Efficacy of Time-Frequency Domain System Identification Scheme Using Transformed Singular Value Decomposition

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ABSTRACT: Structures in real wind environments often experience winds that are characterized by sudden changes in wind direction and wind speed, such as those in thunderstorms that cannot be analyzed by stationary system identification techniques. Accordingly, techniques that can identify properties in the time-frequency domain are being sought. For the time-frequency domain, Wavelet Transform based techniques for system identification have been developed; however, these are limited to stationary processes. In this paper, an extension of this approach that utilizes the Wavelet Transform and Singular Value Decomposition in tandem (WT-TSVD) for applications to non-stationary processes is presented. WT-TSVD permits the transformed data to be processed using conventional time and frequency domain approaches. This paper examines the efficacy of this approach and investigates the effectiveness of these techniques for non-stationary data.

KEYWORDS: Damping, Non-stationary SI, Wavelet Transform, Singular Value Decomposition

1 INTRODUCTION

Full-scale building response data routinely exhibits bursts of high amplitude response which often correlates with thunderstorm winds [1]. These rapid amplitude modulations of response associated with the rise and fall of wind speed and variations in direction result in acceleration levels that escalate over short durations causing discomfort to building occupants. The significance of these transient wind events and their load effects is evident from the analysis of thunderstorm databases in the U.S. and around the world, which suggests that these winds actually represent the design wind speed for many locations outside of the hurricane belt.

To confirm that these burst are indeed non-stationary and require non-traditional analysis techniques, the time-varying mean and standard deviation were extracted from the example burst and plotted in Figure 1. From the figure, it is clear that this signal is indeed non-stationary. As these response bursts are non-stationary in nature, using a stationary technique to extract the dynamic properties may not be appropriate and may lead to incorrect values, especially if the signal is of short duration. Therefore, the dynamic characteristics of the response bursts in this paper are extracted using a modification to the Wavelet Transform, which is often used to examine non-stationary signals. This modification, Transformed Singular Value Decomposition (TSVD), decomposes the wavelet scalogram matrix or the wavelet coefficient matrix to highlight the important dynamic features and reduces the effects of noise.

The aim of this paper is to examine the efficacy of techniques to identify natural frequency and damping from the transformed signals and seek an effective method for system identification of structures subjected to non-stationary wind loading. Using this optimal method, the nature of tall building response to transient loads is examined using the full-scale data. A detailed time-frequency based analysis of these measured transient responses, in conjunction with similar

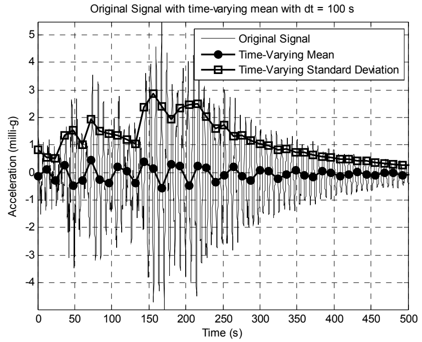

Figure 1. Example data from Chicago Full-Scale Monitoring Project with time-varying mean and standard deviation removed.

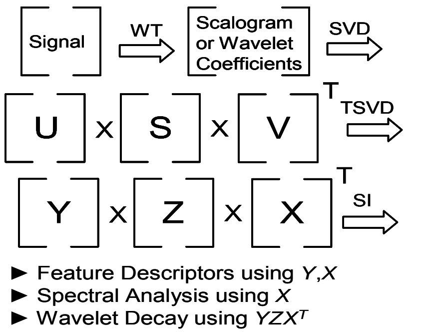

Figure 2. Illustration of system identification of signal transformed with Wavelet Transform, SVD, and TSVD.

analysis of the measured wind conditions, will help in improved understanding of the building response due to transient winds.

2 TRANSFORMED SINGULAR VALUE DECOMPOSITION

2.1 *Singular Value Decomposition*

The TSVD technique utilizes singular value decomposition (SVD) to decompose the wavelet matrix. SVD is defined as:

$$A = USV^T \tag{1}$$

where A is the data matrix to be decomposed, U,V are the left and right singular vectors, respectively, and S is the diagonal matrix of singular values. Table 1 describes these matrices. Due to the orthonormality of the singular vectors, their squared elements can be treated as a distribution function, such as a probability density function (pdf) [e.g., 2]. Thus, by squaring the elements of U and V, the temporal and spectral moments of the process can be obtained and are defined as:

$$\langle t^m \rangle = \int_{-\infty}^{\infty} t^m |s(t)|^2 dt \quad \text{and} \quad \langle f^n \rangle = \int_{-\infty}^{\infty} f^n |S(f)|^2 df \tag{2}$$

where t and f are time and frequency vectors, m and n represent the moment order (i.e. $m, n = 1$ is the first moment), $s(t)$ is the signal and its Fourier Transform is represented by $S(f)$. These moments can then describe the time location, time duration, frequency location, and frequency bandwidth of each identified component.

