

# SIMULATION OF MULTIVARIATE NONSTATIONARY RANDOM PROCESSES: HYBRID DFT AND DIGITAL FILTERING APPROACH

By Yousun Li<sup>1</sup> and Ahsan Kareem<sup>2</sup>

**ABSTRACT:** A numerical simulation scheme is presented that combines the advantages of the discrete Fourier transform algorithm and a digital filtering scheme to generate nonstationary multivariate random processes. The resulting time histories provide piecewise continuous evolutionary spectra and the proposed simulation technique offers significant computational efficiency. The effectiveness of the proposed technique is demonstrated with examples. The simulated records are in excellent agreement with the prescribed probabilistic characteristics. The proposed technique has immediate applications to the simulation of ground motions, evolutionary sea states, and fast-moving gust fronts.

## INTRODUCTION

The nonstationary characteristics are observed in many random environmental load effects, that is, their amplitudes and/or their frequency contents are time dependent. Typical examples include atmospheric turbulence during the passage of a weather front, evolutionary sea states, and seismic excitation. The evolutionary characteristics of such a random process are described by Priestley's model (Priestley 1967)

$$y(t) = \int_{-\infty}^{\infty} A(t, \omega) e^{i\omega t} dZ(\omega) \quad (1)$$

where  $A(t, \omega)$  = a modulating function; and  $dZ(\omega)$  = an orthogonal increment process satisfying

$$E[dZ(\omega)] = 0$$

$$E[dZ(\omega_1) dZ(\omega_2)] = \Phi(\omega_1) \delta(\omega_1 - \omega_2) d\omega_1 d\omega_2$$

Accordingly, the definition of the evolutionary power spectral density (PSD) is given by

$$S(t, \omega) = |A(t, \omega)|^2 \Phi(\omega) \quad (2)$$

Following the preceding definition of the evolutionary PSD, several models have been proposed for generating nonstationary processes. The numerical integration of (1) by discretizing the evolutionary spectrum in the frequency domain is an obvious choice. However, this is not a numerically efficient approach.

The simulation of nonstationary time series can be simplified by describing a nonstationary time history in terms of processes modulated by time-varying functions. Following Grigoriu et al. (1988), Yeh and Wen (1990) expressed earthquake-induced ground motion as a frequency-modulated process multiplied by a deterministic time-varying function. It is noted that these models, which describe frequency modulations, do not necessarily have evolutionary PSDs, rather they are represented by instantaneous spectra. Saragoni and Hart (1974) partitioned the processes into several segments and proposed a piecewise stationary model. This representation introduced abrupt changes that are difficult to justify in the light of physical processes. An extension of the Saragoni-Hart model was proposed by Der Kiureghian and Crempien (1988).

They described the model as a summation of modulated banded white noise. Li and Kareem (1991) also used the modulated stationary time series concept. A nonstationary process was expressed as a sum of mutually correlated stationary processes modulated by a deterministic time function. The spectra of the stationary processes and the deterministic modulating function can be obtained by matching a prescribed evolutionary spectrum. The time simulation can be performed by using a numerically efficient fast Fourier transform (FFT) algorithm. Besides the preceding examples, the concept of a modulated stationary process centered at narrow-banded frequencies to model ground motion has been used by others, including Kamada (1975), Scherer et al. (1982), Preumont (1985), and Scherer (1994). Grigoriu (1993) presented a new probabilistic model to simulate realizations of general nonstationary Gaussian processes using random trigonometric polynomials.

Recently, the popularity of wavelets due to their retention of both time and frequency information has led to the simulation of nonstationary processes using wavelet transforms (Kareem et al. 1993; Gurley and Kareem 1994). The method is again based on the concept of a modulated stationary process centered at narrow-banded frequencies to model ground motion. In this representation, each component process is modulated by a different modulating function. This approach is used to simulate ground motion records. The statistical and spectral features of the parent and simulated records exhibit good comparisons (Gurley and Kareem 1994).

The application of modulation-based methods may be limited because the modulating functions may not be easily found, or a large number of functions may be required to match the given time records or prescribed evolutionary spectra.

The digital filtering approach offers an efficient alternative means of simulation. The nonstationary time series can be generated by a digital impulse or by a filtered white noise (Lin and Yong 1987; Shinozuka and Deodatis 1989). Autoregressive and moving-averages models offer improved accuracy and efficiency [e.g., Kozin (1988), Conte et al. (1992), Cakmak et al. (1985), Polhemus and Cakmak (1981), Deodatis and Shinozuka (1989), and Gersch and Kitagawa (1985)]. However, the determination of parameters in the digital filters may become difficult for accurate representation of the evolutionary spectra. Other models based on the output of a time-variant linear filter to stationary white noise are available [e.g., Safak and Borre (1986)]. Simulation based on these models involves numerical difficulty, particularly when the target spectra are characterized by multiple peaks. The simulation of response-spectrum-consistent ground motion is presented in Spanos (1983).

The theoretical background of random processes with nonstationary probabilistic characteristics is omitted here for the sake of brevity. For additional background, a sample of related references is provided: Priestley (1967), Mark (1986), Lin and

<sup>1</sup>Sr. Res. Engr., Shell E&P Co., Houston, TX 77001-0481.

<sup>2</sup>Prof., Dept. of Civ. Engrg. and Geol. Sci., Univ. of Notre Dame, Notre Dame, IN, 46556-0767.

Note. Associate Editor: Mahendra P. Singh. Discussion open until May 1, 1998. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on January 17, 1996. This paper is part of the *Journal of Engineering Mechanics*, Vol. 123, No. 12, December, 1997. ©ASCE, ISSN 0733-9399/97/0012-1302-1310/\$4.00 + \$.50 per page. Paper No. 12455.

Yong (1987), Lin (1967), Li and Kareem (1988), Sun and Kareem (1989), Madsen and Krenk (1981), Borino et al. (1988), Roberts and Spanos (1990), and Soong and Grigoriu (1993).

The present paper concerns the development of a simulation technique for nonstationary processes based on a recently developed technique for stationary processes that uses a hybrid combination of discrete Fourier transform (DFT) and digital filters (Li and Kareem 1993). This approach is especially suited for efficient simulation of a large number of sample time records of nonstationary processes. In the time domain analysis of structural response to nonstationary excitation, a large number of input time histories are needed for obtaining statistical averages due to lack of ergodicity. This requires ensemble averages in contrast with temporal averages used in ergodic cases. Therefore, an efficient simulation scheme becomes an essential tool for analysis. The time histories are generated directly from the evolutionary spectra, that is, the modulating functions or model parameters are not explicitly evaluated as noted earlier for the techniques based on modulated processes.

## THEORETICAL BACKGROUND

For the sake of illustration, let us consider a univariate stationary Gaussian process  $y(t)$  with a single-sided spectral density function  $G(f)$

$$y(t) \stackrel{\text{Re}}{=} \sum_{k=0}^{\infty} \sqrt{2G(f_k)\delta f} \epsilon_k e^{j2\pi f_k t} \lim_{\delta f \rightarrow 0} \quad (3)$$

where  $f_k = k\delta f$ ;  $\delta f$  = frequency resolution; and  $\epsilon_k$  = a complex Gaussian random process that satisfies the following:

$$E[\epsilon_k \epsilon_k^*] = \delta_{kk} \quad (4)$$

and  $\delta_{kk}$  represents the Dirac delta function.

