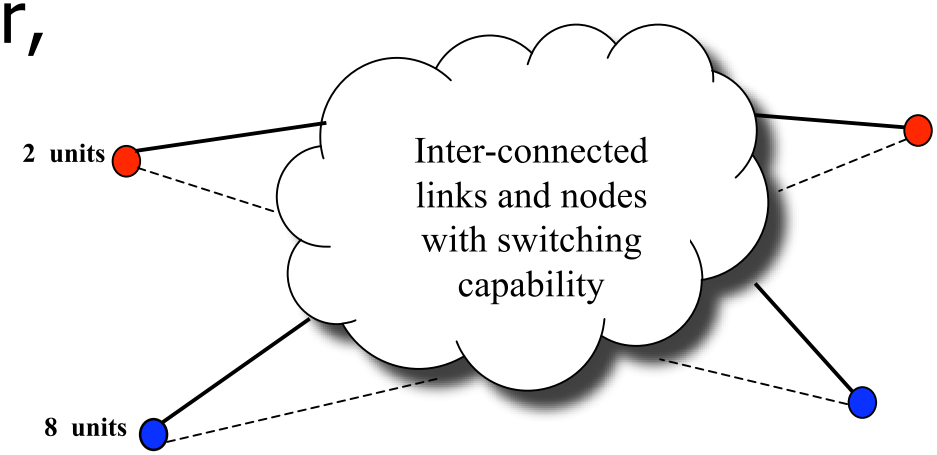
Robust Network Planning

Design a network and place in sufficient link and node capacity to satisfy the expected demand for point-to-point connections in a cost-effective manner,



while hedging against the prominent operating risks.

Planning Decisions

Scheduling the expansion of link and node capacity during planning horizon

- Routes for pt-to-pt connections
- Timing / sizing of (equipment) purchases
- Technology decisions
- Adjustments to network topology

Demand Uncertainty Risk

- Demand may not arise as anticipated.
- Demand levels naturally fluctuate throughout the planning horizon – “Churn”.
- Prior data determines a *distribution* of potential values.
- Correlation / elasticity of demand for services across locations are hard to identify.

Unexpected demands may not be served, resulting in a loss of potential revenue.

Connection Risk

Some link or node (equipment) may fail during network operation, resulting in:

- Temporary disruption of some services
- Loss of customer revenue
- Financial penalties, lawsuits
- Exposure of customers to danger,
  e.g., loss of 911 service, home heat, electricity
- Potential defection of customers

Utilization Risk

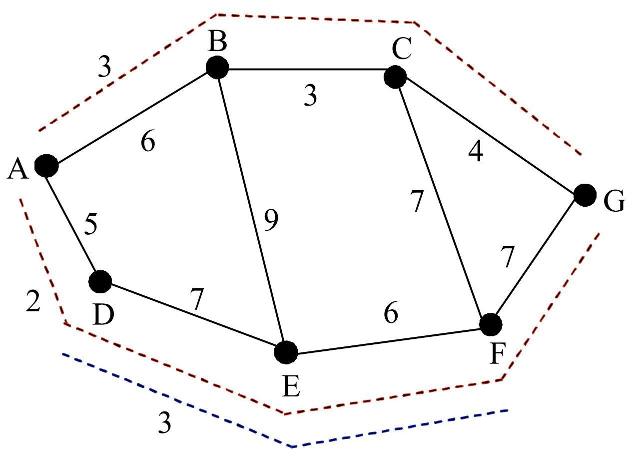
Cost of lost opportunity associated with:

- Over-estimating potential demand
- Purchasing/placing capacity too early
- Cost / Revenue uncertainty
- Providing traffic protection to customers not paying for it
- Dedicating protection capacity to unlikely or benign failure scenarios

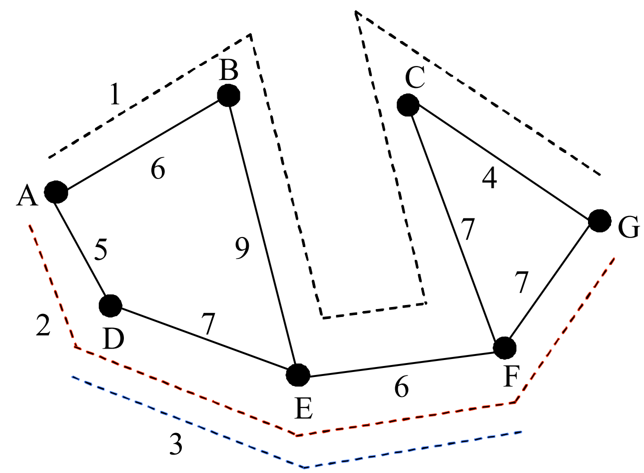
Address this risk to keep plan economical

Hedging Connection Risk

If the network is 2-connected and has accessible extra capacity, then network is “Survivable”: demands can still be satisfied even if some link or node is rendered useless.



Working Routes



Protection Routes

Hedging Strategies

- Route Diversity: Split traffic over multiple (diverse) paths to satisfy demand for a pt-to-pt connection.