Table 1 Description of Singular Value Decomposition Matrices

U [r x r]	V [c x c]	S [r x c]
Eigenvectors of AA^T	Eigenvectors of A^TA	Singular values: $\sigma_i = \sqrt{\lambda_i}$
Orthogonal matrices Columns called left singular vectors of A	Orthogonal matrices Columns called right singular vectors of A	Diagonal matrix Only non-zero entries are $S_{ii} = \sigma_i$ ($i = 1 \ldots k$)

2.2 *Transformed Singular Value Decomposition*

Although the SVD filtering provides a better portrait of the process, SVD does introduce additional components to the analysis that are mathematical artifacts and are not necessarily physically meaningful. To correct this, the TSVD method is employed in which the singular vectors and values are further transformed into new bases in an attempt to concentrate the densities into the smallest regions. Thus, the singular vectors are rotated into a set of principal axes. The transformation matrix is determined by maximizing the means in order to reduce the spread of the frequencies and to concentrate information. The TSVD is defined as:

$$A = YZX^T \tag{3}$$

where *Y*, *Z*, and *X* are the transformed singular vector and singular value matrices. These matrices are transformed with specific transformation matrices defined as:

$$U = YC^T \quad \text{and} \quad Z = C^T SD \quad \text{and} \quad V = XD^T \tag{4}$$

where *C,D* are transformation matrices. The transformation matrices are found by:

$$E[Y] = C^T MC \tag{5}$$

where *C* are the eigenvectors of *M* that maximize the mean and *M* is a matrix of the singular vectors. Figure 2 illustrates how a signal is transformed using the Wavelet Transform, SVD, and TSVD techniques.

3 ESTIMATING NATURAL FREQUENCY AND DAMPING

A range of existing time and frequency domain based techniques could be utilized to estimate frequency and damping after using WT-TSVD due to the independence of the time and frequency components. In addition to feature descriptors, the wavelet modulus decay envelope and maximum likelihood estimators may be employed to extract dynamic properties [3, 4]. Other possibilities would be to use the right singular vector and apply spectral methods such as Half-Power Bandwidth (HPBW) or Spectral Moments Method (SMM) to the instantaneous spectra [5, 6]. As part of this paper, appropriate methods to estimate dynamic properties based on WT-TSVD method are assessed. Each method is discussed and used to estimate the dynamic properties of several simulated signals and selected full-scale data.

One application of using TSVD is to reduce the effects of noise in a signal. In has been shown that SVD and, more so, TSVD is capable of removing noise in a signal [2]. As the aim of this paper is to determine an adequate technique for determining the dynamic properties of a non-stationary signal, the effects of noise in different techniques are investigated. Several analyzed signals contain various amounts of added white noise and the resulting dynamic estimates are compared to the actual values.

For this analysis, several example signals are investigated. First, an impulse response function of a linear one degree of freedom (1DOF) system is analyzed. Second, signals with closely spaced modes are investigated. These include responses of a two degree of freedom system (2DOF) subjected to a sine wave, white noise and initial conditions. Finally, an example non-stationary full-scale record shown in Figure 1 is analyzed. For the full-scale record (Signal #11 in Table 3), the finite element analysis of the building predicted frequency is 0.14 Hz and the assumed damping level for design is 1%. Selected signals are reported and discussed here. The signals are summarized in Table 2, while the frequency and damping estimates from each technique are shown in Table 3.

Table 2. Description of signals analyzed

Signal Number	Signal Type	Noise Level	Actual Parameters			
			Mode 1		Mode 2	
			f_n (Hz)	ζ (%)	f_n (Hz)	ζ (%)
1	1DOF Impulse	0	0.2	1.0	NA	NA
2		0.1				
3		0.25				
4	2DOF Impulse	0	0.2	5.0	0.205	5.0
5		0	0.2	1.0	0.205	1.5
6		0.1				
7	2DOF Impulses at well separated time locations	0				
8	2DOF Impulse at overlapping time locations	0				
9	2DOF Sine wave input	0	0.2	5.0	0.205	5.0
10	2DOF White noise input	0				