Similarly, a nonstationary random process with evolutionary PSD,  $G(f, t)$ , can be expressed as [e.g., Shinozuka and Deodatis (1988)]

$$y(t) \stackrel{\text{Re}}{=} \sum_{k=0}^{\infty} (\sqrt{2G(f_k, t)\delta f}) \epsilon_k e^{j2\pi f_k t} \lim_{\delta f \rightarrow 0} \quad (5)$$

The preceding expression for the simulation of nonstationary univariate processes can be conveniently extended to multivariate processes following the approach used for multivariate stationary processes. For example, stationary multivariate ( $N$ ) correlated processes can be simulated based on the prescribed cross-power spectral density matrix  $\mathbf{G}(f)$  of dimension ( $N, N$ ) [e.g., Shinozuka (1963)]

$$y_i(t) \stackrel{\text{Re}}{=} \sum_{j=1}^N \sum_{k=0}^{\infty} \sqrt{2\delta f} D_{ij}(f_k) \epsilon_{jk} e^{j2\pi f_k t} \lim_{\delta f \rightarrow 0} \quad (6)$$

where  $E[\epsilon_{jk} \epsilon_{jk}^*] = \delta_{jj} \delta_{kk}$ . In the preceding equation,  $D_{ij}(f)$  = the element of the  $i$ th row and  $j$ th column of the complex matrix  $\mathbf{D}(f)$

$$\mathbf{G}(f) = \mathbf{D}(f) \mathbf{D}^*(f) \quad (7)$$

where the asterisk denotes conjugate and transpose of a matrix. Accordingly, the evolutionary cross-spectral density matrix  $\mathbf{G}(f, t)$  leads to the following expression for a nonstationary process:

$$y_i(t) \stackrel{\text{Re}}{=} \sum_{j=1}^N \sum_{k=0}^{\infty} \sqrt{2\delta f} D_{ij}(f_k, t) \epsilon_{jk} e^{j2\pi f_k t} \lim_{\delta f \rightarrow 0} \quad (8)$$

where  $D_{ij}(f, t)$  = an element of  $\mathbf{D}(f, t)$

$$\mathbf{G}(f, t) = \mathbf{D}(f, t) \mathbf{D}^*(f, t) \quad (9)$$

For numerical implementation (8) is recast by truncating the summation to a finite number

$$y_i(t) \stackrel{\text{Re}}{=} \sum_{j=1}^N \sum_{k=0}^{N_f} \sqrt{2\delta f} D_{ij}(f_k, t) \epsilon_{jk} e^{j2\pi f_k t} \quad (10)$$

For the nonstationary process given in (8), the cross-correlation between two components is given by

$$R_{ii'}(t, \tau) \stackrel{\text{Re}}{=} E[y_i(t) y_{i'}(t + \tau)] \quad (11)$$

After some mathematical manipulations the preceding cross-correlation can be expressed in terms of the evolutionary spectral matrix as

$$R_{ii'}(t, \tau) = \sum_{j=1}^N \sum_{k=0}^{N_f} D_{ij}(f_k, t + \tau) D_{i'j}^*(f_k, t) e^{j2\pi f_k \tau} \delta f \quad (12)$$

The computational effort involved in the simulation of a realization of a nonstationary process is quite significant due to the summation of a large number of complex functions in (8). The need for a large number of frequency intervals for accurate representation of spectral contents results in additional computational demand. This computational difficulty is further compounded by the need to simulate many realizations for subsequent response analysis due to the lack of ergodicity. Simulation of vector fields would add to the overall computational effort.

One of the drawbacks of this approach is that the simulated process is periodic, with a period of  $1/\delta f$  (Shinozuka 1972). The periodicity can be eliminated by using a variable frequency interval. Another shortcoming stems from the major computational effort needed for a multivariate process. From spectral considerations, the PSD of the simulated process does not duplicate the target spectral density. The resulting estimated evolutionary spectral matrix,  $\hat{\mathbf{G}}(f, t)$ , is given by

$$\hat{\mathbf{G}}(f, t) = \sum_{k=1}^{N_f} \mathbf{G}(f_k, t) \delta(f - f_k) \quad (13)$$

It is obvious that the simulated spectrum has a spiky structure. A lack of continuity in the spectral description may limit the use of generated data.

In the case of stationary processes, the preceding simulation scheme may be realized by the FFT in which  $\delta f = 1/T$ , where  $T$  is the total duration of the process to be simulated. In the limiting case, the discrete spikes approach the continuous spectral description. However, due to the time-variant nature of the evolutionary process, the application of the straightforward FFT is not possible. To facilitate the use of the FFT algorithm, Li and Kareem (1991) proposed to decompose the evolutionary spectra in terms of a trigonometric or polynomial expansion as given here

$$D_{ij}(f, t) = \sum_{j=1}^N \sum_{r=1}^{N_r} \psi_{jr}(t) D_{ijr}(f) \quad (14)$$

In this manner, the evolutionary spectra are decomposed into products of the deterministic time functions  $\psi_{jr}(t)$  and spectra of stationary random subprocesses,  $D_{ijr}(f)$ . Details are omitted here. For additional information, interested readers may refer to Li and Kareem (1991). The desired time history of a component of a multivariate nonstationary process according to (8) is given by

$$y_i(t) = \sum_{j=1}^N \sum_{r=1}^{N_r} \psi_{jr}(t) Y_{ijr}(t) \quad (15)$$

where  $Y_{ijr}(t)$  = a stationary random process that can be simulated by an FFT-based algorithm utilizing  $D_{ijr}(f)$ . In this manner, a

nonstationary process becomes the sum of a stationary subprocess  $Y_{jr}(t)$  modulated by a deterministic time function  $\psi_{jr}(t)$ . Theoretically, almost any evolutionary spectral density matrix can be decomposed according to (14). In Li and Kareem (1991), examples of the El Centro and Nagata earthquakes are given to illustrate this methodology. Also included in Li and Kareem (1991) is a numerical scheme that facilitates such a decomposition in the case where only data points are available to describe the evolutionary spectral characteristics. In exceptional cases where complicated amplitude and frequency modulation is present, the decomposition order  $N_r$  may become large, which may influence the computational efficiency of this FFT-based approach. However, for such events, even the simple trigonometric summation approach would require extremely fine discretization, which causes a very high computational demand.

In this paper, a new approach based on the concept of modulated discrete Fourier series that uses a hybrid combination of the FFT algorithm and a digital filtering scheme is presented. This approach has already been successfully implemented for the simulation of continuous stationary multivariate random processes of very long duration (Li and Kareem 1993). The simulation of such long records otherwise imposes a very heavy demand on the computer memory (Li and Kareem 1993).

