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# AERODYNAMIC RESPONSE OF STRUCTURES WITH PARAMETRIC UNCERTAINTIES

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## ABSTRACT

*Uncertainties associated with the load effects and dynamic characteristics of wind-excited structures have been identified and discussed. Based on the available experimental data from laboratory and field study measurements the variability of the parameter space categorized as wind environment and meteorological data, wind–structure interaction and structural properties has been assessed. The probabilistic dynamic response of a wind-excited structure has been expressed in terms of uncertain parameters. The influence of uncertainty in these parameters has been propagated in accordance with the functional relationships that relate them to the structural response. In this study, the propagation of uncertainty has been obtained by employing the Second-Moment and Monte Carlo simulation techniques. The dynamic response of a chimney subjected to aerodynamic loads is presented to illustrate the treatment of uncertainty in the parameter space. The mean and coefficient of variation of the peak chimney response, in terms of top deflections and associated base bending moments, exhibit close agreement between the Second-Moment and simulation techniques. A sensitivity analysis has been conducted which delineates the relative significance of uncertainty in the several parameters, related to both load effects and structural characteristics, on the overall uncertainty in the aerodynamic response of a chimney. The uncertainty associated with both response components suggests a need for further improvement in the modeling of wind–structure interaction, prediction of natural frequencies and damping, and reduction in the variability of extreme wind estimates. There are immediate applications of the procedures discussed in this paper for a variety of wind-sensitive structures.*

## INTRODUCTION

Uncertainties associated with both wind loads and structural characteristics introduce variability in the dynamic response of wind-excited structures. The predicted response of structures

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based on mathematical models with imperfect knowledge tends to be different from reality. In this study, analyses based on the imperfect knowledge are carried out to establish uncertainty associated with response predictions in a probabilistic framework utilizing different procedures.

The uncertainties may be broadly classified into four categories. First, there are uncertainties originating from physical sources, e.g., inherent uncertainty of the physical property of the object such as turbulent fluctuations in the wind field. Other physical sources of uncertainty generally arise from measuring devices such as an anemometer, or an accelerometer. These uncertainties may be expressed in terms of probabilistic models such as the empirical distributions. Second, statistical uncertainties originate from lack of sufficiently large samples of data for establishing empirical distribution of the data. Classical statistical methods, in terms of confidence intervals, introduce conditional statements which are not conveniently integrated in the analysis of propagation of uncertainty or a reliability analysis. Models based on the Bayesian statistics are more suitable. For example, the effect of sampling and estimation errors in the evaluation of an asymptotic extreme value distribution of the extreme winds may be incorporated utilizing a predictive model [1–3]. Third, uncertainty may be introduced due to a lack of knowledge about a process itself. Fourth, there are uncertainties due to deliberate simplifications in our modeling procedures for operational convenience. These errors may be significantly curtailed through an improved understanding of the mechanisms involved in the process being modeled.

In the context of aerodynamic response of structures, a combination of the aforementioned uncertainties arise from variability in the wind environment meteorological data, wind–structure interaction and structural properties, e.g., stiffness and damping. The intrinsic complexity of the dynamic wind load effects compounded by a lack of understanding of the mechanisms that relates them to the far-field wind fluctuations, and a paucity of both experimental and full-scale data contribute significant levels of variability in their quantification. Previous studies related to the analysis of wind effects on structures have addressed the influence of parametric variability in the context of structural reliability, gust factors, and lifetime wind loads [2,4–12]. In this study, the influence of parametric uncertainties on the aerodynamic response of structures is examined.

In the following sections, the description of aerodynamic loads and associated probabilistic response are discussed in the light of uncertainties. Next, the propagation of uncertainty and the influence of parametric uncertainties on the loading and structural response are analyzed. Finally, an example is presented to illustrate the influence of uncertainties on the dynamic response of a tall reinforced concrete chimney.

## PROBABILISTIC AERODYNAMIC LOADS

Contrary to the traditional practice of visualizing wind effects as static in nature, the wind induces unsteady loads that fluctuate with significant energy. The fluctuations in the approach wind field are transformed to unsteady pressures on the outer surface of a building that are a function of both position and time. The instantaneous pressure at a point may be decomposed into mean and randomly fluctuating components that may be superimposed by a periodic contribution from aerodynamic instabilities. The pressure fluctuations over the surface of a structure introduce intense localized load fluctuations and collectively impose overall aerodynamic loads on the structure. The design of structural cladding is strongly influenced by local pressures, whereas, the structural response is dependent on the integrated effect of pressures over the surface.

The unsteady aerodynamic forces on structures may be attributed to several mechanisms that include buffeting, body motion and wake effects. Buffeting primarily results from turbulence or the wake of an upstream structure. The wake developed by the instabilities arising from fluid–structure interaction introduces significant aerodynamic loads. In addition, aeroelastic forces may be introduced by motion of the structure as it oscillates in the air in reaction to the external loads. These mechanisms do not always take place in isolation, rather, more than one may collectively contribute to the response of a structure. The aerodynamic loads are expressed as

$$\text{Aerodynamic loading} = F_{\mathrm{T}}(t) + F_{\mathrm{W}}(t) + F_{\mathrm{I}}(t) + F(x, \dot{x}, \ddot{x}) \tag{1}$$

in which $F_{\mathrm{T}}(t)$ = forces induced by incident turbulence, $F_{\mathrm{W}}(t)$ = forces induced by wake fluctuations, $F_{\mathrm{I}}(t)$ aerodynamics forces due to interference of upstream and adjacent structures, $F(x, \dot{x}, \ddot{x})$ = motion-induced loading expressed in terms of respective aerodynamic derivatives. A wind-sensitive structure, therefore, admits fluctuating energy present in the loading around its characteristic eigenfrequencies and is set to vibrate in rectilinear and/or torsional modes.

Despite the recent advances in the knowledge of wind load effects on structures, our understanding of the mechanisms that relate the random far-field velocity fluctuations to the associated load effects has not developed sufficiently for functional relationships or computational schemes to be formulated. Not only is the incident turbulent boundary-layer wind field complex, but the near-field flow generated around a structure is further complicated due to distortion and amplification of vortical structure of turbulence, the flow separation and possible reattachment, the vortex formation, and the wake development. Due to a lack of predictive methods that relate the random velocity field to the pressure field, the probabilistic wind load description in terms of spectral framework is generally obtained experimentally. One exception of the alongwind load effects exists in which, following the strip and quasi-steady theories, the fluctuating pressure field is assumed to be linearly related to the fluctuating velocity field [13–15].

One of the experimental approaches utilized to quantify aerodynamic loads involves mapping and synthesis of the random pressure field acting on the surface of the structure [16]. A covariance integration scheme may be utilized for synthesizing the space–time fluctuations of the random pressure field in terms of the power spectral density of the point pressure fluctuations as well as the co-spectral description between any two locations over the surface of the structure which are often nonhomogeneous. This requires measurement of multiple point pressure fluctuations. An alternative to mapping and synthesis of random pressure fields based on the point pressure measurements is to focus on locally averaged spatial loads through pneumatic manifolding or sensitive piezopolymer surface transducing techniques [16]. Synthesis of locally averaged random fields permits evaluation of the mode-generalized aerodynamic loads.

Force balance techniques and aeroelastic model tests may be utilized to determine the dynamic wind-induced loads on structures directly [17–19]. Both approaches have their shortcomings [17]. The structural motion may also induce additional loads that may be expressed in terms of aerodynamic damping [20–22]. In the case of aeroelastic model tests, motion-induced loads are explicitly included in the measurements.