3.1 *Time and Frequency Domain Analysis: Feature Descriptors*

Defining the moments of time and frequency, as in Equation 2, allows the time and frequency information to be described in terms of a feature descriptor, which is defined as:

$$F_i = \left(\frac{\sigma_i}{\sigma_1}, \langle t \rangle_i, \langle f \rangle_i, \sqrt{\langle t^2 \rangle_i - \langle t \rangle_i^2}, \sqrt{\langle f^2 \rangle_i - \langle f \rangle_i^2} \right) \tag{6}$$

where σ_i, σ_1 are singular values of the i^{th} and features $\langle t \rangle_i$,$\langle f \rangle_i$ are the first moments of time and frequency associated with the i^{th} feature and $\langle t^2 \rangle_i$,$\langle f^2 \rangle_i$ are the second moments [2]. For the example signal of a 2DOF impulse response and the full-scale signals, Figure 3 plots the singular vectors and singular values from the WT-TSVD analysis along with the feature descriptors defined above. TSVD allows for a decomposition of the signal in the time and frequency domain, separation of modes and subsequent feature extraction. In order to extract system information, additional establish techniques, like half-power bandwidth, spectral moment or maximum likelihood can be introduced. These are described in the following.

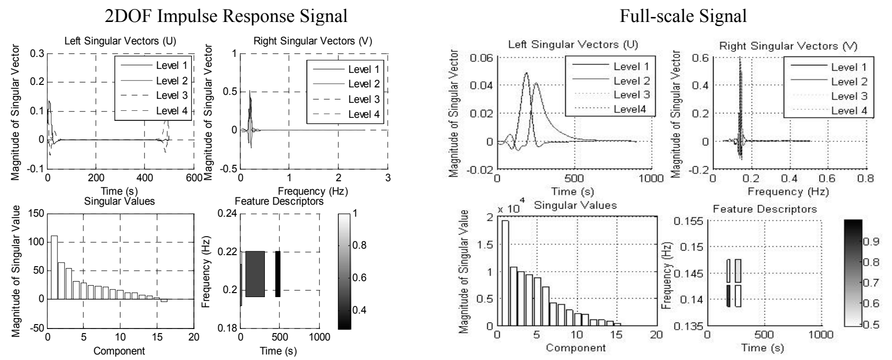

Figure 3. Results from WT-TSVD for first four feature descriptors of 2DOF impulse response and full-scale signal: Left & right singular vectors (top), singular values (bottom left), and feature descriptors (bottom right)

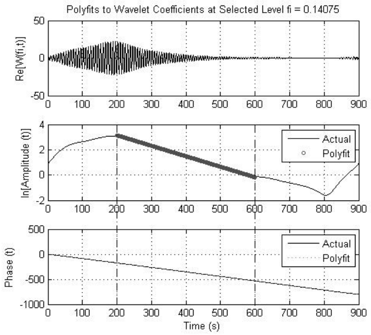

Figure 4. Wavelet modulus decay of example signal from full-scale data: Wavelet coefficients at level i (top); fitted amplitude & phase of wavelet coefficient (middle & bottom)

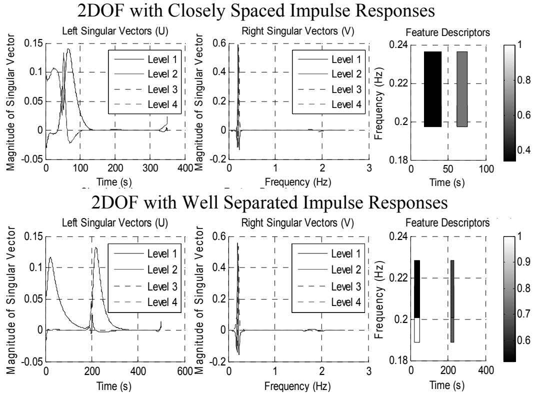

Figure 5. Feature Descriptor plots for a 2DOF system subjected to impulses at differing times: close together (top) and far apart (bottom).

3.2 *Time Domain Analysis: Wavelet Modulus Decay*

In this time domain technique, the frequency and damping are estimated at a particular frequency value of the wavelet coefficients [4]. For an impulse response function, the time-varying envelope and phase of the signal can be determined from the modulus and phase of the wavelet coefficients at each wavelet frequency. Figure 4 plots the wavelet coefficient at the frequency line corresponding to the largest singular value, which should be the natural frequency of the system. By utilizing the slopes of the phase and natural log of the amplitude of the wavelet coefficients, the frequency and damping of the system are estimated (See Table 3). From Figure 4, it is apparent that the modulus decay does suffer from end effects [4]. For these examples, there is sufficient length of unaffected data by the end effects which precludes the need for any corrective measures for this example.

