Stochastic Response of Offshore Platforms by Statistical Cubicization[a]

Discussion by Ahsan Kareem,[4] Member, ASCE, Michael A. Tognarelli,[5] Student Member, ASCE, and Jun Zhao[6]

The discussers wish to commend the authors on this effort in the field of frequency-domain analyses of offshore structures. The discussers, however, wish to dispel some misconceptions about their own work in this area, which seems incompletely and inaccurately cited by the authors in the literature review found in their introduction.

In the authors' treatment of the history of quadratization, they mention a technique introduced by Kareem and Zhao (1994) whereby a nonlinearity was replaced by the sum of linear and cubic polynomials. In fact, this was a form of cubicization, not quadratization, the development of which was prompted by the discussers' recognition that statistically symmetric nonlinearities, i.e., nonlinearities whose odd-order central moments are zero, cannot be treated by quadratization methods for reasons noted by the authors. This technique was employed specifically in cases where a statically symmetric nonlinearity was present, e.g., Morison drag force without current, and not in cases of statistically nonsymmetric nonlinearities where, in fact, quadratization was used by the discussers.

The introduction suggests that the authors incompletely understand the approach of Kareem and Zhao (1994), which utilizes separate quadratization analyses for offshore platforms subjected to Morison drag force when nonzero current is present. This quadratization technique is outlined in that paper as well as in Kareem and Zhao (1993, 1994b), Kareem et al. (1995), and Tognarelli et al. (1995), and is applicable in such scenarios. Though the quadratization and cubicization approaches of the discussers are not used in tandem, they have been shown in the aforementioned references to be very effective tools, separately, in predicting extreme statistics of response with the same limitations noted by the authors.

Although available elsewhere, a brief synopsis of the discussers' work is given here to provide a complete presentation for the readers of this journal. The following is a comparison of the discussers' quadratization and cubicization techniques for the particular case of Morison drag force with and in the absence of current. Consider the following single-degree-of-freedom nonlinear system model for the surge response, $x(t)$, of a tension leg platform (TLP):

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = K_m\dot{u}(t) + K_d[u(t) + U - \dot{x}(t)]|u(t) + U - \dot{x}(t)| \quad (114)$$

where M, C, and K = structural mass, damping, and stiffness, respectively; K_m = added mass coefficient; K_d = drag coefficient; U = current velocity; and $u(t)$ = water particle velocity, which is assumed stationary and Gaussian. The surge response of the TLP is also assumed stationary, but not, in general, Gaussian. If $U \neq 0$, $x(t)$ is expressed as a Volterra series:

$$x(t) = x_0 + \int_{-\infty}^{\infty} h_1(\tau)f(t-\tau)\,d\tau + \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_2(\tau,\sigma)f(t-\tau)f(t-\sigma)\,d\tau\,d\sigma \quad (115a)$$

$$x(t) = x_0 + x_1(t) + \frac{1}{2}x_2(t) \quad (115b)$$

where $f(t)$ = input process; and $h_1(\tau)$ and $h_2(\tau, \sigma)$ = first- and second-order response functions, respectively. If, on the other hand, $U = 0$, $x(t)$ is expressed as

$$x(t) = \int_{-\infty}^{\infty} h_1(\tau)f(t-\tau)\,d\tau + \frac{1}{6}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_3(\tau,\sigma,\theta)f(t-\tau)f(t-\sigma)f(t-\theta)\,d\tau\,d\sigma\,d\theta \quad (116a)$$

$$x(t) = x_1(t) + \frac{1}{6}x_3(t) \quad (116b)$$

where $h_3(\tau, \sigma, \theta)$ = third-order response function. The argument t will be dropped hereafter for convenience.