## PROBABILISTIC DYNAMIC RESPONSE

The wind-excited response of structures consists of alongwind, acrosswind and torsional components. In general, each component is comprised of a mean and a fluctuating part resulting from the several excitation mechanisms discussed in the preceding section. Description of the wind loads in each direction as well as associated dynamic response in the respective directions for different structural systems may be found in Refs. [13,23,24]. Besides the parametric uncertainties associated with aerodynamic loading, uncertainties related to the structural properties impart variability in the prediction of the overall response. Uncertainty in the system parameters such as mass, stiffness and damping may arise either from spatially random variations in the material, its fabrication, or its mathematical idealization. For example, the contribution of partition walls and some cladding components of high-rise buildings introduces uncertainty in the overall system stiffness estimates. The level of uncertainty associated with damping is significant, and introduces marked variability in the wind-excited response. Once the spatial randomness in the structural properties becomes sizeable it becomes essential to incorporate these uncertain characteristics in the analysis as random variables. In the following sections, a discussion of uncertainty in the parameter space and its propagation is presented.

## UNCERTAINTY ANALYSIS

Uncertainty in the quantification of the wind loads, compounded by the variability in the structural parameters is reflected in the dynamic response. These uncertainties are examined here systematically under three categories: (a) wind environment and meteorological data, (b) parameters reflecting wind–structure interactions, and (c) structural properties.

### Wind environment and meteorological data

In any design application, the expected maximum response of a wind-sensitive structure is estimated based on the extreme wind speed over the lifetime of the structure. The selection of the extreme value distribution may be made based on Gumbel's classical method, or statistical inference utilizing extreme order statistics [24,25]. The estimation of design wind speed has inherent modeling, sampling, and observation errors [2,3,24]. Additional uncertainty is introduced as a result of adjustment in the averaging period of wind from the fastest-mile wind speed to the mean hourly wind speed, the effect of local topography at the anenometer site, and the transformation of wind speed from one terrain to another.

The parameters of the mean wind flow field, e.g., the power law exponent, which represent the variation of wind speed along the height, and the surface drag coefficient used to represent the terrain roughness in the logarithmic variation of mean wind exhibit variability that influences the description of the mean wind field. The single-point description of wind velocity fluctuations is given by the turbulence intensity and the power spectral density function. There are several descriptions of the power spectral density functions over a variety of terrains available in the literature [24,26]. In general, the spectral forms tend to agree in that they approach the Kolmogorov limit at high frequencies; all differ in their treatment of low frequencies [26]. Therefore, for the land-based structures generally characterized by a relatively high fundamental natural frequency the variability introduced by the choice of the spectral description is relatively small as compared with the compliant structures with intrinsically low natural frequencies. The length scale of turbulence that bears functional relationship with the spectral description has important influence on the sensitivity of a structural system to wind. It also exhibits variability in magnitude, and sensitivity to the methods of estimation. Regarding the multi-point representation of random wind field the description of spatial coherence is essential for any functional description. The decay model, which is most generally utilized, is described in terms of exponential functions and the decay constants used in these formulations are sensitive to the terrain features, height above the ground, or the sea surface and the relative distance between the points of interest.

### Wind–structure interaction

The wind–structure interaction parameters may be classified as those related to the random pressure field around the structure and the ones that describe the overall integral load effects. In the former category, the multiple-point random pressure field is represented by the local pressure fluctuations that are described by a spectral relationship and the correlation or a lack of it between various locations given by a coherence function. The functional relationships utilized to describe these functions and their parameters exhibit variability. The drag and lift force coefficients, and Strouhal number each depend upon the cross-section of a member, its aspect ratio, surface roughness, turbulence length scale and intensity, and shear in the approach flow. For a curvilinear cross-section, the dependence of drag and lift force coefficients and Strouhal number upon Reynolds number adds additional variability in their estimated values. The loads induced by the structural motion under the action of external loads are generally expressed as aerodynamic damping in terms of aerodynamic derivatives, which also exhibit variability.

The parameters in experimentally derived covariance integration models that represent the description of the space–time variation of the wind loads are generally assumed to be deterministic. However, there is a considerable variability in the values of these parameters which leads to uncertainty in the overall estimation of the wind loads. The directly measured loads obtained by employing force balance or aeroelastic tests include uncertainties stemming from modeling errors to measurement errors that introduce variability in the measured loads. Spectral estimates of wind loads obtained from wind tunnel measurements at different wind tunnel laboratories exhibit significant variability [24].

Uncertainty in the aerodynamic loading is also introduced by future surroundings of a structure, e.g., a nearby new structure or a topographical variation may significantly modify the flow around the structure and the associated loads.

### Structural properties

Structural properties (for example: mass, stiffness and damping) may exhibit uncertainty in their description arising either from spatial random variation in the material, its fabrication, lack of knowledge, or its mathematical idealization. These structural properties influence the dynamic characteristics of structures, i.e. the natural periods and mode shapes.

Many previous studies have assumed that structural systems have deterministic mechanical characteristics or have implied that the variations in these properties were considerably smaller than those associated with the loading. Recently, the dynamic response analysis of systems with material uncertainties has received considerable interest. These efforts are focused on the development of procedures for the dynamic analysis based on probabilistic methods to implement uncertainties in the parameter space.

A rigorous treatment of the material uncertainty may be accomplished by means of analytical and numerical techniques, for example, a perturbation, Second-Moment, stochastic finite element, and Monte Carlo simulation [27,28]. In a simplistic approach for building systems, the stiffness and mass matrices may be expressed as

$$[\boldsymbol{K}] = K^*[\bar{\boldsymbol{K}}] \quad \text{and} \quad [\boldsymbol{M}] = M^*[\bar{\boldsymbol{M}}] \tag{2}$$

in which $[\bar{\boldsymbol{K}}]$ and $[\bar{\boldsymbol{M}}]$ are deterministic matrices consisting of mean values of the stiffness and mass matrices, respectively, $K^*$ and $M^*$ are random variables with mean values equal to unity and coefficients of variation $\Omega_{K^*}$ and $\Omega_{M^*}$ equal to $\Omega_{K_{ij}}$ and $\Omega_{M_i}$, respectively. This formulation implies that the mass and stiffness at two adjacent levels are perfectly correlated with equal coefficients of variation (COVs). The quality of material utilized in fabrication of structural members in buildings is generally the same, which permits equal COVs and perfect correlation between the stiffness of members used at different levels. For distributed systems in which the material variability exhibits spatial dependence, alternate description of uncertain stiffness and mass based on the make-up of the medium being modeled becomes essential. The former representation has been utilized by Portillo and Ang [29], Rojiani and Wen [2], and Kareem and Hsieh [3]. The natural frequencies of the system are then expressed as

$$f_i = f^*\bar{f}_i \tag{3}$$

in which $f^*$ = random variable with mean value equal to unity, its coefficient of variation, $\Omega_{f^*}$, is expressed in terms of $\Omega_{K^*}$ and $\Omega_{M^*}$ following Rayleigh's method and $\bar{f}_i$ = mean value of the $i$th natural frequency. The simplistic approach considered here results in deterministic eigenvectors. A prediction error may be introduced to account for the limitation of this representation. For a perturbation analysis, the uncertain stiffness and mass may be expressed as a sum of unperturbed mean, or a base value and a small random fluctuation [27].

Increasingly, damping is being recognized as an important factor in the design of structures that are sensitive to wind excitation. The ability to estimate damping values accurately at the design stage, would certainly alleviate a major source of uncertainty from the design of wind-sensitive structures. However, the selection of an appropriate damping value is a subject of discussion and controversy. Although it is a general consensus that damping values change with amplitude, their functional descriptions are rather limited [30]. The methods employed to ascertain damping of full-scale structures, and the analysis and interpretation of data introduce additional uncertainty. Information available from full-scale measurements for analyzing the variability of damping has been assembled by Haviland [31], Jeary [30], and Davenport and Hill-Carroll [32]. Haviland [31] reported estimates of the means and coefficients of variation of damping values of a wide class of structural systems, e.g. steel and concrete buildings of several heights, for different levels of response amplitudes. The log–normal and gamma distributions were shown to provide the best fit to the data.