### MODULATED DISCRETE FOURIER SERIES APPROACH

The concept of modulated discrete Fourier series is introduced by considering first a univariate nonstationary process. The target spectrum is expressed in terms of a weighted summation of triangle-shaped spectra

$$\hat{G}(f, t) = \sum_{r=0}^{N_r} G(r\Delta f, t) G'_r(f) \quad (16)$$

where  $G'_r(f)$  is a triangle-shaped spectrum with a unit height at the frequency  $r\Delta f$ . The frequency axis is divided into  $N_r$  equal nonoverlapping segments. The spectrum  $G'_r(f)$  is described by

$$G'_r(f) = 1 - \frac{r\Delta f - f}{\Delta f} \text{ for } (r-1)\Delta f \leq f < r\Delta f \quad (17a)$$

$$G'_r(f) = 1 - \frac{f - r\Delta f}{\Delta f} \text{ for } r\Delta f \leq f < (r+1)\Delta f \quad (17b)$$

$$G'_r(f) = 0 \text{ otherwise} \quad (17c)$$

As noted in Fig. 1, the summation of these triangles according to (16) form a piecewise continuous spectral density function  $\hat{G}(f, t)$ . For large  $N_r$ , the target and the modeled spectra become very close. The task in hand is to simulate processes that conform to the spectrum described in (16). The simulation would involve a summation of  $N_r$  uncorrelated subprocesses. The  $r$ th subprocess is a product of a deterministic time function  $\sqrt{G(r\Delta f, t)}$  and a stationary process  $y'_r(t)$ , consistent with the spectrum  $G'_r(f)$ . Accordingly, the desired nonstationary process is given by

$$y(t) = \sum_{r=0}^{N_r} \sqrt{G(r\Delta f, t)} y'_r(t) \quad (18)$$

Direct simulation of  $y'_r(t)$  from  $G'_r(f)$  is not convenient. Therefore, an alternative approach is proposed. Let us define stationary complex random processes  $A_r(t)$  with  $r = 0, 1, \dots, N_r$ . The one-sided PSD of their real and imaginary parts is characterized by a right triangle shape given by

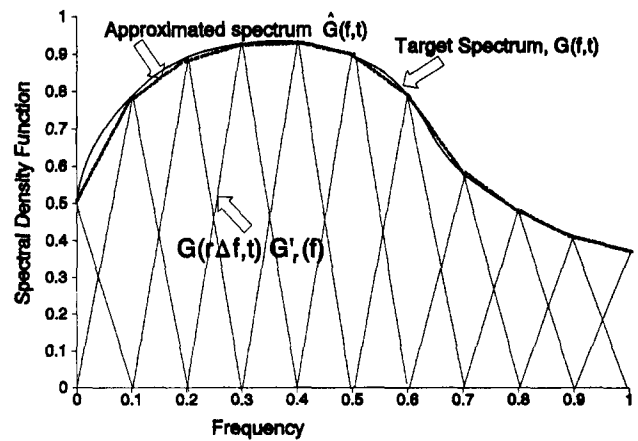


FIG. 1. Spectrum Represented by Modulated Fourier Series

$$G_A(f) = 2 \left( 1 - \frac{f}{\Delta f} \right) \text{ for } 0 \leq f \leq r\Delta f$$

and

$$G_A(f) = 0 \text{ otherwise} \quad (19)$$

Both the real and imaginary parts of  $A_r(t)$  are statistically independent stationary Gaussian processes that satisfy

$$E[A_r(t)A_s^*(t)] = 2\Delta f \delta_{rs} \quad (20)$$

A frequency shift can transform  $A_r(t)$  into  $y'_r(t)$

$$y'_r(t) \stackrel{\text{Re}}{=} A_r(t) e^{j2\pi r \Delta f t} \text{ for } r \neq 0 \quad (21)$$

$$y'_r(t) \stackrel{\text{Re}}{=} 0.5A_r(t) \text{ for } r = 0 \quad (22)$$

In this manner,  $y'_r(t)$  is a stationary Gaussian process by virtue of the real and imaginary parts of  $A_r(t)$  being stationary Gaussian processes and mutually orthogonal as described by (20). The single-sided spectra of  $y'_r(t)$  and  $A_r(t)$  are related according to the following

$$G'_r(f) = \frac{1}{2} G_A(|f - r\Delta f|)$$

for  $r \neq 0$  and  $(r-1)\Delta f < f < (r+1)\Delta f$ , and

$$G'_0(f) = \frac{1}{2} G_A(f)$$

for  $r = 0$  and  $f < \Delta f$ . Outside the preceding frequency range

$$G'_r(f) = 0 \quad (23)$$

The shift in frequency contents of the spectra  $G_A(f)$  and  $G'_r(f)$  by  $e^{j2\pi r \Delta f t}$  is illustrated in Fig. 2. Now, the process  $y(t)$  given by (18) can be simulated by

$$y(t) \stackrel{\text{Re}}{=} \sum_{r=0}^{N_r} \sqrt{G(r\Delta f, t)} A_r(t) e^{j2\pi r \Delta f t} \quad (24)$$

The preceding mathematical manipulations have resulted in (24), which has a form similar to (5), except that function  $A_r(t)$  modulates both the amplitude and the phase in such a way that the target spectrum is piecewise continuous rather than spiky as in (5).

Let the time increment of the desired simulation be  $\delta t$ . The corresponding  $\Delta f$  is chosen such that

$$M = \frac{1}{2\Delta f \delta t} \quad (25)$$

where  $M$  is an integer and is used to calculate a complex coefficient vector  $S_m$

$$S_m = e^{j\pi(m/M)}; \quad m = 0, 1, 2, \dots, 2M - 1 \quad (26)$$

Expressing (22) in discrete time steps and substituting (24) provides

$$y(n\delta t) \stackrel{\text{Re}}{=} \sum_{r=0}^{N_r} \sqrt{G(r\Delta f, n\delta t)} A_r(n\delta t) S_m \quad (27)$$

where  $m =$  the remainder of  $(nr)/(2M)$ . The purpose of manipulating (24) to obtain (27) in which  $e^{j2\pi r\Delta f t}$  is replaced by  $S_m$  is to reduce unnecessary repetition in the evaluation of trigonometric functions, since  $e^{j2\pi r\Delta f t} = e^{j2\pi r\Delta f(t+2M\delta t)}$ . Therefore, we need to evaluate certain trigonometric functions once and they can be subsequently used for different time steps.

The preceding formulation for a univariate process can be extended to multivariate processes. Let us consider the case of  $N$  multicorrelated random processes with prescribed spectral characteristics, e.g.,  $\mathbf{G}(f, t)$ , the evolutionary cross-spectral density matrix ( $N, N$ ). This matrix can be factorized as given in (9). This concept of decomposition has been used by Li and Kareem (1991, 1993, 1995) for the modeling of random linear and quadratic systems. Central to this technique is the decomposition of a set of correlated random processes into component random processes, such that the relationship between any two subprocesses  $y_{i\mu}(t)$  and  $y_{j\lambda}(f)$  is fully coherent for  $\mu = \lambda$  and noncoherent if  $\mu \neq \lambda$ . In the present context, fully coherent and noncoherent imply coherence equal to unity or zero, respectively. The decomposed processes are related to the parent process by

$$y_i(f) = \sum_{\mu=1}^N y_{i\mu}(f) \quad (28)$$

where  $y_{i\mu}(f)$  represents the  $\mu$ th component process associated with the  $i$ th element. Hence, each random process  $y_i(f)$  is viewed as a summation of mutually noncoherent component subprocesses. In the frequency domain, the component processes are expressed in terms of a decomposed spectrum represented by the elements of matrix  $\mathbf{D}(f, t)$  in (9).