The right-hand-side nonlinearity is first expanded as a Taylor series as follows, depending on the type of nonlinearity considered:

$$|u + U - \dot{x}|(u + U - \dot{x}) \cong |u + U - \dot{x}_1|(u + U - \dot{x}_1) - 2E[|u + U - \dot{x}_1|]\frac{\dot{x}_2}{2} \quad (117)$$

or

$$|u - \dot{x}|(u - \dot{x}) \cong |u - \dot{x}_1|(u - \dot{x}_1) - 2E[|u - \dot{x}_1|]\frac{\dot{x}_3}{6} \quad (118)$$

where $E[\cdot]$ denotes the expectation operator. Note that $\dot{x}_1$ is Gaussian since it is the response velocity for a linear system with Gaussian input. It is assumed as well that the second-order and third-order response velocities are small compared to the first-order response velocity and terms involving their higher-order powers may thus be neglected. By expanding as in (117) and (118), a nonlinear function of Gaussian processes remains along with an additional damping term.

Since the nonlinearities in the right-hand sides of (117) and (118) are not cast in polynomial forms and as such are not yet tractable by the Volterra approach, the quadratization or cubicization procedure is now invoked to approximate them in terms of the relative fluid-structure velocity as follows:

$$|u + U - \dot{x}_1|(u + U - \dot{x}_1) \approx \alpha_0 + \alpha_1(u - \dot{x}_1) + \frac{\alpha_2}{2}(u - \dot{x}_1)^2 \quad (119)$$

or

$$|u - \dot{x}_1|(u - \dot{x}_1) \approx \alpha_1(u - \dot{x}_1) + \frac{\alpha_3}{6}(u - \dot{x}_1)^3 \quad (120)$$

The polynomial approximations of (119) and (120) may then be tailored by minimizing the mean-square of one of the following error terms:

$$\varepsilon_{quad} = |u + U - \dot{x}_1|(u + U - \dot{x}_1) - \alpha_0 - \alpha_1(u - \dot{x}_1) - \frac{\alpha_2}{2}(u - \dot{x}_1)^2 \quad (121)$$

or

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$$\varepsilon_{cub} = |u - \dot{x}_1|(u - \dot{x}_1) - \alpha_1(u - \dot{x}_1) - \frac{\alpha_3}{6}(u - \dot{x}_1)^3 \quad (122)$$

The minimization of $E[\varepsilon^2_{quad}]$ produces a system of three equations for the unknowns, α_i, if the current is nonzero (i.e., using quadratization)

$$\begin{bmatrix} 1 & 0 & \sigma^2 \\ 0 & \sigma^2 & 0 \\ \sigma^2 & 0 & 3\sigma^4 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2/2 \end{bmatrix} = \begin{bmatrix} E[|u + U - \dot{x}_1|(u + U - \dot{x}_1)] \\ E[(u - \dot{x}_1)|u + U - \dot{x}_1|(u + U - \dot{x}_1)] \\ E[(u - \dot{x}_1)^2|u + U - \dot{x}_1|(u + U - \dot{x}_1)] \end{bmatrix} \quad (123)$$

which, when solved, yields

$$\alpha_0 = 2U\sigma(rb_1 + b_2); \quad \alpha_1 = 4\sigma(rb_1 + b_2); \quad \alpha_2 = 4b_1 \quad (124a\text{–}c)$$

where

$$b_1 = \frac{1}{\sqrt{2\pi}} \int_0^r \exp\left(-\frac{y^2}{2}\right) dy; \quad b_2 = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{r^2}{2}\right); \quad r = \frac{U}{\sigma};$$

and $\sigma^2 = E[(u - \dot{x}_1)^2]$

If the current is zero (i.e., using cubicization), only a two-by-two system must be solved to minimize $E[\varepsilon^2_{cub}]$ for the coefficients, α_i. This yields $\alpha_1 = \sigma\sqrt{2/\pi}$ and $\alpha_3 = \sigma^{-1}\sqrt{8/\pi}$. Turning attention back to the system (123), an important advantage of the present techniques may be noted in the fact that all of the expected values taken involve only Gaussian quantities and functions thereof.

Now, the equations of motion for wave excitation when $U \neq 0$ can be expressed as

$$M\ddot{x}_1 + (C + a_1)\dot{x}_1 + Kx_1 = K_m\dot{u} + a_1 u \quad (125a)$$

$$M\ddot{x}_2 + (C + a_1)\dot{x}_2 + Kx_2 = a_2(u - \dot{x}_1)^2 \quad (125b)$$

where $a_0 = K_d\alpha_0$; $a_1 = K_d\alpha_1$; and $a_2 = K_d\alpha_2$. The static response of this system may be given as $x_0 = a_0/K$.