The uncertainty associated with damping introduces variability in the response of a system. The damping uncertainty may be expressed in terms of the coefficients that appear in the modeling of damping, e.g., Rayleigh's damping. Alternatively, uncertainty may be assigned to the critical damping ratios. In view of the impracticality of determining damping coefficients and the general engineering practice of expressing structural damping in terms of critical damping ratios, it is often convenient to assign uncertainty in damping to the critical damping ratios.

## PROPAGATION OF UNCERTAINTY

The probabilistic dynamic response of a wind-excited structure is expressed in terms of uncertain parameters associated with structural properties and aerodynamic loading. The uncertainty in the foregoing parameters has been identified in the previous section and it is customarily expressed in terms of the coefficient of variation. The influence of uncertainty in these parameters is propagated in accordance with the functional relationships that relate them to the structural response. The propagation of uncertainty may be accomplished by employing one or a combination of the following approaches: perturbation techniques; stochastic finite element methods; Galerkin-based weak-form discretization of stochastic fields; Monte Carlo simulation; and Second-Moment approaches (e.g. [27,28,33–36]). In this paper, the propagation of uncertainty is carried out in the context of finite element discretization by means of Second-Moment and Monte Carlo simulation approaches. In the following, a brief discussion of these techniques is presented.

### Second Moment analysis

The Second-Moment techniques have provided practical and efficient means of analyzing probabilistic engineering mechanics problems [37–39]. The attractiveness of these approaches rests on the limited statistical information needed to analyze a problem, e.g., only the first two statistical moments of a random variable are sufficient for the analysis. The expression for response is expanded in terms of the Taylor series; only up to the first- or second-order terms are retained. In this case, only the first-order terms are retained, the approximation is referred to as the First-Order Second-Moment (FOSM) approach. The coefficient of variation of structural response $R = g(X_1, X_2, X_3, \ldots, X_n)$ which is a function of a number of variables, $X_i$, in the First-Order Second-Moment format is given by

$$\Omega_g = \frac{1}{\bar{R}} \left( \sum_{i=1}^{N} \left( \left. \frac{\partial g}{\partial x_i} \right|_{\bar{x}_i} \right)^2 \bar{x}_i^2 \Omega_{x_i}^2 + \sum\sum_{i \neq j} \rho_{ij} \left( \left. \frac{\partial g}{\partial x_i} \right|_{\bar{x}_i} \right) \left( \left. \frac{\partial g}{\partial x_j} \right|_{\bar{x}_j} \right) \bar{x}_i \bar{x}_j \Omega_{x_i} \Omega_{x_j} \right)^{1/2} \tag{4}$$

in which $\rho_{ij}$ is the correlation between $x_i$ and $x_j$, $\Omega_{x_j}$ is the COV of variable $x_j$, and $\partial g/\partial x_i \,|_{\bar{x}_i}$ is the derivative of $g(\cdot)$ evaluated at the mean value of $x_i$.

Following a random vibration-based modal superposition technique, the mean value of the structural response with uncertain mass and stiffness under spatiotemporally varying wind field is given by

$$\bar{\sigma}_{x_n^{(r)}} = \left( \sum_{i}^{M} \bar{\phi}_{ni}^2 \bar{\sigma}_{q_i^{(r)}}^2 \right)^{1/2} \tag{5}$$

$$\bar{\sigma}_{q_i^{(r)}}^2 = \int_0^\infty \frac{(2\pi f)^{2r} \,|\, H_i(2\pi f) \,|^2 S_{F_i}(f)\, \mathrm{d}f}{(2\pi \bar{f}_i)^4} \tag{6}$$

in which $\phi_{ni}$ = normalized mode shaped with respect to mass matrix, $r = 0$, 1, 2, and 3 represents displacement, velocity, acceleration and jerk components of response, respectively, and $H_i(2\pi f)$ = transfer function. The integration in the preceding equation for lightly damped systems may be carried out by means of the residue theorem if the excitation is idealized as white noise near the resonant frequency. Alternatively, the symbolic manipulation code MACSYMA may be used for filtered white noise processes such as wind loads [40]. For the sake of illustration, the mean value of the RMS response to idealized white noise excitation is given by the following expression

$$\bar{\sigma}^2_{q_i^{(r)}} = \left( \frac{\pi f S_{F_i}(\bar{f}_i)(2\pi \bar{f}_i)^{r-4}}{4\xi_i} \right) \tag{7}$$

where $\xi_i$ is the damping ratio in the $i$th mode. The variance of the response estimate expressed in the preceding equation is given by

$$\mathrm{Var}(\sigma_{X_n^{(r)}}) = \sum_i^M \left\{ \bar{\phi}^2_{ni} \sigma^2_{q_i^{(r)}} \left[ \bar{\sigma}^2_{q_i^{(r)}} \sigma^2_{\phi_{ni}} + \bar{\phi}^2_{ni} \frac{f_n^{2r-8}(2\pi)^{2r-6}}{256\xi^2} \left( (r-3) S_F(\bar{f}_i) + \bar{f}_i S'_F(\bar{f}_i) \right)^2 \sigma^2_{f_i} \right] \right\}$$
$$\times \left( \sum_i^M \bar{\phi}^2_{ni} \bar{\sigma}^2_{q_i^{(r)}} \right)^{-1} \tag{8}$$

in which the parameters have been defined previously.

The variance of the structural response utilizing the complete spectral approach, eqn. (6), involves complex expressions describing the response derivatives with respect to uncertain variables. The derivatives involved are given in Appendix A and have been utilized in the example presented later in the paper.

The mean and variance of the peak response based on the Second-Moment format may be estimated by the following equation [41]

$$(\bar{X}_n^{(r)})_{\max} = \left( \chi(\nu t_d) + \frac{\gamma}{\chi(\nu t_d)} \right) \sigma_{X_n^{(r)}} \tag{9}$$

$$\mathrm{Var}((X_n^{(r)})_{\max}) = \left( \frac{\pi}{\sqrt{6}} \frac{\sigma_{X_n^{(r)}}}{\chi(\nu t_d)} \right)^2 \tag{10}$$

in which $\gamma$ = Euler's constant, $\chi(\nu t_d)) = \sqrt{2 \ln(\nu t_d)}$, and $\nu = (1/2\pi)(\sigma_{X_n}(1)/\sigma_{X_n}(0))$. The coefficient of variation of $(X_n^{(r)})_{\max}$ is given by

$$\Omega^2_{(X_n^{(r)})_{\max}} = \frac{1}{\left(\bar{X}_n^{(r)}\right)_{\max}} \left\{ \sigma^2_{(X_n^{(r)})_{\max}} + \sum_{i=1}^N \left( \left. \frac{\partial (X_n^{(r)})_{\max}}{\partial X_i} \right|_{\bar{X}_i} \right)^2 \Omega^2_{X_i} \bar{X}_i^2 \right.$$
$$\left. + \sum_{i \neq j} \sum \rho_{x_i x_j} \left. \frac{\partial (X_n^{(r)})_{\max}}{\partial X_i} \right|_{\bar{X}_i} \left. \frac{\partial (X_n^{(r)})_{\max}}{\partial X_j} \right|_{\bar{X}_j} \Omega_{X_i} \Omega_{X_j} \bar{X}_i \bar{X}_j \right\} \tag{11}$$

$$\frac{\partial (X_n^{(r)})_{\max}}{\partial X_i} = \frac{1}{2\sigma_{X_n^{(r)}}} \left\{ (A(X_i) - A'(X_i)) \frac{\partial \sigma^2_{X_n^{(r)}}}{\partial X_i} + A'(X_i) \frac{\sigma^2_{X_n^{(r)}}}{\sigma_{X_n^{(1)}}} \frac{\partial \sigma^2_{X_n^{(1)}}}{\partial X_i} \right\} \tag{12}$$

$$A(X_i) = \left( \chi(\nu t_d) + \frac{\gamma}{\chi(\nu t_d)} \right); \quad A'(X_i) = \left( \chi(\nu t_d) - \frac{\gamma}{\chi^3(\nu t_d)} \right) \tag{13}$$

in which the derivatives of the RMS response components with respect to the problem variables are given in Appendix A.