However, a major drawback with this technique is that it does not extract system information for any signal other than an impulse response or free decay. For application to full-scale analysis, wavelet modulus decay is not generally applicable as wind-excitation is not an impulse. Despite the fact that wavelet modulus decay requires an impulse response, previous work with TSVD has used wavelet modulus on non-impulse response signals [2]. Although the author in [2] achieved reasonable estimates of natural frequency, damping values were not reported. Therefore, the accuracy of damping estimates cannot be ascertained.

3.3 *Spectral Approaches: Half-Power Bandwidth Method, Spectral Moments Method, and Maximum Likelihood Estimation*

While HPBW can be used effectively for stationary signals, this technique breaks down for non-stationary signals, especially short signals and those exhibiting slow variations in frequency as this feature results in enhancing the bandwidth [7]. HPBW and SMM are based on a power spectral density function, which requires a certain number of data points for its reliable estimate [6]. Thus, using HPBW and SMM techniques for a short signal is difficult as there are not enough data points for reliable spectral estimates. However, it has been shown that there is an analogy between a normalized spectral density function and a pdf [5], which implies that the right singular vector can be treated as a density function. Accordingly, this allows spectral analysis techniques to be used for system identification. This analysis is also complemented by the maximum likelihood estimates (MLE) derived on the basis of fitting data to a prescribed linear system transfer function.

Table 3 Natural frequency estimates after using WT-TSVD

Wavelet Transform-Transformed Singular Value Decomposition

	Signal Number	FD		WM		HPBW		SMM		MLE	
		f_n (Hz)	Z (%)	f_n (Hz)	ζ (%)	f_n (Hz)	ζ (%)	f_n (Hz)	ζ (%)	f_n (Hz)	ζ (%)
Mode 1	1	0.201	0.89	0.200	1.00	0.201	1.46	0.201	0.02	0.201	0.78
	2	0.201	1.08	0.199	1.03	0.201	1.68	0.201	0.02	0.201	0.86
	3	0.202	4.51	0.199	0.65	0.202	1.75	0.202	0.12	0.202	1.06
	4	0.203	1.65	0.200	5.04	0.203	3.26	0.203	0.08	0.203	1.53
	5	0.199	1.48	0.200	1.00	0.198	1.60	0.199	0.01	0.198	0.84
	6	0.201	2.57	0.202	1.11	0.200	1.67	0.202	0.23	0.200	1.01
	7	0.202	0.88	0.200	1.00	0.202	1.54	0.202	0.02	0.202	0.79
	8	0.203	1.03	0.201	3.07	0.203	1.74	0.204	0.03	0.203	0.87
	9	0.213	1.76	NA	NA	0.213	2.90	0.210	0.06	0.213	1.49
	10	0.203	2.25	NA	NA	0.203	1.94	0.204	0.14	0.203	1.04
	11	0.141	1.42	0.143	0.88	0.141	1.47	0.141	0.04	0.141	0.77
Mode 2	4	0.173	6.31	0.201	6.12	0.220	4.15	0.206	0.71	0.219	3.01
	5	0.208	2.18	0.199	1.00	0.210	1.93	0.209	0.06	0.200	1.38
	6	0.217	3.62	0.219	1.65	0.217	4.22	0.215	0.24	0.217	2.31
	7	0.209	1.02	0.200	1.00	0.209	1.57	0.209	0.03	0.202	1.75
	8	0.211	1.24	0.203	2.31	0.211	2.84	0.211	0.04	0.203	1.47
	9	0.189	5.31	NA	NA	0.194	5.31	0.190	0.53	0.232	2.38
	10	0.194	3.98	NA	NA	0.196	1.61	0.196	0.41	0.214	1.56

4 DISCUSSION OF RESULTS

4.1 *Analysis of Signals with a Single Mode*

For all estimation techniques, the natural frequency is estimated quite well, even in cases with noise. Although, as the noise level increases, there is some deviation in the natural frequency estimates from the actual value, but these deviations are minimal. Damping, however, is not as accurate. Damping is estimated accurately by WM and reasonably well by the MLE and the feature descriptors; however, the spectral methods do not provide good estimates of damping. The ML estimates are accurate in estimating damping even in cases with noise. For the signals with little or no noise, WM estimates damping accurately. However, for the signal with increased noise, MLE provides the best damping estimates.

For the example signals, the spectral analysis techniques have been applied to the WT-TSVD transformed signal. By using the singular vectors as spectral density function, conventional analysis procedures used for stationary data can now be employed to analyze non-stationary signals that have been transformed. Of the two spectral analysis techniques used in this paper (HPBW and SMM), HPBW obtains more reasonable damping estimates, although the damping is over-estimated; SMM, on the other hand, overly under-estimates damping. From this initial analysis, it is evident that SMM is of little value when estimating damping for this application.

