# Understanding the Underlying Physics of Multimode Coupled Bridge Flutter Based on Closed-Form Solutions

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ABSTRACT: This study presents a framework with closed-form solutions for the bimodal coupled bridge aerolastic system, and points out the control parameters which critically influence the generation of inter-modal coupling and aerodynamic damping. Accordingly, this information helps in developing a guideline for the selection of most important structural modes in a coupled flutter analysis. The role of each flutter derivative and the potential influence of the self-excited drag force on bridge flutter are also discussed. Finally, a closed-form formula for quantifying the critical flutter speed is introduced which can be regarded as an extension of the empirical Selberg's formula for generic bridges.

KEYWORDS: Flutter, Wind loads, Aerodynamics, Aeroelasticity, Dynamics, Bridges.

## 1 INTRODUCTION

Many analysis examples have shown that coupled bridge flutter is dominated by the aerodynamic coupling of a few important structural modes (e.g., Jones et al. 1998; Chen et al. 2000). From the point of view of flutter prediction, the clarification of the most important modes may not be an important issue, because the analysis even involving a larger number of modes, e.g., 50 modes, is actually not computational expensive. However, improved understanding of the most important modes helps in better capturing the underlying physics of coupled flutter. It also offers equally valuable guidance for design and interpretation of wind tunnel studies using full aeroelastic bridge models. While information on the modal participation in flutter has been obtained from a variety of bridge examples, a simple and physically insightful guidance on the selection of bridge modes has not been reported in the literature.

Another important issue in flutter analysis is concerning the modeling of self-excited forces. It has generally been understood that the self-excited lift and pitching moment on a bridge deck caused by vertical and torsional motions are most important in the generation of coupled flutter. However, since the interesting finding concerning the noticeable contribution of the self-excited drag force on the coupled flutter of Akashi Kaikyo Bridge (Miyata et al. 1994), the force modeling with total eighteen flutter derivatives instead of the traditional eight flutter derivatives is becoming increasingly prevalent (Sarkar et al. 1994; Chen and Kareem 2002; Chen et al. 2002). While this modeling increases the accuracy of flutter prediction, it considerably complicates and undermines the efforts of better capturing the fundamental flutter characteristics of bridge deck sections with a minimal number of flutter derivatives (Matsumoto 1999; Chen and Kareem 2006). It becomes an important issue to clarify why and under what conditions these additional flutter derivatives can be excluded in a flutter analysis.

In this paper, the aerodynamic coupling of two modes are discussed using closed-form formulations, which reveals the control parameters most influencing the generation of inter-modal coupling and aerodynamic damping. This discussion leads to improved understanding of the most important modes in flutter. Based on the closed-form solutions of bimodal coupled flutter, the role of each flutter derivative on coupled flutter is identified, which clarifies why and under what conditions the modeling of self-excited forces can be simplified with a small number of most important flutter derivatives. Finally, a closed-form formula for estimating the critical flutter speed is introduced which serves an extension of Selberg's formula (Selberg 1961) for generic bridges with bluff deck sections.

## 2 IMPORTANT MODES IN FLUTTER

From the closed-form formulations for estimating the modal frequencies, damping ratios and inter-modal coupling of a bimodal coupled bridge aeroelastic system developed in this study, it is clear that the control parameters influencing the inter-modal coupling, especially, the modal damping are the frequency ratio, damping ratios, mass parameters, coupled aerodynamic stiffness and damping terms between these two modes. Therefore, a mode comprising of large values of coupled aerodynamic stiffness and damping terms with the fundamental torsional mode is more likely to be important for coupled flutter. Furthermore, this coupling will be enhanced when its damping is low and its frequency is close to the torsional modal frequency. For example, the fundamental vertical bending mode often has higher similarity in shape with the fundamental torsional mode, which leads to larger coupled aerodynamic terms between these two modes. Therefore, the fundamental vertical bending mode is more likely to be coupled with the torsional mode than other higher vertical bending modes whose mode shapes have less similarity with the fundamental torsional mode, although their frequencies may be closer to the torsional modal frequency. In this context where the fundamental torsional mode is anti-symmetric, the corresponding fundamental bending mode is referred to as the fundamental anti-symmetric mode. Otherwise, both are referred to as fundamental symmetric modes. Some modes may become locally important at a certain wind speed region where their frequencies are close to the torsional modal frequency.