Once the target process is decomposed into the component subprocesses, which are either fully coherent or noncoherent, the simulation of correlated processes can be accomplished by applying (28). The  $i$ th component of the multivariate process is given by

$$y_i(n\delta t) \stackrel{\text{Re}}{=} \sum_{j=1}^N \sum_{r=0}^{N_r} D_{ij}(r\Delta f, n\delta t) A_r(n\delta t) S_m \quad (29)$$

where  $D_{ij}(\cdot)$  is defined in (9), and  $A_r(n\delta t) =$  a complex modulating function whose real and imaginary parts are Gaussian random processes. Their spectral description is given in (19) and the processes are statistically independent

$$E[A_{jr}(t)A_{j'r'}^*(t)] = 2\delta_{rr'}\delta_{jj'}\Delta f \quad (30)$$

## DOUBLE-SUBSCRIBED DISCRETE CONVOLUTION

The efficiency of simulation using (29) depends on the generation of the stationary processes,  $A_r(n\delta t)$  for  $r = 0, 1, \dots, N_r$  and  $j = 1, 2, \dots, N$ , and  $n = 0, 1, 2, \dots, \infty$ . One efficient way of simulating  $A_r(n\delta t)$  is with a double-subscribed discrete convolution to filter a white-noise process to obtain the desired spectral features. Li and Kareem (1993) developed this technique for simulating a stationary Gaussian vector process. A detailed derivation of this approach can be found in Li and Kareem (1993). In this paper, simulation appropriate in the context of nonstationary signals is presented.

The basic form of the double-subscribed discrete convolution is given by

$$A_{jr}[(\beta M + \alpha)\delta t] = \sum_{k=-N_k+1}^{N_k} C_{\alpha k} \xi_r(\beta + k) \quad (31)$$

where  $\xi =$  a complex Gaussian white-noise process with statistically independent real and imaginary parts  $E[\xi_r(K)\xi_r'(K')] = \delta_{rr'}\delta_{kk'}$ ;  $C_{\alpha k} =$  weights of the double-subscribed convolution with  $\beta = 0, 1, 2, \dots, \infty$ ;  $\alpha = 0, 1, 2, \dots, M - 1$  (i.e.,  $n = \beta M + \alpha$ ) and  $k = -N_k + 1, -N_k + 2, \dots, 0, 1, 2, \dots, N_k$ ;  $M$  has been defined earlier; and  $N_k =$  order of the discrete convolution. The convolution weights  $C_{\alpha k}$  are derived based on the desired target spectrum. For the spectral description given in (19), the following weights are derived by taking the DFT of (31)

$$C_{\alpha k} \cong \frac{1}{N_k} \sqrt{\frac{\Delta f}{2}} \left[ 1 + 2 \sum_{l=1}^{N_k-1} \sqrt{1 - l/N_k} \cos\left(\pi \frac{(kM - \alpha)l}{N_k M}\right) \right] \quad (32)$$

Detailed derivation of (32) is given in Appendix I. Like any discretized operator, an agreement between the target and prescribed spectra is improved as  $N_k$  increases. Numerical experiments suggest that for  $N_k = 2$ , a sufficiently good agreement is obtained that involves only four multiplications in (31). Generally, the double-subscribed convolution results in a nonstationary process. However, if the convolution coefficients satisfy certain conditions,  $A_r(t)$  results in a stationary, ergodic, Gaussian process. The coefficients given in (32) satisfy these conditions. In Li and Kareem (1993), details concerning the conditions under which a double-subscribed convolution leads to a stationary random process are presented.

## NUMERICAL EXAMPLES

In this section, examples concerning simulation of earthquake ground motions are used to demonstrate the effectiveness of the simulation technique presented. The first example concerns the north-south (N-S) component of the Dec. 30, 1934, El Centro earthquake. The single-point evolutionary PSD is given by (Liu 1970; Deodatis and Shinozuka 1988)

$$G_{ii}(f) = \left\{ \frac{\exp(-at) - \exp(-bt)}{\max[\exp(-at) - \exp(-bt)]} \right\}^2 \hat{K}(f) \quad (33)$$

where  $a = 0.25$ ;  $b = 0.5$ ; and  $\hat{K}(f) =$  Kanai-Tajimi spectrum expressed by

$$\hat{K}(f) = 4\pi S_o \frac{f_g^4 + 4\xi_g^2 f_g^2 f^2}{(f^2 - f_g^2)^2 + 4\xi_g^2 f_g^2 f^2} \quad (34)$$

where the parameters are  $S_o = 0.1 \text{ cm}^2 \text{ s}^{-3}$ ;  $f_g = 15/2\pi \text{ Hz}$ ; and  $\xi_g = 0.25$ .

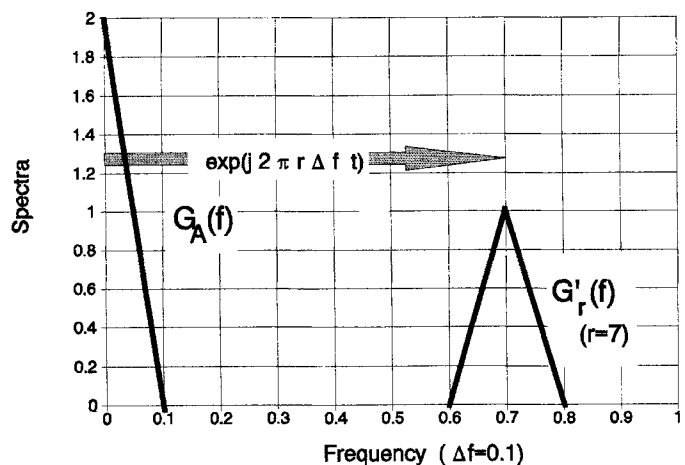


FIG. 2. Frequency Shift

Similarly for the east-west (E-W) component of the same earthquake (Liu 1970), the single-point evolutionary spectrum was analytically described by Deodatis and Shinozuka (1989)

$$G_{ii}(f, t) = \left\{ \frac{\exp(-at) - \exp(-bt)}{\max[\exp(-at) - \exp(-bt)]} \sqrt{\hat{K}_1(f)} + \exp \left[ -\frac{(t-m)^2}{2\sigma^2} \right] \sqrt{\hat{K}_2(f)} \right\}^2 \quad (35)$$

where  $\hat{K}_1(f)$  and  $\hat{K}_2(f)$  satisfy the Kanai-Tajimi spectrum in (32). The parameters in the Kanai-Tajimi spectrum remain the same with the exception that  $f_g = 30/2\pi$  Hz in  $\hat{K}_2(f)$ ;  $a$  and  $b$  have the same values as in the preceding example;  $m = 5.0$  s; and  $\sigma = 1.0$  s.

To introduce multipoint (multivariate) features, the model is extended based on the empirical relationship for the cross-spectral density function given by Harichandran and Vanmarcke (1986)

$$G_{ij}(f) = G_{ii}(f)\rho(v_{ij}, f)\exp \left( -j2\pi f \frac{v_{ij}}{V_{ij}} \right) \quad (36)$$

where  $v_{ij}$  denotes the distance between the  $i$ th and the  $j$ th locations;  $V_{ij}$  represents the apparent wave propagation velocity along the direction between the  $i$ th and  $j$ th locations; and  $\rho(v_{ij}, f)$  = the coherence function, given by

$$\rho(v_{ij}, f) = a \exp \left[ -\frac{2v_{ij}}{2\theta(f)} (1 - a + \alpha a) \right] + (1 - a) \exp \left[ -\frac{2v_{ij}}{\theta(f)} (1 - a + \alpha a) \right]$$

and  $\theta(f) = k[1 + (f/f_0)^c]^{-1/2}$  (37a,b)

where the parameters are  $a = 0.736$ ;  $c = 2.78$ ;  $\alpha = 0.147$ ;  $k = 5,210$  m/s; and  $f_0 = 1.09$ . The simulation scheme presented here is not restricted to the preceding description of multivariate processes. Any other formulation of this feature is equally applicable.