  (Low Cost; Low Effectiveness)

- Protection: Back-up paths with capacity to be used only during a network failure. May be dedicated to specific connections or to specific links.

  (High Cost; High Effectiveness)

- Restoration: Use smart switching to access available capacity to recover traffic lost to a network failure.

  (Cost and effectiveness dependent on traffic distribution at failure time and on the quality of capacity decisions)

"Infinite Severity" Model

Connection Risk Assumptions:

- Any loss of traffic due to a link or node failure is unacceptable.
- The relative probability of the potential network failure events is irrelevant.

Hedging Strategy: "100% Survivability"

All of the network traffic can be recovered despite *any* link or node failure in the network.

Infinite Severity Model

- Providing 100% Survivability may be impossible or prohibitively expensive.
- Cost-benefit analyses are irrelevant.

  (<u>Math Prog. Model</u>: Min Cost Objective
  w/ Survivability Constraints)
- Some customers' traffic may not warrant the expense of protection/restoration.
- Some failure scenarios are more likely / more severe than others.

Model Formulation

Let *z* represent the capacity assigned to the links.

Let *x(k)* represent the allocation of connection paths to satisfy demand *k*.

Network capacity should be sufficient to accommodate all demands under normal conditions:

$$\sum x_k = d_k \quad \textit{all demands } k$$

$$\sum_k \mathbf{P}_k x_k \le z$$

Path-arc incidence matrix for connections satisfying k

Formulation

For each potential network failure *f:*

$$x_k^f = \mathbf{I}^f x_k$$

Some working path assignments for demand k are lost.

Let y*(k,f)* represent the use of some paths to recover some of demand *k* after failure *f.*

$$\sum (x_k^f + y_k^f) = d_k - loss_k^f \quad all\ k$$

$$\sum_k \mathbf{P}_k^f (x_k^f + y_k^f) \le \mathbf{D}^f z$$

Unmet demand

Modified path-arc incidence matrix for demand k during failure f

Some network capacity is lost to the failure

Formulation

Controlling the level of survivability:

Objective function: $$\textit{Minimize } \mathbf{c}^{\mathrm{T}} z$$

Capacity expansion

100% Survivability Constraint:

$$loss_k^f = 0 \quad all\ f,\ all\ k$$

Flexible Alternatives

Objective: Effective use of capital

- Allow lower levels of Survivability
- Apply targeted survivability constraints

  Specific customers' demand or failure scenarios
- Perform cost / benefit analyses for risk hedging
  - Models for marginal cost of restoration capacity
  - Valid (dollar-based) measures for connection risk
- Integrated models for hedging both demand uncertainty and connection risk

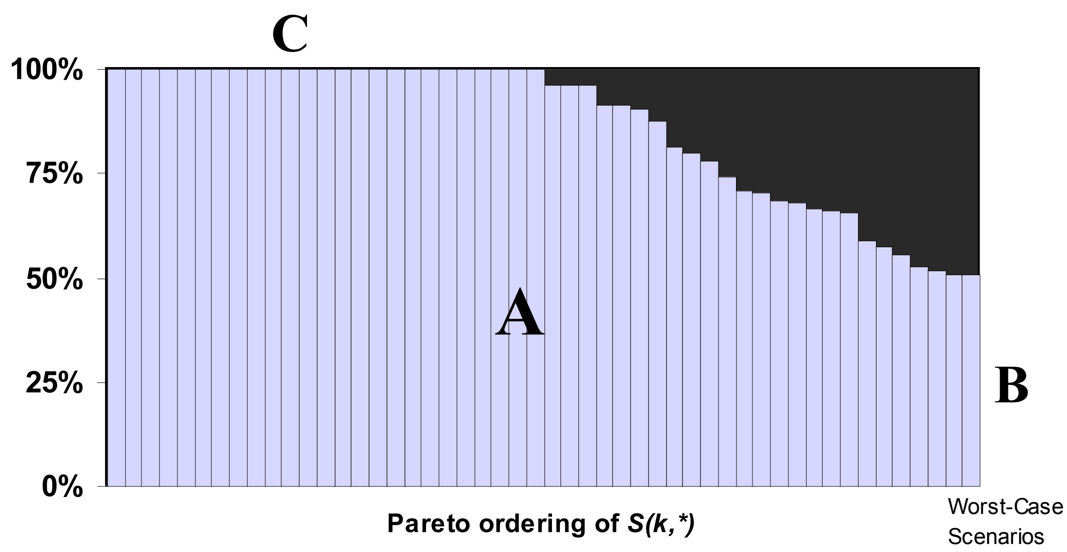
Lower Survivability Levels

What does it mean if a network is *90% Survivable*?

- At least 90% of the total traffic survives any failure.
- Each customer is guaranteed that at least 90% of their traffic survives any failure.
- In 90% of the possible failure scenarios, all of the traffic survives.
- The probability that any unit of traffic will be lost to some failure is less than 10%.
- The expected proportion of traffic to survive, over the possible failure events is 90%.