## Monte Carlo simulation

This approach may be viewed as a synthetic, or computer-generated experiment in which a problem is analyzed numerically through a sampling experiment. The simulation procedure is generally described in three steps (i) simulation of sufficiently representative samples of random variables, (ii) solution of the problem for a large number or realizations aimed at obtaining samples of the output process, e.g., the moment capacity, and (iii) statistical analysis of results. The first item requires generating a sequence of sample values of a stochastic variable with a prescribed distribution.

Once all the random variables are generated, then each experiment consists of choosing a set of input values performing numerical evaluation of the desired function and thus obtaining a set of output quantity, e.g., the structural response. The numerical experiment is repeated $n$ times, and the statistics of the output quantity are calculated from the generated sample output. Like other experiments, the foregoing simulation technique unfortunately shares the problems of sampling errors. These errors are minimized by making the number of trials large which in turn significantly influences computational effort. The sampling error introduced by limited sample size may be improved by means of variance reduction techniques, e.g., importance sampling, antithetic variates and stratified sampling [42–44]. These techniques are quite dependent on the type of model under study and do not provide an apriori estimate of the variance reduction.

## EXAMPLE

In this section, the peak response of a chimney to aerodynamic loads is presented to illustrate the foregoing treatment of uncertainty in the parameter space. A 598 ft (182 m) tall reinforced-concrete chimney was employed in this example. The details of structural dimensions and other related information are given in Ref. [3]. The chimney was discretized into 13 elements along the height, with a translational and a rotational degree-of-freedom at each node. The system stiffness matrix in global coordinates was assembled from the element stiffness matrices of the system. The mass matrix was formulated utilizing a consistent mass description. Only the first three modes were included in the dynamic analysis. The mean value of the natural frequencies in the first three modes were computed to be 0.48, 1.86 and 4.71 Hz. The mean value of the structural damping in the fundamental mode was assumed to vary from 1% of the critical to 4% with an increment of 1%. The damping values in the higher modes were estimated following Kareem and Hsieh [3].

The uncertainty in the design wind speed that corresponds to the lifetime extreme wind speed was evaluated from data pertaining to an arbitrarily selected industrial site in the U.S. The extreme value Type I, Type II, and Rayleigh distributions were used to model the annual maximum wind speed distribution. The data provided the best fit to the Type I extreme value distribution based upon a maximum probability plot correlated coefficient (MPPCC) criterion [3,24,45]. The estimates of the mean value and the COV for various flow-related parameters were made from the experimental and field study data [3]. In Table 1, the parameters of the Type I distribution, and the sampling and observational errors for the annual and the lifetime extreme wind are given.

Analysis of the data base suggested values of 0.7 and 0.15 for the mean values, and 0.14 and 0.27 for the COVs of the drag and RMS acrosswind force coefficients, respectively. The mean

TABLE 1

Parameters in the wind distribution

| | |
|---|---|
| *Extreme Value Type I* | |
| location $U = 38.66$ | $\Omega_U = 0.019$ |
| scale $A = 3.26$ | $\Omega_A = 0.219$ |
| | $\Omega_{AU} = 0.129$ |
| *Annual Extreme Wind Speed* | |
| $V = 40.55$ mph | $\Omega_V = 0.10$ |
| *Lifetime Extreme Wind Speed* | |
| $V_T = 52.91$ mph | $\Omega_{se} = 0.062$ |
| | $\Omega_{V_T} = 0.101$ |
| | $\Omega_{obs} = 0.025$ |

value and the COV of the Strouhal number were estimated to be 0.2 and 0.11, respectively. The computed values of the mean and COV of the acrosswind spectral bandwidth were 0.25 and 0.3, respectively. In this study, the chimney diameter and thickness, drag coefficient, Strouhal number like mass and stiffness (eqn. 2) are expressed as a product of a random and a deterministic part, e.g., $D(z) = D^* \overline{D}(z)$ in which $D^*$ is a random variable with mean value equal to unity and COV equal to that of the drag coefficient, and $\overline{D}(z)$ is the mean value of the diameter at height $z$. The aerodynamic damping, derived on the basis of equivalent amplitude-dependent damping, is a function of a number of variables. Thus, the uncertainty in aerodynamic damping described in Appendix B was obtained based on the First-Order Second-Moment approach

$$\Omega_{\xi_a} = \frac{1}{\xi_a^2}\left[\left(\frac{\partial \xi_a}{\partial D^*}\right)^2 \Omega_{D^*}^2 + \left(\frac{\partial \xi_a}{\partial m^*}\right)^2 \Omega_{m^*}^2 + \left(\frac{\partial \xi_a}{\partial K_{ao}}\right)^2 \overline{K}_{ao}^2 \Omega_{K_{ao}}^2 + \left(\frac{\partial \xi_a}{\partial a}\right)^2 \bar{a}^2 \Omega_a^2 + \left(\frac{\partial \xi_a}{\partial \sigma_y}\right)^2 \bar{\sigma}_y^2 \Omega_{\sigma_y}^2 + \left(\frac{\partial \xi_a}{\partial N}\right)^2 N^2 \Omega_N^2\right]^{1/2} \tag{14}$$

in which $K_{ao}$, $\sigma_y$, and $N$ are aerodynamic coefficient, variance of response, and a constant, respectively, and other variables have been defined earlier in the text. The parameter $K_{ao}$ depends on the acrosswind force coefficient, hence it was assigned the same uncertainty as that of the acrosswind force coefficient. The overall uncertainty in the aerodynamic damping, following eqn. (14) was found to be equal to 0.3 [3]. The uncertainty in the stiffness matrix was estimated on the basis of uncertainty in the flexural rigidity, EI, of the tubular concrete section. The uncertainty of EI expressed in terms of COV is given by

$$\Omega_{EI} = \frac{1}{\left(1 - \rho_s + \frac{E_s}{E_c}\rho_s\right)}\left[\left(3 - 2\rho_s + 2\frac{E_s}{E_c}\rho_s\right)^2 \Omega_r^2 + \Omega_t^2 + \left(1 - \rho_s^2\right)\Omega_{E_c}^2 + \left(\frac{E_s}{E_c}\rho_s\right)^2 \Omega_{E_s}^2 + \left(\frac{E_s}{E_c}\rho_s - \rho_s\right)^2 \Omega_{A_s}^2\right]^{1/2} \tag{15}$$