Both earthquake records are simulated at a time increment of 0.02 s for 20 s duration. The frequency resolution is 0.5 Hz, and 16 nonoverlapping segments are used to describe the desired spectrum. The order of the double-subscripted convolution used is equal to 2. On the average, at each time step of each time history, less than 100 multiplications and one random number are required. Accordingly, less computational effort is needed than in the summation of the large number of trigonometric functions used in conventional simulation.

This approach facilitates the efficient simulation of hundreds of ground motion time records used to estimate ensemble averages of corresponding structural response estimates. The simulated records are validated by comparing the correlation structure of the simulated and target processes. The estimated correlations represent ensemble averages of the correlations at each time instant involving 100 simulated time histories. The target correlations are directly obtained from (12).

In Figs. 3 and 4, the N-S and E-W components of the El Centro earthquake are presented at locations 1 and 2, which are 150 m apart and  $v_{ij} = 2000$  m/s. In Figs. 5 and 6 the auto- and cross-correlation functions for various time lags are presented for both the N-S and E-W components, respectively. Fig. 5 details auto- and cross-correlation for the N-S component for time lags of 0.0, 0.04, 0.1, 0.3, and 0.5 s. Also plotted are the corresponding target functions; the results show very good agreement. The estimated correlations can be further smoothed by increasing the number of sample time histories used for ensemble averaging.

Similar agreement between the correlations of the target and simulated processes is found for the E-W component and reported in Fig. 6.

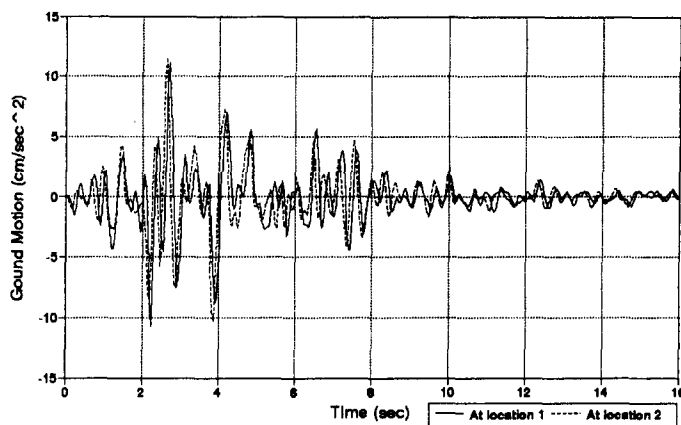


FIG. 3. Sample Time Histories of Ground Motion of N-S 1934 El Centro Earthquake

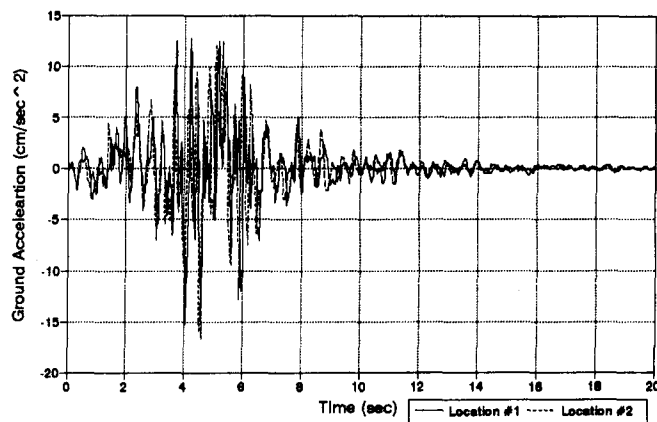


FIG. 4. Sample Time Histories of Ground Motion of E-W 1934 El Centro Earthquake

It has been noted in the case of AR( $P$ ) models that the simulated nonstationary time series have the exact cross-correlation matrices up to the time lag of  $(P - 1)\Delta t$ . At the remaining time lags, the cross-correlation matrices may be estimated through the maximum entropy approach. Hence only an AR model with suitably large values of  $P$  will provide the evolutionary auto- and cross-correlation functions close to the prescribed one. The straightforward application of the AR models for the evolutionary processes may have limitations. The approach presented in this paper offers an excellent match between the simulated and target correlations for large time lags as noted in Figs. 5 and 6. For nonstationary random processes that are characterized by a very narrow peak in the evolutionary spectral density, the number of segments needed to accurately describe the spectral description increases, which may compromise the computational efficiency. Such narrow spectral peaks are evidenced in seismic activity like the 1964 Nigata earthquake, in which soil liquefaction led to low-frequency oscillations in the signal that resulted in a narrow peak in the spectrum. It is noted that accurate simulation of such processes by conventional approaches also requires considerable additional computational effort.

## COMPARISON BETWEEN SIMULATION SCHEMES

In this section a comparison between simulation of nonstationary processes by summation of trigonometric functions and simulation by the hybrid DFT and digital filtering approach is made. In the time domain analysis of structural response to nonstationary excitation, a large number of input time histories is needed for obtaining statistical averages due to the noner-

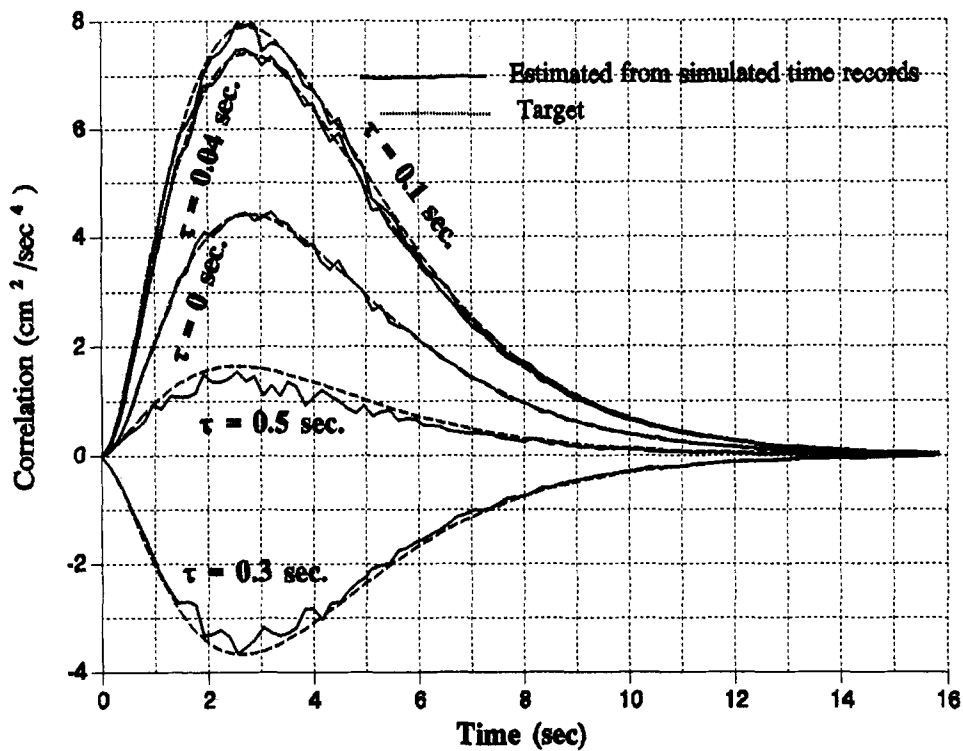
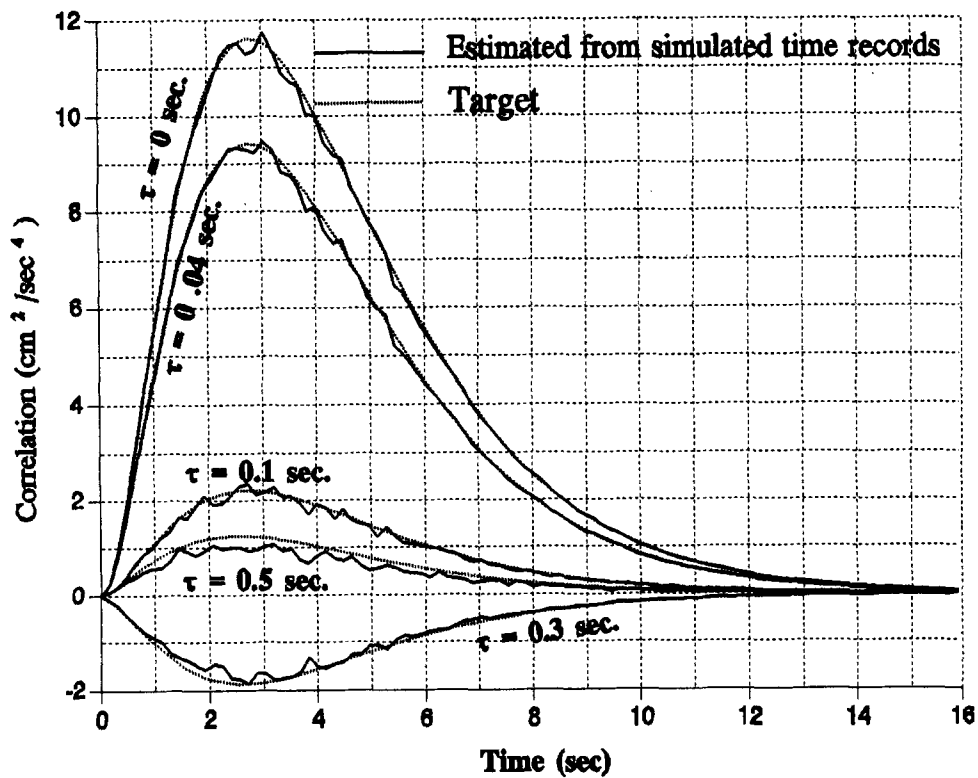