## 3 IMPORTANT FLUTTER DERIVATIVES

For a bimodal coupled bridge system involving only the fundamental vertical and torsional modes, when only the self-excited lift and pitching moment are considered, the closed-form formulations explicitly unveil the role of different force components on modal damping (Chen and Kareem 2006). The uncoupled aerodynamic forces, i.e., the lift caused by vertical motion and the pitching moment caused by torsion in terms of the flutter derivatives $H_1^*$, $H_4^*$, $A_2^*$ and $A_3^*$, result in positive damping to the vertical and torsional modal branches, respectively. The effects of the coupled forces, i.e., the lift caused by torsion and the pitching moment caused by vertical motion in terms of the flutter derivatives $H_2^*$, $H_3^*$, $A_1^*$ and $A_4^*$, however, produce negative damping to the torsional modal branch. The flutter derivatives $H_3^*$, $A_1^*$, $A_2^*$ and $A_3^*$ are most influential to coupled flutter.

Similar discussion on the role played by each flutter derivative can also be made when the self-excited forces are modeled in terms of eighteen flutter derivatives. For the aforementioned bimodal coupled system, for example, the aerodynamic stiffness and damping terms relevant to the torsional mode can be expressed as

$$A_{s22} = (2k^2)(P_4^* G_{p_2p_2} + bP_3^* G_{p_2\alpha_2} + bA_6^* G_{p_2\alpha_2} + b^2 A_3^* G_{\alpha_2\alpha_2}) \tag{1}$$

$$A_{d22} = (2k)(P_1^* G_{p_2p_2} + bP_2^* G_{p_2\alpha_2} + bA_5^* G_{p_2\alpha_2} + b^2 A_2^* G_{\alpha_2\alpha_2}) \tag{2}$$

where $P_i^*$ and $A_i^*$ are the flutter derivatives; $G_{r_is_j} = \int_{span} r_i(x)s_j(x)dx$ (where $r, s = h, p, \alpha$) are the modal integrals; $B = 2b$ is the bridge deck width; $k = \omega b/U$ is the reduced frequency; $U$ is the mean wind speed; $\omega$ is the frequency of motion.

Obviously, the importance of additional flutter derivatives on bridge flutter depends on their values and associated mode shape integrals. For instance, the contribution of $P_1^*$, $P_2^*$ and $A_5^*$ in $A_{d22}$ is equivalent to a change in $A_2^*$ given by $P_1^* G_{p_2p_2}/(b^2 G_{\alpha_2\alpha_2}) + P_2^* G_{p_2\alpha_2}/(bG_{\alpha_2\alpha_2}) + A_5^* G_{p_2\alpha_2}/(bG_{\alpha_2\alpha_2})$. While this change is generally small in magnitude, it may have a markable effect on the critical flutter speed when the value of $A_2^*$ itself is low. Similar statements apply to other flutter derivatives.

To demonstrate the potential influence of drag force component on a coupled flutter, the following parametric study on a suspension bridge with a center span of about 2000 m is carried out. Three cases with different values of $H_1^* \sim H_4^*$ and $A_1^* \sim A_4^*$ are considered. Case A: $H_1^* \sim H_4^*$ and $A_1^* \sim A_4^*$ are determined through Theoderson function; Case B: $H_1^* \sim H_4^*$ are the values of Case A divided by 5, and $A_2^*$ and $A_3^*$ are the values of Case A divided by 10; and Case C: $H_1^* \sim H_4^*$, $A_2^*$ and $A_3^*$ take the values of

Case A divided by 10. The self-excited drag force is the same and modeled by invoking the quasi-steady theory in all cases.