TABLE 2

Uncertainties of various basic variables

| Variable | Mean | Cov | Distribution |
|---|---|---|---|
| Wind speed | 52.91 | 0.101 | Extreme Value Type I |
| Drag coefficient | 0.7 | 0.14 | lognormal |
| RMS acrosswind force coefficient | 0.15 | 0.27 | lognormal |
| Strouhal number | 0.20 | 0.11 | lognormal |
| Spectral bandwidth | 0.25 | 0.30 | lognormal |
| Aerodynamic damping | – | 0.30 | lognormal |
| Natural frequency | – | 0.17 | lognormal |
| Structural damping | – | 0.35 | lognormal |
| Element of mass matrix | 1.0 | 0.094 | normal |
| Flexural rigidity | – | 0.18 | normal |
| Element of stiffness matrix | 1.0 | 0.27 | normal |
| Diameter | – | 0.04 | normal |
| Thickness | – | 0.04 | normal |
| Specific weight of concrete | 150 $lb/ft^3$ | 0.30 | normal |
| $f_c'$ Compressive stress | 4000 psi/3390 psi | 0.18 | normal |
| in concrete | 5000 psi/4028 psi | 0.15 | normal |
| $E_c$ 4000 psi | 3320 ksi | 0.09 | normal |
| 5000 psi | 5000 psi | 0.075 | normal |
| $E_s$ | 29200 ksi | 0.033 | normal |

in which $\rho_s$, $r$, $t$, $E_c$, $E_s$ and $A_s$ represent steel ratio, radius of chimney, thickness of shell, modulus of elasticity of concrete and steel, respectively, and area of steel [3]. Due to modeling assumptions an additional 10% uncertainty was included that resulted in a total value of uncertainty equal to 0.18 [46]. Therefore, the variability of stiffness of an element which is proportional to EI was set equal to 0.18. Additional uncertainty of 0.2, arising from idealization, neglecting shear deformation and formulation of the stiffness matrix was included that resulted in a COV of 0.27 for the overall stiffness matrix. The uncertainty in the mass matrix was estimated to be 0.09. Utilizing the COVs of the stiffness and mass matrices and including an additional uncertainty of 0.1 to include the influence of possible soil–structure interaction, the COV of the natural frequency was computed to be 0.17. Based on the analysis of structural damping data related to the reinforced concrete chimneys the COV was found to be 0.35. Due to a lack of data the same coefficient of variation was assumed for the damping in the higher modes.

Initially, a total of twenty-five basic variables associated with parameters reflecting the wind environment and meteorological data, wind–structure interactions and structural properties were considered. A sensitivity analysis of the contribution of the uncertainty of various variables to the overall uncertainty suggested that the number of variables could be reduced to those which significantly influence the overall uncertainty in the response. In Table 2, a summary of the mean values of the parameters and their COVs and probability distributions are reported.

The static deflection at the top of the chimney is given by

$$Y_{st} = \sum_{i=1}^{N} \frac{\int_0^H q(z)\phi_i(z)\,dz}{(2\pi f_i)^2 M_i} \tag{16}$$

in which

$$q(z) = \tfrac{1}{2}\rho_A C_D(z) D(z) U^2(z)$$

$$M_i = \int_0^H m(z)\phi_i^2(z)\,\mathrm{d}z$$

$$U(z) = U_{\mathrm{ref}}(z/z_{\mathrm{ref}})^{\alpha}$$

$\rho_A$ = air density; $C_D(z) = C_D^* \overline{C_D(z)}$; $D(z) = D^*\overline{D(z)}$; $m(z) = m^*\overline{m(z)}$; and $f_i = f_i^* \overline{f}_i$. Based on the FOSM approximation the coefficient of variation of $Y_{\mathrm{st}}$ is given by

$$\Omega_{Y_{\mathrm{st}}} = \left[\Omega_{C_D^*}^2 + \Omega_{D^*}^2 + \Omega_{m^*}^2 + 4\Omega_{f^*}^2 + 4\Omega_{U_{\mathrm{ref}}}^2 + \frac{1}{\overline{Y}_{\mathrm{st}}^2}\left(\frac{\partial Y_{\mathrm{st}}}{\partial \alpha}\right)^2 \Omega_\alpha^2 \overline{\alpha}^2 + (0.05)^2\right]^{1/2} \tag{17}$$

where an additional uncertainty of 0.05 is introduced to incorporate uncertainty due to mathematical modeling [46], and $\Omega_x$ = coefficient of variation of variable $x$.

The uncertainty associated with the RMS and peak response components was evaluated following the full spectral approach (eqn. 6) and the expressions presented in eqns. (9)–(13). Expressions for the spectral description of wind loads are described in Appendix B. The fluctuating moment at any height on the chimney is composed of the static and dynamic components. The static component at height $z_a$ is given by

$$M_{\mathrm{st}}(z_a) = \int_{z_a}^H q(z)(z - z_a)\,\mathrm{d}z \tag{18}$$

The COV of $M_{\mathrm{st}}(z_a)$ is given by

$$\Omega_{M_{\mathrm{st}}(z_a)} = \left[\Omega_{C_D^*}^2 + \Omega_{D^*}^2 + 4\Omega_{U_{\mathrm{ref}}}^2 + \frac{1}{\overline{M}_{\mathrm{st}}^2}\left(\frac{\partial M_{\mathrm{st}}}{\partial \alpha}\bigg|_{\overline{\alpha}}\right)^2 \Omega_\alpha^2 \overline{\alpha}^2 + (0.05)^2\right]^{1/2} \tag{19}$$

In the preceding expression an additional uncertainty of 0.05 has been introduced to account for modeling error. The RMS value of the moment and its derivatives is expressed by

$$\{\sigma_M^{(r)}\} = \sum_{i=1}^{K} \frac{\left[\int_0^\infty (2\pi f)^{2r}\,|H_i(2\pi f)|^2 S_{F_i}(f)\,\mathrm{d}f\right]^{1/2} [\boldsymbol{T}][\boldsymbol{M}]\{\phi_i\}}{(2\pi \bar{f}_i)^4 M_i} \tag{20}$$

in which $|H_i(2\pi f)|^2$ = system transfer function in the $i$th mode; $S_{F_i}(f)$ = generalized spectrum of wind force in the $i$th mode for the alongwind, or the acrosswind direction; $[\boldsymbol{T}]$ and $[\boldsymbol{M}]$ = system transformation (relates nodal forces to associated bending moment) and mass matrices, respectively; $\{\phi_i\}$ = $i$th mode shape; $M_i$ = generalized mass in the $i$th mode; and $K$ = number of modes included in the analysis. The COV of the maximum top deflection and moment at a level were evaluated following eqns. (9)–(13). The derivatives involved in these expressions are documented in Appendix A to facilitate their use in other applications related to wind excitation. The foregoing expressions were evaluated on a computer utilizing a Monte Carlo integration scheme for expedient computation of the multiple integrals. The FOSM format for the correlation between two response components e.g., $f$ and $g$, which are a function of other variables is given by

$$\mathrm{COV}[f, g] = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial f}{\partial X_i}\frac{\partial g}{\partial X_j}\rho_{ij}\overline{X}_i\Omega_{X_i}\overline{X}_j\Omega_{X_j} \tag{21}$$

where $\rho_{ij}$ is the correlation coefficient between the basic variables $X_i$ and $X_j$.

TABLE 3

Deflection at top

| Mean value of damping in the first mode (%) | Alongwind response | | Acrosswind response | |
|---|---|---|---|---|
| | Mean (ft) | COV | Mean (ft) | COV |
| 1 | 0.4029 | 0.773 | 0.8142 | 1.0801 |
| | (0.3165) | (0.602) | (0.7249) | (0.7936) |
| 2 | 0.2889 | 0.695 | 0.4684 | 0.8125 |
| | (0.2238) | (0.602) | (0.4416) | (0.7936) |
| 3 | 0.2371 | 0.696 | 0.3632 | 0.7677 |
| | (0.1827) | (0.612) | (0.3469) | (0.7936) |
| 4 | 0.2076 | 0.727 | 0.3065 | 0.7764 |
| | (0.1582) | (0.602) | (0.2938) | (0.7936) |

The peak alongwind and acrosswind chimney displacements at the top and associated bending moments at different levels along the chimney height were simulated utilizing a Monte Carlo simulation technique [3]. The computer-generated response estimates were statistically analyzed to obtain the means and COVs. The complexity associated with the evaluation of the aerodynamic loading, involving a double integration for each sample value, and the subsequent estimation of the response including the first three modes in each orthogonal direction involved significant computational effort. On an AS9000 computer, six hours of CPU time were required to generate 14,000 samples of data. The results were not influenced by the sample size, once the number of simulated values reached 10,000. The sampling error introduced by limited sample size may be improved without increasing the sample size by utilizing variance reduction techniques, e.g., importance sampling, antithetic variates, conditional expectations and stratified sampling [5,42].