FIG. 5. Ground Motion for N-S of 1934 El Centro Earthquake: (a) Autocorrelation; (b) Cross Correlation

godic nature of the processes involved. This requires ensemble averages in contrast with temporal averages that are used in ergodic cases. For explanation of computational efficiency, we compare the average number of multiplications per time step required by the present method and the conventional summation of trigonometric functions method. A single-point simulation is considered for simplicity in the discussion.

By the present method, the amplitude  $\sqrt{G(r\Delta f, n\delta t)}$  and trigonometric function  $S_m$  [see (26)] can be evaluated at the

beginning of the simulation and repeatedly used for all the sample time records. The time used in computing these two parameters is negligibly small after averaging per time step. The number of multiplications at each time step in (29) and (31) is only  $(N_r + 1)(4N_k + 5)$ , and the average number of random numbers generated is  $2(N_r + 1)/M$ . In terms of computational effort, generation of one random number is approximately equivalent to 20 multiplications. For our example, we use  $N_r = 16$ ,  $N_k = 2$ , and  $M = 50$ . Then, on the average, 221

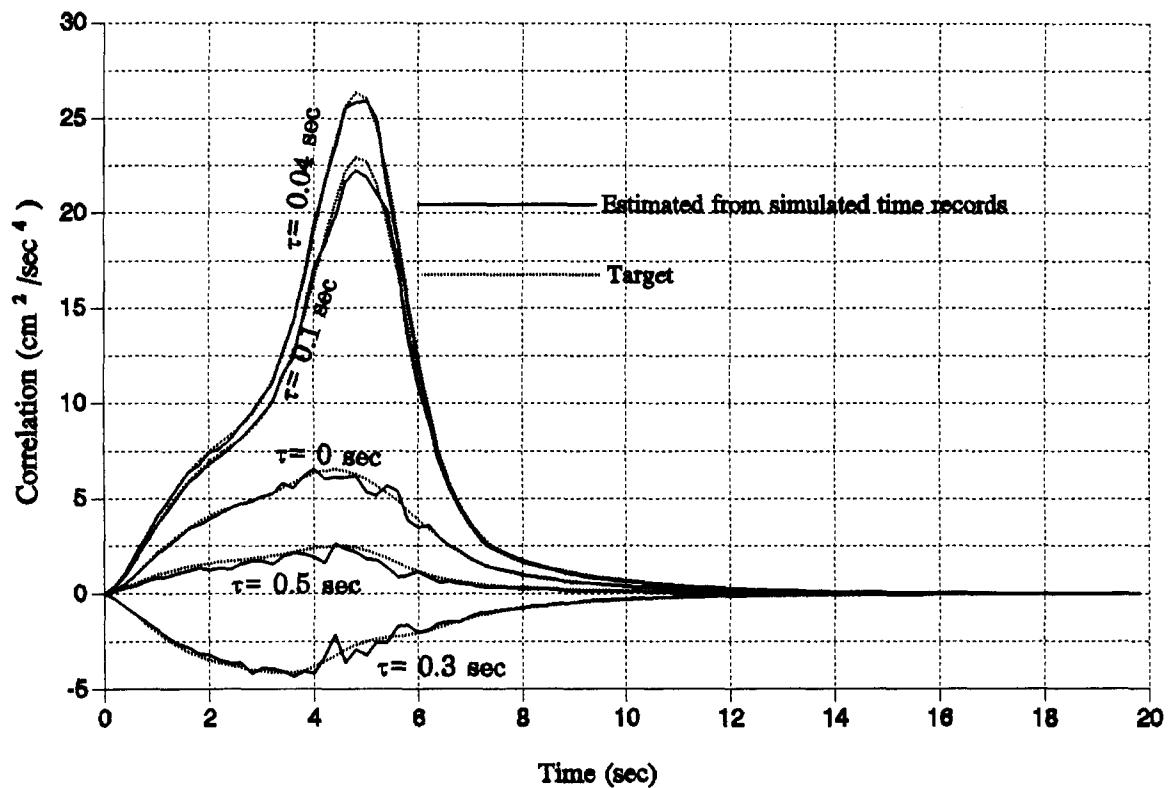
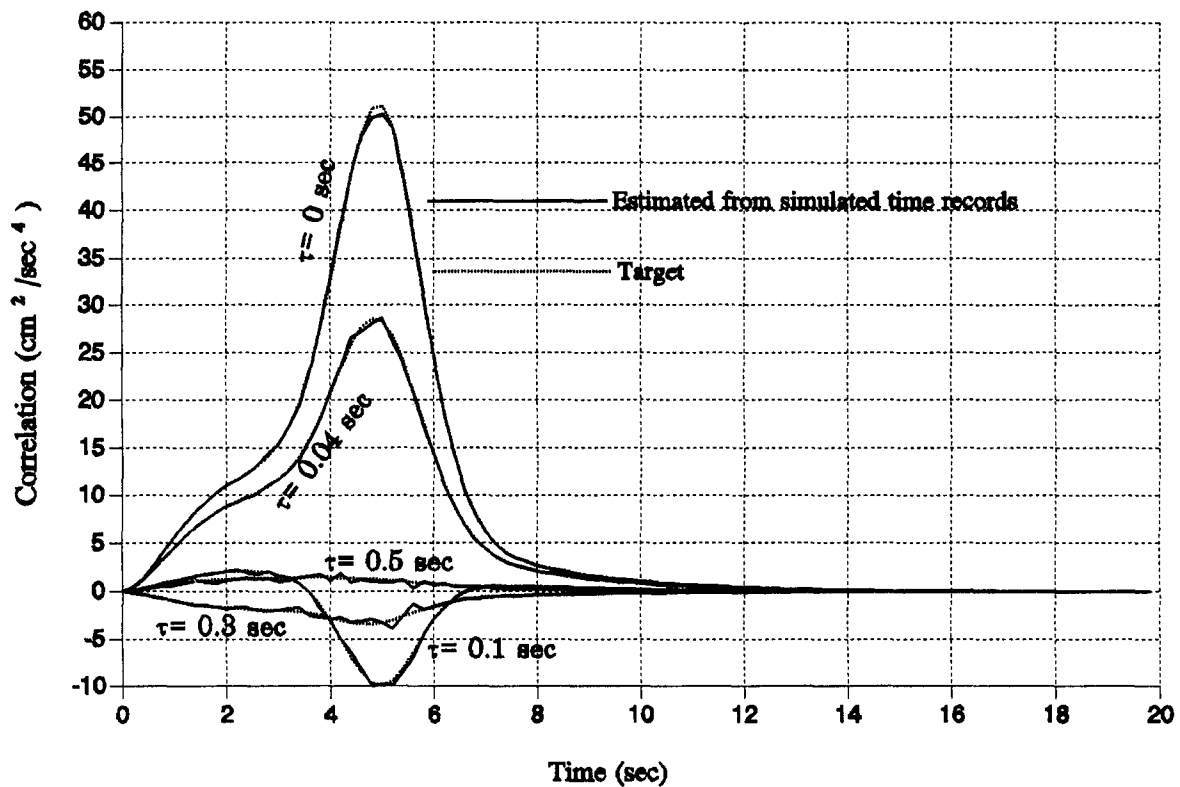