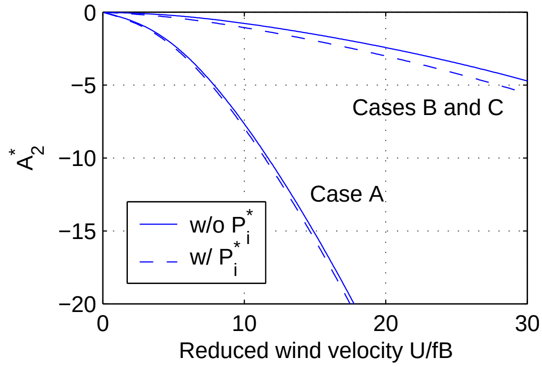


Fig. 1 Equivalent changes in $A_2^*$ due to $P_i^*$

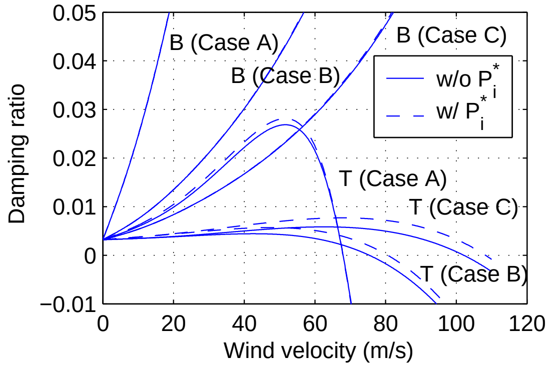


Fig. 2 Effect of $P_i^*$ on the modal damping

Fig. 1 shows the equivalent change in $A_2^*$ as the result of $P_i^*$. Fig. 2 portrays the effect of $P_i^*$ on the modal damping ratios at varying wind velocities. The potential influence of the self-excited drag force on a bridge flutter can be discussed in light of the flutter characteristics featured by the rate of change in modal damping with increasing wind speed. In the case of a soft-type coupled flutter where the level of damping is low and it changes slowly with increasing wind speed around the critical flutter speed, even a little influence of additional self-excited drag force on the modal damping may result in a markable change in the predicted critical flutter speed as indicated by Cases B and C. Consequently, careful modeling of the self-excited forces with the consideration of drag force may become critical for an accurate flutter prediction. The structural damping also becomes considerably beneficial to this type of flutter. However, majority of bridges are considered to exhibit a hard-type flutter characterized by modal damping that changes rapidly with increasing wind speed around the flutter onset as indicated by Case A. This type of flutter typically associated with large values of self-excited lift and pitching moment caused by the vertical and torsional motion of bridge deck. For this type of flutter, the structural damping and the additional damping caused by the self-excited drag force or auxiliary damping through a tuned mass damper generally have little effects on the critical flutter speed (e.g., Chen and Kareem 2003).

## 4 CLOSED-FORM FORMULA FOR CRITICAL FLUTTER SPEED

In the case of well separated modal frequencies and a low level of damping, the critical flutter speed can be estimated by

$$U_{cr} = \gamma \omega_{s2} b \sqrt{(1-\eta^2)\left(\frac{mr}{\rho b^3}\right)} \tag{3}$$

$$\gamma = \{[(-kA_2^*) + 2k\xi_{s2}(1+\upsilon A_3^*)^{1/2}/\upsilon]/[(-k^2H_3^*)(kA_1^*)(k^2A_3^*)]\}^{1/4}/\sqrt{2D} \tag{4}$$

where $\rho$ is the air density; $\eta = \omega_{s1}/\omega_{s2}$ = frequency ratio; $\omega_{s1}$ and $\omega_{s2}$ are the circular frequencies of the vertical and torsional modes, respectively; $\xi_{s2}$ is the damping ratio of the torsional mode; $\upsilon = \rho b^4/I$; $m = m_1/G_{h_1h_1}$ and $I = m_2/G_{\alpha_2\alpha_2} = mr^2$ are the effective mass and polar moment of inertia per unit span, respectively; $r$ is the radius of gyration of the cross-section; $D = G_{h_1\alpha_2}/(G_{h_1h_1}G_{\alpha_2\alpha_2})^{1/2}$ is the similarity factor between the vertical and torsional mode shapes. Eq. (3) with $k_0^2 = (\omega_{s2}b/U_{cr})^2 = k^2(1+\upsilon A_3^*)$ provides an alternative format for estimating the critical flutter speed and flutter frequency.