The top displacement and base moment statistics, derived from the FOSM and simulation techniques, in terms of mean values and their COVs are presented in Tables 3 and 4. The numbers given in parentheses are generated by employing a Monte Carlo simulation scheme. The results exhibit a good agreement. These estimates of uncertainty in the response may provide a useful input to establish a limit-state design procedure, or reliability analysis of structures to ensure their safety and serviceability.

TABLE 4

Base bending moment

| Mean values of damping in the first mode (%) | Alongwind moment | | Acrosswind moment | |
|---|---|---|---|---|
| | Mean Value (lb-ft) | COV | Mean Value (lb-ft) | COV |
| 1 | $0.1011906 \times 10^{9}$ | 0.774 | $0.2051884 \times 10^{9}$ | 1.073 |
| | $(0.9946820 \times 10^{8})$ | (0.749) | $(0.18122762 \times 10^{9})$ | (0.823) |
| 2 | $0.7255618 \times 10^{8}$ | 0.695 | $0.1182936 \times 10^{9}$ | 0.8038 |
| | $(0.6619817 \times 10^{8})$ | (0.744) | $(0.1103972 \times 10^{9})$ | (0.823) |
| 3 | $0.5977762 \times 10^{8}$ | 0.716 | $0.9188896 \times 10^{8}$ | 0.761 |
| | $(0.5588672 \times 10^{8})$ | (0.744) | $(0.8649999 \times 10^{8})$ | (0.823) |
| 4 | $0.5215667 \times 10^{8}$ | 0.728 | $0.7761918 \times 10^{8}$ | 0.773 |
| | $(0.4973306 \times 10^{8})$ | (0.745) | $(0.73471448 \times 10^{8})$ | (0.823) |

TABLE 5

Uncertainty in moment at the node 8 due to the uncertainties in various parameters for the wind velocity of 40 mph at the reference height

| Parameter $X_i$ | $\Omega_{X_i}$ | $\Omega_{st}$ | $\Omega_{al}$ | $\Omega_{ac}$ |
|---|---|---|---|---|
| $D$ | 0.04 | 0.04 | 0.095 | 0.157 |
| $M$ | 0.094 | 0.0 | 0.128 | 0.125 |
| EI | 0.18 | 0.0 | 0.125 | 0.122 |
| $\xi_s$ | 0.35 | 0.0 | 0.129 | 0.369 |
| $f^*$ | 0.17 | 0.0 | 0.521 | 0.172 |
| $C_D$ | 0.14 | 0.14 | 0.164 | 0.0 |
| $a$ | 0.10 | 0.119 | 0.117 | 0.083 |
| $C_z$ | 0.15 | 0.0 | 0.098 | 0.0 |
| $k$ | 0.11 | 0.0 | 0.102 | 0.0 |
| $C_L$ | 0.27 | 0.0 | 0.0 | 0.282 |
| $S$ | 0.11 | 0.0 | 0.0 | 0.213 |
| $B$ | 0.30 | 0.0 | 0.0 | 0.103 |
| $\xi_a$ | 0.30 | 0.0 | 0.0 | 0.153 |
| $V_O$ | 0.066 | 0.132 | 0.183 | 0.101 |
| overall | – | 0.230 | 0.597 | 0.575 |

A sensitivity analysis was carried out in which the influence of uncertainty of each parameter to the overall uncertainties of the resistance and the load effects were analyzed. The computed values of uncertainty in the several moment components at node 8 for the reference-height velocity of 40 mph (64 km/h) are given in Table 5. These results suggest that the uncertainty of the alongwind fluctuating moment at the node 8 is very sensitive to the uncertainties in the natural frequency, damping value and the wind velocity. The uncertainty of the acrosswind fluctuating moment at the node 8 is sensitive to the lift force coefficient, Strouhal number, wind velocity, natural frequency and structural damping. These results suggest that significantly more information is needed to reduce part of the uncertainties associated with both aerodynamic load effects and structural characteristics.

The influence of different approaches utilized to incorporate uncertainty associated with the variability in the design wind speed estimation on the overall uncertainty is also evaluated. Three different approaches are considered which are referred to as a complete-FOSM approach, a modified-FOSM approach and a predictive model. In the complete-FOSM approach the aerodynamic load effects are computed on the basis of the mean lifetime extreme wind speed. The uncertainty in the wind speed estimate, which includes errors associated with the variability of the wind speed, sampling error and observational error, is estimated based on the statistical analysis of wind data according to the selected wind distribution. In the modified-FOSM approximation the uncertainty in the load effects is estimated for a range of wind speeds and the overall uncertainty is obtained through the convolution of the conditional uncertainties with the PDF of the lifetime wind speed as indicated by the following equation

$$\Omega_s = \int_0^{\infty} \Omega_s|_v f_{v_T}(v)\,dv \tag{22}$$

in which $\Omega_s|_v$ = conditional coefficient of variation of the load effects for a given wind speed and $f_{v_T}(v)$ = PDF of the lifetime wind speed. The introduction of the wind speed distribution

TABLE 6

Uncertainty in the total moment [a]

| Node | Modified FOSM model | | | Complete FOSM model | | | Predictive model | | |
|---|---|---|---|---|---|---|---|---|---|
| | Type I | Type II | Rayleigh | Type I | Type II | Rayleigh | Type I | Type II | Rayleigh |
| 8 | 0.395 | 0.428 | 0.387 | 0.397 | 0.405 | 0.382 | 0.387 | 0.397 | 0.376 |
| 10 | 0.386 | 0.418 | 0.378 | 0.385 | 0.395 | 0.375 | 0.379 | 0.389 | 0.369 |
| 12 | 0.382 | 0.414 | 0.374 | 0.380 | 0.391 | 0.373 | 0.375 | 0.385 | 0.366 |
| base | 0.385 | 0.417 | 0.377 | 0.383 | 0.394 | 0.375 | 0.378 | 0.388 | 0.368 |

[a] $M_t = \{(M_{st} + M_{al})^2 + M_{ac}^2\}^{1/2}$

identifies only the error associated with the variability of wind, hence the sampling and observational errors need to be included. In the predictive approach, the sampling and observational errors are included by treating the parameters of the lifetime extreme winds as random variables [1–3]. Therefore, the modified PDF of the lifetime extreme wind is given by

$$f_{v_T}(v) = \iint f_{v_T} \mid_{A,B}(v \mid a, b) f_{AB}(a, b) \, \mathrm{d}a \, \mathrm{d}b \tag{23}$$

in which $f_{v_T} \mid_{A,B}(v \mid a, b) =$ PDF of the lifetime extreme wind speed for given values of the parameters $A = a$ and $B = b$, and $f_{AB}(a, b) =$ joint probability density function of random variables $A$ and $B$. Generally, $f_{AB}(a, b)$ is assumed to be a bivariate normal density function, however, the statistics of $A$ and $B$ may be estimated from the wind data at the site. The predictive distribution model provides a useful alternative to the conventional confidence interval method for incorporating sampling and estimation errors. The integration in the preceding equation may be efficiently carried out by employing a fast probability integration scheme based on the first-order reliability method [39]. The uncertainty in the bending moment at the base and a few other nodes obtained from these three approaches is compared in Table 6. In addition to the Type I extreme wind distribution, Type II and Rayleigh distributions were also included to study the influence of different distributions on the uncertainty associated with aerodynamic load effects. The results suggest that different procedures for the propagation of uncertainty associated with the variability in wind do not significantly influence the estimates of uncertainty in aerodynamic load effects for Type I and Rayleigh distributions. Conversely, for the extreme value Type II distribution, the uncertainty analysis is influenced by the procedure used to incorporate uncertainty associated with wind [3].