FIG. 6. Ground Motion for E-W of 1934 El Centro Earthquake: (1) Autocorrelation; (b) Cross Correlation

multiplications are required for the simulation of each time step for each record.

In conventional simulation by (10), as has been pointed out,  $f_k$  cannot be uniformly distributed. Otherwise, the simulated time records present  $N_f$  spectral peaks. For better statistics obtained from ensemble averaging,  $f_k$  should be altered with different sample records. Hence, at each time step of each sample record, both  $D(f_k, t)$  and  $\epsilon_k e^{j2\pi f_k t}$  for  $k = 1, \dots, N_f$  must be

recomputed. The former may involve 20 multiplications. The latter can be computed as a cosine function. Invoking the built-in cosine function requires a CPU time equivalent to six to 50 multiplications, including address searching. Let  $N_f$  in (10) be 100. Then at each time step, the equivalent number of multiplications is on the order of 5,000. The present method can be 20 times faster than the conventional one. This drastic improvement in computational effort becomes more significant

for multivariate simulation used for long-span structures, where the spatial coherence of random fields is important for simulation of loads on structures (e.g., earthquakes).

## CONCLUDING REMARKS

A computationally efficient FFT-based procedure for the simulation of evolutionary multivariate random processes is presented. A modulated Fourier series representation of the evolutionary characteristics combined with the double-subscripted convolution concept is used in the simulation process. The resulting simulated process not only provides a piecewise continuous spectrum as opposed to a spiky spectrum obtained from conventional simulation based on the summation of trigonometric functions, but also offers significant improvement in computational efficiency. The effectiveness of the proposed technique is demonstrated by means of two examples using analytical description of ground-motion-encompassing characteristics of actual earthquakes. The simulated records exhibit excellent agreement with the prescribed probabilistic characteristics (e.g., correlation structure). The simulation procedure is computationally efficient, particularly for simulating large numbers of multiply-correlated nonstationary random processes needed for ensemble averaging to evaluate response statistics. Applications of this procedure are immediate for the simulation of evolutionary sea states and gusts associated with a fast-moving gust front and in conditional simulation of nonstationary processes.

## ACKNOWLEDGMENTS

The support for this research was provided in part by the National Science Foundation Grant BCS90-96274 and CMS 95-03779 and several industrial sponsors. Their support is gratefully acknowledged.

## APPENDIX I. DERIVATION OF EQ. (32)

In this section a derivation of (32) is provided to detail the background and procedure so that the paper becomes self-explanatory. For the sake of illustration, rewriting (31) for a univariate case

$$A[(\beta M + \alpha)\delta t] = \sum_{k=-N_k+1}^{N_k} C_{\alpha k} \varepsilon(\beta + k) \quad (38)$$

where  $\alpha = 0, 1, 2, \dots, M-1$ ;  $\beta = 0, 1, 2, \dots, N_\beta-1$ ; and  $\varepsilon$  is a complex Gaussian White noise with statistically independent real and imaginary parts  $E[\varepsilon(K)\varepsilon(K')] = \delta_{kk}$ . Transforming both sides of (38) provides

$$\hat{A}(p) = \hat{C}(p)\hat{\varepsilon}(p) \quad (39)$$

where

$$\hat{A}(p) = \sum_{\beta=0}^{N_\beta-1} \sum_{\alpha=0}^{M-1} A[(\beta M + \alpha)\delta t] \exp \left[ j2\pi \frac{(\beta M + \alpha)p}{N_\beta M} \right] \quad (40a)$$

$$\hat{C}(p) = \sum_{k=-N_k+1}^{N_k} \sum_{\alpha=0}^{M-1} c_{\alpha k} \exp \left[ j2\pi \frac{(\alpha - kM)p}{N_\beta M} \right] \quad (40b)$$

$$\hat{\varepsilon}(p) = \sum_{\beta=0}^{N_\beta-1} \varepsilon(\beta) \exp \left[ j2\pi \frac{\beta p}{N_\beta} \right] \quad (40c)$$

Let us define  $\delta f = 1/(N_\alpha M \delta t)$  and the spectrum of  $\hat{A}$  and  $\hat{\varepsilon}$

$$S_A(p\delta f) = \frac{E[\hat{A}(p)\hat{A}^*(p)]}{(N_\beta M)^2 \delta f} \quad (41a)$$

$$S_\varepsilon(p\delta f) = \frac{E[\hat{\varepsilon}(p)\hat{\varepsilon}^*(p)]}{N_\beta^2 \delta f} \quad (41b)$$

We can express

$$S_A(p\delta f) = H(p\delta f)S_\varepsilon(p\delta f)H^*(p\delta f) \quad (42)$$

where  $H(p\delta f) = \hat{C}(p)/M$  as the transfer function of the double-subscripted convolution. Therefore

$$C_{\alpha k} = \frac{1}{N_k} \sum_{p=0}^{MN_k-1} H(p\delta f) \exp \left[ \frac{-j2\pi(\alpha - kM)p}{N_\beta M} \right] \quad (43)$$

Eq. (43) can be simplified as

$$C_{\alpha k} \cong \frac{1}{2N_k} \sum_{l=-MN_k-1}^{MN_k} H(l\delta f) \exp \left[ \frac{-j2\pi(\alpha - kM)p}{2N_k M} \right] \quad (44)$$

where  $\delta f = [N_\beta/(2N_k)]\delta t$ . Following (19) and  $G_\varepsilon(f) = 1/\Delta f$  and

$$H^2(l\delta f) = 2\Delta f \left( 1 - \frac{|l\delta f|}{\Delta f} \right) \quad (45)$$

Recall that  $\Delta f = 1/(2M\delta t)$  [from (25)], hence

$$\delta f = \frac{\Delta f}{N_k} \quad (46)$$

Therefore, the transfer function can be expressed

$$H(l\delta f) = \sqrt{2\Delta f \left( 1 - \frac{|l|}{N_k} \right)}; \quad 0 \leq |l| \leq N_k = 0; \quad \text{otherwise} \quad (47)$$

This leads to

$$C_{\alpha k} \cong \frac{1}{N_k} \sqrt{\frac{\Delta f}{2}} \left[ 1 + 2 \sum_{l=1}^{N_k-1} \sqrt{1 - \frac{l}{N_k}} \cos \left( \frac{\pi(kM - \alpha)l}{N_k M} \right) \right] \quad (48)$$

Following the preceding procedure, an extension to a multivariate case is immediate.