As such, the resulting damping estimates from SMM are not discussed in the following, but are presented for completeness. Although over-estimated, HPBW performs moderately well for this simple signal. It should be noted that without the use of TSVD, it is not possible to use HPBW for very short signals.

4.2 *Analysis of Signals with Two Modes*

For the 2DOF impulse response with small damping, all methods determine the natural frequency reasonably well for the first mode, except in the case of Signal #9 (sine wave input). WM accurately estimates the damping in nearly all cases and MLE estimates the damping reasonably well for signals with low damping. On the other hand, feature descriptors and HPBW slightly over-estimate the damping in the first mode and MLE underestimates the damping for signals with higher damping. For the second mode, FD, MLE, and SMM estimate the natural frequency reasonable well, while HPBW slightly overestimates the natural frequency in most cases. WM, however, cannot distinguish between the two closely-spaced modes. In most cases, MLE did a better job than FD. For this example signal, noise does affect all the estimates.

To examine the effects of time-varying signals, a signal was generated that included two impulse responses with different natural frequencies and damping values occurring at different times (See Figure 5). The both the FD and MLE techniques were able to estimate the natural frequency in both modes reasonable well and clearly indicated that two frequencies were present. Damping was not as accurate, but near the true value. In addition, FD and MLE both showed the trend in the damping between the two modes, i.e. mode 2 had higher damping than mode 1. For the well separated impulses, wavelet modulus was able to accurately estimate the natural frequency and damping in mode 1. However, for the overlapping impulse responses, damping is significantly over-estimated, indicating that the WD cannot separate the two modes. This example shows that wavelet modulus is not a good technique for estimating dynamic properties in signals that have changing properties with time. While not very accurate, FD and MLE are able to provide a clearer picture as to the true characteristics of the signal. In addition, FD is able to determine the location of both impulse responses in the time vector.

Finally, the 2DOF system with higher damping was subjected to a sine wave and white noise. For these output signals, the input conditions violate the conditions of wavelet modulus and, as such, WD cannot be used to estimate the dynamic properties. Again, these examples indicate the utility of FD and MLE over wavelet modulus. Although, FD and MLE do not accurately estimate the dynamic properties in all cases, they do provide a clear indication of the trends in respective modes. The estimates for the white noise input case are closer than those for the sine wave input case.

4.3 *Analysis of Full-Scale Data*

Previous analysis of full scale data from the building involving very long records during extra-tropical wind events have revealed the building frequency to be 0.14 Hz and the average damping level around 0.9% [1]. A comparison of the WT-TSVD based estimation techniques with the stationary records analysis results shows that the estimated natural frequency is accurate for all five techniques; however, there is variation in the damping estimates. The estimate from SMM is very low and the estimates from both FD and HPBW are above the average damping value of 0.9%, which agree with the trends noted in the previous examples. Finally, both MLE and WM estimated the damping slightly below the average value of 0.9%. Although WM was shown to be very accurate with data from impulse responses, it is not valid with other types of signals. However, a re-examination of Figure 1 and the accuracy of the WM estimates of dynamic properties, suggests that signal may indeed be close to a free-decay response as building oscillates freely after experiencing a large gust. Thus, for this particular full-scale transient response the dynamic features are close to those during a long duration stationary wind event. In general, WM and MLE provided the most consistent and reliable results from the transformed non-stationary data for the examples presented in this paper.

5 CONCLUDING REMARKS

This paper discusses several approaches to extract frequency and damping from a signal that has been decomposed and transformed using Wavelet Transform-Transformed Singular Value Decomposition. This technique is particularly useful for non-stationary signals and for signals that are embedded in noise. The system identification techniques that are utilized include extracting the structural properties from feature descriptors based on the moments of the singular vectors, utilizing the right singular vectors in conjunction with maximum likelihood estimates, determining the properties from a decay envelope of the wavelet coefficients, Half-Power Bandwidth, and Spectral Moments Method. A suite of example signals were analyzed to determine the effectiveness of each technique. Spectral Moments Method was found to be inadequate as all damping values were extremely low. In addition, wavelet modulus decay was only valid under free vibration decay which is not generally the case in full-scale data and, therefore this may not hold much promise for full-scale analysis. Finally, feature descriptors and maximum likelihood estimates proved to be the most promising techniques with a few exceptions.

6 ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of the National Science Foundation through grant CMS 00-85109 and CMMI 06-01143.

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