## CONCLUDING REMARKS

The uncertainties associated with the load effects and dynamic characteristics of wind-excited structures have been identified and discussed. Based on the available experimental data from laboratory and field study measurements the variability of the various parameters categorized as wind environment and meteorological data, wind–structure interaction and structural properties has been assessed. The probabilistic dynamic response of a wind-excited structure has been expressed in terms of uncertain parameters. The influence of uncertainty in these parameters has been propagated in accordance with the functional relationships that relate them to the structural response. In this study the propagation of uncertainty has been obtained by employing the

Second-Moment and Monte Carlo simulation techniques. The random vibration based approach utilized in this study is applicable to other wind-sensitive structures.

The dynamic response of a chimney subjected to wind loads is presented to illustrate the treatment of uncertainty in the parameter space. The uncertainty of response exhibits close agreement between the Second-Moment and simulation approaches. The simulation approach provided a basis for the validation of the Second-Moment approximation which is computationally efficient for this class of problems. A sensitivity analysis helped to delineate the relative significance of uncertainty in the several parameters, related to both load effects and structural characteristics, on the overall uncertainty in the aerodynamic response of the chimney. The analysis also suggests that the overall uncertainty in the dynamic response is insensitive to the procedure utilized in propagating uncertainty associated with the wind speed. The COVs for both components of response suggest a need for further improvement in the modeling of wind–structure interaction, prediction of natural frequencies and damping, and a reduction in the variability of extreme wind estimates.

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## APPENDIX A

### Response derivatives

The derivatives of RMS response components with respect to the uncertain parameters are presented in this appendix. The fluctuating response may be obtained utilizing the mode-displacement approach

$$\sigma^{(r)^2}(z) = \sum_{i=1}^{N} h_i^{(r)^2}\phi_i^2(z); \quad h_i^{(r)^2} = \int_0^\infty \frac{(2\pi f)^{2r}\,|\,H_i(2\pi f)\,|^2 S_{F_i}(f)\,\mathrm{d}f}{M_i^2(2\pi f_i)^4} \tag{A.1}$$

$$\frac{\partial\sigma^{(r)^2}(z)}{\partial x_i} = \sum_{i=1}^{N} \frac{\partial h_i^{(r)^2}}{\partial x_i}\phi_i^2(z) \tag{A.2}$$

$$\frac{\partial h_i^{(r)^2}}{\partial C_D^*} = \frac{2}{C_D^*}h_i^{(r)^2}; \quad \frac{\partial h_i^{(r)^2}}{\partial D^*} = \frac{2}{D^*}h_i^{(r)^2}; \quad \frac{\partial h_i^{(r)^2}}{\partial m^*} = \frac{-2}{m^*}h_i^{(r)^2} \tag{A.3}$$

$$\frac{\partial h_i^{(r)^2}}{\partial f^*} = \frac{-4}{f^*}\int_0^\infty (2\pi f)^{2r}\,|\,H_i(2\pi f)\,|^2 S_{F_i}(f)\,|\,H_i(2\pi f)\,|^2\left[1-\left(\frac{f}{f_i}\right)^2 + 2\xi_i^2\left(\frac{f}{f_i}\right)^2\right]\mathrm{d}f \tag{A.4}$$

$$\frac{\partial h_i^{(r)^2}}{\partial \xi^*} = \frac{-8}{f^*}\int_0^\infty \frac{(2\pi f)^{2r}\,|\,H_i(2\pi f)\,|^2 S_{F_i}(f)\,|\,H_i(2\pi f)\,|^2}{M_i^2(2\pi f_i)^2}\xi_i^2\left(\frac{f}{f_i}\right)^2\mathrm{d}f \tag{A.5}$$

$$\frac{\partial h_i^{(r)^2}}{\partial \kappa} = \frac{1}{\kappa}h_i^{(r)^2}; \quad \frac{\partial h_i^{(r)^2}}{\partial C_z} = \int_0^\infty \frac{(2\pi f)^{2r}\,|\,H_i(2\pi f)\,|^2}{M_i^2(2\pi f_i)^4}\frac{\partial S_{F_i}(f)}{\partial C_z}\,\mathrm{d}f \tag{A.6}$$

$$\frac{\partial h_i^{(r)^2}}{\partial \alpha} = \int_0^\infty \frac{(2\pi f)^{2r}\,|\,H_i(2\pi f)\,|^2}{M_i^2(2\pi f_i)^4}\frac{\partial S_{F_i}(f)}{\partial \alpha}\,\mathrm{d}f \tag{A.7}$$

$$\frac{\partial h_i^{(r)^2}}{\partial V_{\mathrm{ref}}} = \int_0^\infty \frac{(2\pi f)^{2r}\,|\,H_i(2\pi f)\,|^2}{M_i^2(2\pi f_i)^4}\frac{\partial S_{F_i}(f)}{\partial V_{\mathrm{ref}}}\,\mathrm{d}f \tag{A.8}$$

$$\frac{\partial h_i^{(r)^2}}{\partial C_L^*} = \frac{2}{C_L^*}h_i^{(r)^2}; \quad \frac{\partial h_i^{(r)^2}}{\partial D^*} = \int_0^\infty \frac{(2\pi f)^{2r}\,|\,H_i(2\pi f)\,|^2}{(2\pi f_i)^4 M_i^2}\frac{\partial S_{F_i}(f)}{\partial D^*}\,\mathrm{d}f \tag{A.9}$$

$$\frac{\partial h_i^{(r)^2}}{\partial m^*} = \frac{-2}{m^*}h_i^{(r)^2}; \quad \frac{\partial h_i^{(r)^2}}{\partial B} = \int_0^\infty \frac{(2\pi f)^{2r}\,|\,H_i(2\pi f)\,|^2}{(2\pi f_i)^4 M_i^2}\frac{2S_{F_i}(f)}{\partial B}\,\mathrm{d}f \tag{A.10}$$

$$\frac{\partial h_i^{(r)^2}}{\partial S^*} = \int_0^\infty \frac{(2\pi f)^{2r} \mid H_i(2\pi f) \mid^2}{(2\pi f_i)^4 M_i^2} \frac{\partial S_{F_i}(f)}{\partial S^*} \, \mathrm{d}f \tag{A.11}$$

$$\frac{\partial h_i^{(r)^2}}{\partial \alpha} = \int_0^\infty \frac{(2\pi f)^{2r} \mid H_i(2\pi f) \mid^2}{(2\pi f_i)^4 M_i^2} \frac{\partial S_{F_i}(f)}{\partial \alpha} \, \mathrm{d}f \tag{A.12}$$

$$\frac{\partial h_i^{(r)^2}}{\partial f^*} = \frac{-4}{f^*} \int_0^\infty \frac{(2\pi f)^{2r} \mid H_i(2\pi f) \mid^2}{(2\pi f_i)^4 M_i^2} S_{F_i}(f) \left\{ 1 - \left( \frac{f}{f_1} \right)^2 + 2\xi_i \left( \frac{f}{f_i} \right)^2 \right\} \mathrm{d}f \tag{A.13}$$

$$\frac{\partial h_i^{(r)^2}}{\partial f^*} = \frac{-1}{\xi^*} \int_0^\infty \frac{(2\pi f)^{2r} \mid H_i(2\pi f) \mid^2}{(2\pi f_i)^4 M_i^2} S_{F_i}(f) 8\xi_i^2 \left( \frac{f}{f_i} \right)^2 \mathrm{d}f \tag{A.14}$$

$$\frac{\partial h_i^{(r)^2}}{\partial V_{\mathrm{ref}}} = \int_0^\infty \frac{(2\pi f)^{2r} \mid H_i(2\pi f) \mid^2}{(2\pi f_i)^4 M_i^2} \frac{\partial S_{F_i}(f)}{\partial V_{\mathrm{ref}}} \, \mathrm{d}f \tag{A.15}$$