For 100% all of these meanings are equivalent!

Pictorial Model – Demand k

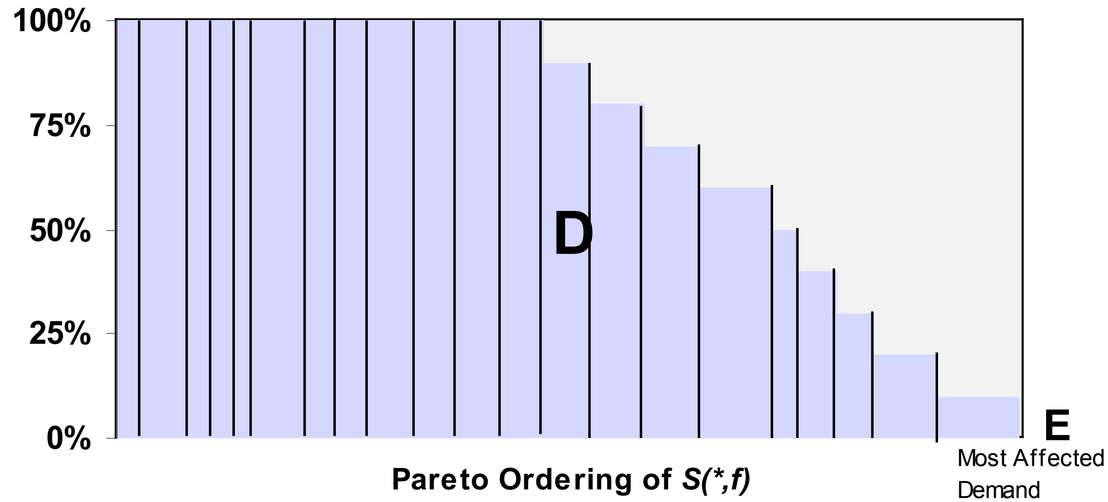


A: Average protection of demand k from potential failures

B: Worst-case = Survivability guarantee for k

C: Prob. that demand k is insulated from some random failure

The Severity of a Failure



D: Proportion of total traffic surviving the failure.

E: Minimum survivability guarantee provided to customers.

Modified Formulations

Controlling the level of survivability:

Through the objective function:

$$\text{Minimize } \underbrace{\mathbf{c}^{\mathbf{T}} z}_{\textit{Capacity expansion cost}} + \lambda \times \underbrace{p(loss)}_{\textit{Penalty for losing service}}$$

and/or through the constraints:

$$loss_k^f \leq s(d_k)$$

Functions to set appropriate limits on loss

Modified Formulations

Constraints on loss of demand *k*:

$$\sum_f loss_k^f / F \le (.1) d_k \qquad \textit{(Area A in chart is 90\%)}$$

$$loss_k^f \le \mathrm{M} w_k^f, \quad where\ w_k^f \in \{0,1\}$$

$$\sum_f w_k^f / F < (.5) \qquad \textit{(Length C in chart is 50\%)}$$

Constraint on severity of failure *f:*

$$\sum_k loss_k^f \le (.1) \sum_k d_k \qquad \textit{(Area D in chart is 90\%)}$$

These can be applied flexibly to specific demand/failures.

Costs/Benefits of Hedges

- Penalty function must capture (true) costs of disrupted connections – beyond revenue loss.
- The parameter λ allows planner to control tradeoff between survivability and expansion costs.

Note: The MILP problem as formulated is quite large and complex

Integrated Models

Restoration (protection) capacity also provide a hedge against demand uncertainty.

- Develop planning models that measure the combined effectiveness of hedging strategies
- Survivability measures when demand is uncertain
- Measuring the marginal costs associated with hedging capacity – which capacity is extra?!
- MILP formulation is even more complex!

Integrated Evaluations

- At least 90% survivability is provided (to the offered services) in every demand scenario.
- The average network survivability over the demand scenarios is at least 90%.
- In at least 90% of the demand scenarios there is 100% survivability (for the offered services).
- At least 90% of the demands are offered 100% survivability in all of the demand scenarios.

Hedging Uncertainty Risk

If accessible capacity is placed throughout the network at sufficient levels, the network might accommodate a variety of potential demand scenarios.

Deterministic Approach: Apply a solution model based on the *expected* pt-to-pt demand levels

(or some higher percentile).

Stochastic Programming Approach: Maximize some probabilistic profit function based on the service provided *over a set* of demand scenarios.

Simulation-based Approach

- For each demand scenario, determine a routing over the network topology.
- For each node / link, collect statistics about the capacity requirements, e.g.,

Capacity	Prob.
50	5%
60	20%
70	50%
80	80%
90	95%
100	100%

Percentage of scenarios in which the capacity is sufficient to accommodate all of the demands

Approach, cont'd

Determine a capacity level:

(Note similarity to "Newsvendor model")

Revenue index +

shortage penalty = 10 *

Cost index +
overage penalty

Note: Shortage penalty ratio would have to be set above 20 for the min-risk capacity to be the most profitable.