The parameter $\gamma$ is insensitive to the value of the reduced wind speed over the reduced wind speed range of interest, i.e., $U/fB$=10 to 20. This feature makes Eq. (3) very attractive for an expeditious assessment of flutter performance of a given deck section, based on their flutter derivatives but without implementing flutter analysis. This formula provides not only an analytical basis for the well known Selberg's formula, but also is regarded as its extension to generic bridges with bluff deck sections. It clearly points at the significance of structural and aerodynamic characteristics on bridge flutter performance, which helps to better understand how and where the structure features may be tailored for better flutter performance.

## 5 CONCLUDING REMARKS

The fundamental vertical bending and torsional modes are two dominant modes in coupled flutter. The participation of other modes depends on the similarity in mode shapes and their modal frequencies as compared to the fundamental torsional mode.

The flutter derivatives $H_3^*$, $A_1^*$, $A_2^*$ and $A_3^*$ are the most important for characterizing the flutter performance of a section. For a soft-type flutter, additional aerodynamic damping of the drag force may have a markable influence on the critical flutter speed. However, a majority of bridges are characterized by a hard-type flutter, for which the effect of drag force is negligibility small. Therefore, inclusion of eighteen flutter derivatives may not be necessarily important in these cases in light of minimal improvement in flutter prediction.

The closed-form formula for estimating critical flutter speed provided an analytical basis for the well known Selberg's formula and can be regarded as its extension to generic bridges with bluff deck sections. It clearly points at the significance of structural and aerodynamic characteristics on bridge flutter performance, which helps to better understand how and where the structure may be tailored for better flutter performance.

## 6 REFERENCES


1. Chen, A., He, X. and Xiang, H. (2002). "Identification of 18 flutter derivatives of bridge decks." *J. Wind Eng. Ind. Aerodyn.*, 90, 2007-2022.
2. Chen, X., Matsumoto, M., and Kareem, A. (2000). "Aerodynamic coupling effects on flutter and buffeting of bridges." *J. of Eng. Mech.*, 126(1), 17-26.
3. Chen, X., and Kareem, A. (2002). "Advances in the modeling of aerodynamic forces on bridge decks." *J. Eng. Mech.*, 128(11), 1193-1205.
4. Chen, X., and Kareem, A. (2003). "Efficacy of tuned mass damper for bridge flutter control." *J. of Struct. Eng.*, 129(10), 1291-1300.
5. Chen, X., and Kareem, A. (2006). "Revisiting multimode coupled bridge flutter: some new insights."*J. Eng. Mech.*(in press)
6. Jones, N. P., Scanlan, R. H., Jain A., and Katsuchi, H. (1998). "Advances (and challenges) in the prediction of long-span bridge response to wind." *Bridge Aerodyn.*, A. Larsen & S. Esdahl (eds), Balkema, Rotterdam, The Netherlands, 59-85.
7. Matsumoto, M. (1999). "Recent study on bluff body aerodynamics and its mechanism." *Wind Engineering into the 21 Century*, Larsen, Larose & Livesey (eds), Balkema, Rotterdam, 67-78.
8. Miyata, T., Tada, K., Sato, H., Katsuchi, H., and Hikami, Y. (1994). "New findings of coupled flutter in full model wind tunnel tests on the Akashi Kaikyo Bridge." *Proc. of Symp. on Cable-Stayed and Suspension Bridges*, Deauville, France, October 12-15, 163-170.
9. Selberg, A. (1961). "Oscillation and aerodynamic stability of suspension bridge." *Acta Polytechnica Scandinavica*, Civil Eng. and Construction Series, 13.
10. Sarkar, P. P., Jones, N. P., and Scanlan, R. H. (1994). "Identification of aeroelastic parameters of flexible bridges." *J. Eng. Mech.* 120(8), 1718-1742.