$$\begin{aligned} \frac{\partial S_{F_i}(f)}{\partial C_z} &= \frac{1}{C_z} \int_0^H \int_0^H \rho^2 C_D(z_1) C_D(z_2) D(z_1) D(z_2) V(z_1) V(z_2) Co_u(z_1, z_2; f) \\ &\quad \times \Phi_i(z_1) \Phi_i(z_2) \ln(R(z_1, z_2; f)) \, \mathrm{d}z_1 \, \mathrm{d}z_2 \end{aligned} \tag{A.16}$$

$$\begin{aligned} \frac{\partial S_{F_i}(f)}{\partial \alpha} &= \int_0^H \int_0^H \rho^2 C_D(z_1) C_D(z_2) D(z_1) D(z_2) V(z_1) V(z_2) \Phi_i(z_1) \Phi_i(z_2) \\ &\quad \times Co_u(z_1, z_2; f) \left[ \ln\left( \frac{z_1}{z_{\mathrm{ref}}} \right) + \ln\left( \frac{z_2}{z_{\mathrm{ref}}} \right) \right] \mathrm{d}z_1 \, \mathrm{d}z_2 \end{aligned} \tag{A.17}$$

$$\begin{aligned} \frac{\partial S_{F_i}(f)}{\partial V_{\mathrm{ref}}} &= \frac{1}{V_{\mathrm{ref}}} \int_0^H \int_0^H \rho^2 C_D(z_1) C_D(z_2) D(z_1) D(z_2) V(z_1) V(z_2) Co_u(z_1, z_2; f) \\ &\quad \times \Phi_i(z_1) \Phi_i(z_2) \left[ 4 - \frac{2}{3} \left( \frac{3 - x^2}{1 + x^2} \right) - \ln R \right] \mathrm{d}z_1 \, \mathrm{d}z_2 \end{aligned} \tag{A.18}$$

$$\begin{aligned} \frac{\partial S_{F_i}(f)}{\partial D^*} &= \frac{1}{D^*} \int_0^H \int_0^H S_q(z_1, z_2; f) \\ &\quad \times \left[ 3 + \frac{\frac{f}{f_{s_1}} \left( 1 - \frac{f}{f_{s_1}} \right)}{B^2} + \frac{\frac{f}{f_{s_2}} \left( 1 - \frac{f}{f_{s_2}} \right)}{B^2} + \frac{2}{3} \frac{R'(r)}{R(r)} \right] \Phi_i(z_1) \Phi_i(z_2) \, \mathrm{d}z_1 \, \mathrm{d}z_2 \end{aligned}$$

$$R'(r) = \left[ r \sin(\tfrac{2}{3} r) + \frac{r}{3} \cos(\tfrac{2}{3} r) \right] \exp\left[ -\left( \frac{r}{3} \right)^2 \right] \tag{A.19}$$

$$\frac{\partial S_{F_i}(f)}{\partial B} = \frac{1}{B} \int_0^H \int_0^H S_q(z_1, z_2; f) \left[ -1 + \left( \frac{1 - \frac{f}{f_{s_1}}}{B} \right)^2 + \left( \frac{1 - \frac{f}{f_{s_2}}}{B} \right)^2 \right] \Phi_i(z_1) \Phi_i(z_2) \, \mathrm{d}z_1 \, \mathrm{d}z_2 \tag{A.20}$$

$$\frac{\partial S_{F_i}(f)}{\partial S^*} = \frac{1}{S^*}\int_0^H\int_0^H S_q(z_1, z_2; f)\left[-1 - \frac{\frac{f}{f_{s_1}}\left(1 - \frac{f}{f_{s_1}}\right)}{B^2} + \frac{\frac{f}{f_{s_2}}\left(1 - \frac{f}{f_{s_2}}\right)}{B^2}\right]\Phi_i(z_1)\Phi_i(z_2)\,\mathrm{d}z_1\,\mathrm{d}z_2 \tag{A.21}$$

$$\frac{\partial S_{F_i}(f)}{\partial \alpha} = \int_0^H\int_0^H S_q(z_1, z_2; f)\left\{\left[1.5 - \frac{\frac{f}{f_{s_1}}\left(1 - \frac{f}{f_{s_1}}\right)}{B^2}\right]\ln\left(\frac{z_1}{z_{\mathrm{ref}}}\right)\right.$$
$$\times\left.\left[1.5 - \frac{\frac{f}{f_{s_2}}\left(1 - \frac{f}{f_{s_2}}\right)}{B}\right]\ln\left(\frac{z_2}{z_{\mathrm{ref}}}\right)\right\}\Phi_i(z_1)\Phi_i(z_2)\,\mathrm{d}z_1\,\mathrm{d}z_2 \tag{A.22}$$

$$\frac{\partial S_{F_i}(f)}{\partial V_{\mathrm{ref}}} = \frac{1}{V_{\mathrm{ref}}}\int_0^H\int_0^H S_q(z_1, z_2; f)\left[3 - \frac{\frac{f}{f_{s_1}}\left(1 - \frac{f}{f_{s_1}}\right)}{B^2} + \frac{\frac{f}{f_{s_1}}\left(1 - \frac{f}{f_{s_2}}\right)}{B^2}\right]\Phi_i(z_1)\Phi_i(z_2)\,\mathrm{d}z_1\,\mathrm{d}z_2 \tag{A.23}$$

## APPENDIX B

The spectral descriptions of the alongwind and acrosswind forcing functions based on the covariance integration formulation are utilized in this study [3,13,21] and are given by the following:

$$S_{F_i}(f) = \int_0^H\int_0^H S_q(z_1, z_2; f)\Phi_i(z_1)\Phi_i(z_2)\,\mathrm{d}z_1\,\mathrm{d}z_2 \tag{B.1}$$

### Alongwind

$$S_q(z_1, z_2; f) = \rho^2 C_D(z_1)C_D(z_2)D(z_1)D(z_2)U(z_1)U(z_2)Co_u(z_1, z_2; f)$$

$$Co_u(z_1, z_2; f) = R_u(z_1, z_2; f)S_u(z_1, f)S_u(z_2, f)$$

$$S_u(z, f) = S_u(f) = \frac{4\kappa x^2}{f(1 + x^2)^{4/3}}U_{\mathrm{ref}} \tag{B.2}$$

$$R_u(z_1, z_2; f) = \exp\left(-\frac{C_z\,|z_1 - z_2|\,f}{\frac{1}{2}\left[\bar{U}(z_1) + \bar{U}(z_2)\right]}\right)$$

in which $\kappa$ = surface coefficient, $C_z$ = constant, $x = Lf/V_{\mathrm{ref}}$ and $L$ = length scale. Alternate expressions for $S_u(f)$, and $R_u(z_1, z_2; f)$ are available in the literature.

**Acrosswind**

$$S_q(z_1, z_2; f) = \sqrt{S_p(z_1, f) S_p(z_1, f)}\, R_p(z_1, z_2; f)$$

$$S_p(z, f) = \tfrac{1}{2}\rho C_L(z) D(z) \bar{U}^2(z) \frac{1}{f_s \sqrt{\pi}\, B} \exp\left[1 - \left(\frac{1 - f/f_s}{B}\right)^2\right] \tag{B.3}$$

$$R_p(z_1, z_2; f) = \cos\left(\frac{2r}{3}\right) \exp\left[-\left(\frac{r}{3}\right)^2\right]$$

in which $r = 2\,|z_1 - z_2|/[D(z_1) + D(z_2)]$; $f_s = S(z)U(z)/D(z)$; $S(z)$ = Strouhal number, $C_L(z)$ = local RMS lift coefficient. The aerodynamic damping in the acrosswind direction are given by Basu and Vickery [21] as follows:

$$\xi_a = \frac{-\rho D^2}{M_e}\left[C_1 - C_2\left(\frac{\sigma}{D}\right)^N\right] \tag{B.4}$$

in which $M_e$ is the equivalent modal mass per unit length, and $C_1$ and $C_2$ are functions of the mode shape and aerodynamic parameters. The contribution of lateral turbulence to the acrosswind loads was not included in this study.