Similarly, when the current, U, is zero, one has the system

$$M\ddot{x}_1 + (C + a_1)\dot{x}_1 + Kx_1 = K_m\dot{u} + a_1 u \quad (126a)$$

$$M\ddot{x}_3 + (C + 2a_1)\dot{x}_3 + Kx_3 = a_3(u - \dot{x}_1)^3 \quad (126b)$$

where $a_1 = K_d\alpha_1$; $a_3 = K_d\alpha_3$; and because of the statistical symmetry of the nonlinearity, there is no static offset, x_0.

It is then desired to characterize the time-varying system response in the frequency domain. Thus, the following transfer functions are developed which correspond to the system (125), where the discussers have applied quadratization, for the statistically nonsymmetric case when $U \neq 0$. These relate x_1, $(u - \dot{x}_1)$, and x_2, respectively, to the input water particle velocity spectrum:

$$H^{(1)}_x(\omega) = (K_m i\omega + a_1)H(\omega) \quad (127)$$

$$H_v(\omega) = 1 - i\omega H^{(1)}_x(\omega) \quad (128)$$

$$H^{(2)}_x(\omega_1, \omega_2) = a_2 H(\omega_1 + \omega_2)H_v(\omega_1)H_v(\omega_2) \quad (129)$$

where $H(\omega) = [K - \omega^2 M + i\omega(C + a_1)]^{-1}$.

Likewise, for the system (125), where cubicization has been applied,

$$H^{(1)}_x = (K_m i\omega + a_1)H_1(\omega) \quad (130)$$

$$H^{(3)}_x(\omega_1, \omega_2, \omega_3) = a_3 H_3(\omega_1 + \omega_2 + \omega_3)H_v(\omega_1)H_v(\omega_2)H_v(\omega_3) \quad (131)$$

where $H_1(\omega) = [K - \omega^2 M + i\omega(C + a_1)]^{-1}$; and $H_3(\omega) = [K - \omega^2 M + i\omega(C + 2a_1)]^{-1}$.

As the response distribution is no longer Gaussian, higher-order moments or cumulants are necessary to describe the response statistics. The response statistics are considered in terms of the response cumulants, k_i, rather than the moments. The first-order cumulant is the mean of the response, and the second-order cumulant is equal to its variance. The third- and fourth-order cumulants are descriptors of the skewness and kurtosis, respectively, of the process, quantifying its departure from Gaussianity. The skewness and kurtosis are given by $\gamma_3 = k_3/k_2^{3/2}$; and $\gamma_4 = k_4/k_2^2$, where both quantities are zero for a Gaussian process. Frequency domain expressions for the power spectral density of the response and its first four cumulants based on Bedrosian and Rice (1971) can be obtained for each system. These expressions involve the transfer functions outlined herein and are given in detail in the references to this discussion. Due to truncation of the expression for the fourth-order cumulant associated with the discussers' cubicization technique, this procedure exhibits a wider range of accuracy in predicting the second-order cumulant than in predicting the fourth-order cumulant.

Having this statistical information, an estimate of the probability density function of the response may be made by choosing an appropriate model. In the past, the discussers have had much success employing the moment-based Hermite transformation model given by Winterstein (1985), which has been modified slightly to more accurately reflect the desired response statistics.

The discussers' techniques accurately predict the response of offshore systems to Morison wave loadings when current is present as well as when it is not. Further, they introduce additional computational ease as nonlinear terms are expressed as functions of Gaussian stochastic processes. Thus, they represent a contribution that has not been made by other authors in this field and which seems unclear to the authors of this particular paper. It is hoped, therefore, that the material presented above will serve to clarify both the development and the intent of the separate approaches to both quadratization and cubicization introduced by the discussers.

APPENDIX. REFERENCES

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Closure by Ser-Tong Quek[7]

The writers wish to thank the discussers for their interest in the paper. With the accompanying synopsis given by the discussers, the issue of the suitability of quadratization and cubicization for representing statistically symmetric and nonsymmetric nonlinearity is exemplified. The ease with which the Volterra series solution can handle the presence of current velocity is well demonstrated.

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