## APPENDIX II. REFERENCES

- Borino, G., Paola, M., and Muscolino, G. (1988). "Non-stationary spectral moments of base excited MDOF Systems." *Earthquake Engrg. and Struct. Dynamics*, 16, 745-746.
- Cakmak, A. S., Sherit, R. I., and Ellis, G. (1985). "Modelling earthquake ground motions in California using parametric time series methods." *Soil Dynamics and Earthquake Engrg.*, 4(3), 124-131.
- Conte, J. P., Pister, K. S., and Mahin, S. A. (1992). "Nonstationary ARMA modelling of seismic motions." *Soil Dynamics and Earthquake Engrg.*, 11(7), 411-420.
- Deodatis, G., and Shinozuka, M. (1989). "Auto-regressive model for nonstationary stochastic processes." *J. Engrg. Mech.*, ASCE, 114, 1995-2012.
- Der Kiureghian, A., and Crempien, J. (1989). "An evolutionary model for earthquake ground motion." *Struct. Safety*, 6, 235-246.
- Gersch, W., and Kitagawa, (1985). "A time varying AR coefficient model for modelling and simulating earthquakes ground motions." *Earthquake Engrg. and Struct. Dynamics*, 13(2), 124-131.
- Grigoriu, M. (1993). "Simulation of nonstationary Gaussian processes by random trigonometric polynomials." *J. Engrg. Mech.*, ASCE, 119(2), 328-343.
- Grigoriu, M., Ruiz, S. E., and Rosenblueth, E. (1988). "Nonstationary models of seismic ground acceleration." *Tech. Rep. NCEER-88-0043*, Nat. Ctr. for Earthquake Engrg. Res.
- Gurley, K., and Kareem, A. (1994). "On the analysis and simulation of random processes utilizing higher order spectra and wavelet transforms." *Proc., Second Int. Conf. on Computational Stochastic Mech.*, A. A. Balkema Publishers, Amsterdam, Netherlands.
- Harichandran, R. S., and Vanmarcke, E. (1986). "Stochastic variation of earthquake ground motion in space & time." *J. Engrg. Mech.*, ASCE, 112(2).
- Kameda, H. (1975). "Evolutionary spectra of seismogram by multifilter." *J. Engrg. Mech. Div.*, ASCE, 101(6), 787-801.
- Kareem, A., Gurley, K., and Kantor, J. C. (1993). "Time-scale analysis of nonstationary processes utilizing wavelet transforms." *Proc., 6th Int. Conf. on Struct. Safety and Reliability*, Innsbruck, Austria.
- Kozin, F. (1988). "Autoregressive moving average models of earthquake records." *Probabilistic Engrg. Mech.*, 3(2), 58-63.



- Li, Y., and Kareem, A. (1988). "Recursive modelling of dynamic systems." *J. Engrg. Mech.*, ASCE, 114, 1526-1541.
- Li, Y., and Kareem, A. (1991). "Simulation of multivariate nonstationary random processes by FFT." *J. Engrg. Mech.*, ASCE, 117(9), 1037-1058.
- Li, Y., and Kareem, A. (1993). "Simulation of multivariate random processes: Hybrid DFT and digital filtering approach." *J. Engrg. Mech.*, ASCE, 119(5), 1078-1098.
- Li, Y., and Kareem, A. (1995). "Stochastic decomposition and its applications to probabilistic dynamics." *J. Engrg. Mech.*, ASCE, 10(4), 225-240.
- Lin, Y. K. (1967). *Probabilistic theory of structural dynamics*. McGraw-Hill, New York, N.Y.
- Lin, Y. K., and Yong, Y. (1987). "Evolutionary Kanai-Tajimi earthquake models." *J. Engrg. Mech.*, ASCE, 113(8), 1119-1137.
- Liu, S. C. (1970). "Evolutionary power spectral density of strong motion earthquake." *Bull. Seismological Soc. Am.*, 60(3), 891-900.
- Madsen, P., and Krenk, S. (1982). "Stationary and transient response statistics." *J. Engrg. Mech. Div.*, ASCE, 108(8), 622-635.
- Mark, W. D. (1986). "Power spectrum representation for nonstationary random vibration." *Random vibration—Status and recent developments*, Elsevier, New York, N.Y., 211-240.
- Polhemus, N. W., and Cakmak, A. S. (1981). "Simulation of earthquake ground motion using autoregressive moving average (ARMA models)." *Earthquake Engrg. Struct. Dynamics*, 9(4), 343-354.
- Preumont, A. (1985). "The generation of non-separable artificial earthquake accelerograms for design of nuclear power plants." *Nuclear Engrg. Des.*, 88, 59-67.
- Priestley, M. B. (1967). "Power spectral analysis of non-stationary random processes." *J. Sound and Vibration*, 6, 86-97.
- Roberts, J. B., and Spanos, P. D. (1990). *Random vibration and statistical linearization*. John Wiley & Sons, New York, N.Y.
- Safak, E., and Borre, D. M. (1986). "On nonstationary stochastic models for earthquakes." *Proc., Third U.S. Nat. Conf. on Earthquake Engrg.*, Charleston, S.C.
- Saragoni, G. R., and Hart, G. C. (1974). "Simulation of artificial earthquakes." *Earthquake Engrg. Struct. Dynamics*, 2.
- Scherer, R. J. (1994). "On the frequency dependence of the strong ground motion duration." *Structural safety and reliability*, Schueller, Shinozuka, and Yao, eds., Balkema Publishers, Rotterdam, The Netherlands, 2201-2223.
- Scherer, R. J., Riera, J. D., and Schueller, G. I. (1982). "Estimation of the time-dependent frequency content of earthquake accelerations." *Nuclear Engrg. & Des.*, 71, 301-310.
- Shinozuka, M. (1963). "Simulation of multivariable and multidimensional random processes." *J. Acoustic Soc. of Am.*, 49, 376-411.
- Shinozuka, M., and Deodatis, G. (1988). "Stochastic process models for earthquake ground motion." *Probabilistic Engrg. Mech.*, 3, 114-123.
- Soong, T. T., and Grigoriu, M. (1993). *Random vibration of mechanical structural systems*. Prentice-Hall, Inc., Englewood Cliffs, N.J.
- Spanos, P-T. D. (1983). "Digital synthesis of response-design spectrum compatible earthquake records for dynamic analysis." *Shock & Vibration Digest*, 15(3), 21-30.
- Sun, W.-J., and Kareem, A. (1989). "Response of MDOF systems to nonstationary random excitation." *Engrg. Struct.*, 11, 83-91.
- Yeh, C.-H., and Wen, Y. K. (1990). "Modelling of nonstationary ground motion and analysis of inelastic structural response." *Struct. Safety*, 8, 281-298.