On the Veering of Eigenvalue Loci and the Physics of Coupled Flutter of Bridges

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1. INTRODUCTION

The multimode eigenvalues of a bridge under strong winds with aeroelastic effects can be estimated through a complex eigenvalue analysis (e.g., Chen et al., 2000). These eigenvalues are commonly plotted versus mean wind velocity by a family of frequency and damping loci. Depending on the structural and aerodynamic characteristics of the bridge, two of these closely spaced eigenvalues may approach each other at a certain wind velocity range. It is interesting that when two frequency loci approach, the curves repel each other avoiding an intersection, whereas the eigenvectors associated with these two eigenvalues are interchanged as if the curves intersected (Chen et al., 2000; and Chen and Kareem, 2001). This is called as curve veering phenomenon that has also been observed in other structural dynamic problems (Chen and Ginsberg, 1992; and Morand and Ohayon 1995).

A spring supported bridge section model with heaving and torsional degrees of freedom is commonly used in the wind tunnel investigation of flutter behavior of long span bridges. In most cases, the coupled flutter is initiated from the torsional branch. However, a heaving branch coupled flutter may also be observed in some cases when the torsional aerodynamic damping is relatively large (Matsumoto et al., 1999). The underlying physics and conditions of occurrence of this kind of flutter have not been clearly understood.

This paper addresses the physics of curve veering of eigenvalue loci using spring-supported 2-DOF bridge section models and a long span suspension bridge. A closed form solution of 2-DOF flutter is provided based on a perturbation analysis. A criterion under which the eigenvalue loci veer is presented. The underlying physics of a heaving mode branch coupled flutter and multimode coupled flutter is discussed from the curve veering viewpoint.

2. CLOSED FORM SOLUTION OF 2-DOF COUPLED FLUTTER

The equations of motion of a spring supported bridge section model with heaving and torsional degrees of freedom are given as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \frac{1}{2}\rho U^2 (\mathbf{A}_s \mathbf{q} + \frac{b}{U}\mathbf{A}_d \dot{\mathbf{q}}) \tag{1}$$

The eigenvalues and eigenvectors of 2-DOF system can be calculated through a complex eigenvalue analysis (e.g., Chen et al., 2000). Here, a closed form solution is presented based on a perturbation analysis, which provides more physical insight to the effects of coupled self-excited forces on flutter.

The system without the coupled self-excited forces is chosen as the unperturbed system with the following eigenvalues

$$\lambda_{j0} = -\xi_{j0}\omega_{j0} + i\omega_{j0}\sqrt{1-\xi_{j0}^2} \quad (j0 = h0, \alpha 0) \tag{2}$$

$$\omega_{h0}^2 = \omega_h^2 - \rho\omega_{h0}^2 b^2 H_4^*(k_{h0})/m_h \tag{3}$$

$$\xi_{h0} = \xi_h\omega_h/\omega_{h0} - \rho b^2 H_1^*(k_{h0})/(2m_h) \tag{4}$$

$$\omega_{\alpha 0}^2 = \omega_\alpha^2 - \rho\omega_{\alpha 0}^2 b^4 A_3^*(k_{\alpha 0})/I_\alpha \tag{5}$$

$$\xi_{\alpha 0} = \xi_\alpha\omega_\alpha/\omega_{\alpha 0} - \rho b^4 A_2^*(k_{\alpha 0})/(2I_\alpha) \tag{6}$$

The system with the coupled self-excited forces is chosen as the perturbed system. The frequency dependent aerodynamic matrices $\mathbf{A}_s$ and $\mathbf{A}_d$ are approximated as

$$\mathbf{A}_s = \begin{bmatrix} 2k_{h0}^2 H_4^*(k_{h0}) & 2k_{h\alpha 0}^2 b H_3^*(k_{h\alpha 0}) \\ 2k_{h\alpha 0}^2 b A_4^*(k_{h\alpha 0}) & 2k_{\alpha 0}^2 b^2 A_3^*(k_{\alpha 0}) \end{bmatrix} \tag{7}$$

$$\mathbf{A}_d = \begin{bmatrix} 2k_{h0} H_1^*(k_{h0}) & 2k_{h\alpha 0} b H_2^*(k_{h\alpha 0}) \\ 2k_{h\alpha 0} b A_1^*(k_{h\alpha 0}) & 2k_{\alpha 0} b^2 A_2^*(k_{\alpha 0}) \end{bmatrix} \tag{8}$$

where

$$k_{h0} = \omega_{h0} b/U; \quad k_{\alpha 0} = \omega_{\alpha 0} b/U \tag{9}$$

$$k_{h\alpha 0} = \omega_{h\alpha 0} b/U; \quad \omega_{h\alpha 0} = \frac{1}{2}(\omega_{h0} + \omega_{\alpha 0}) \tag{10}$$

By expressing the eigenvectors of the perturbed system in terms of the eigenvectors of the unperturbed system as

$$\mathbf{\Phi}'_j = \mathbf{\Gamma D} = [\mathbf{\Phi}_{h0} \;\; \mathbf{\Phi}_{\alpha 0}]\mathbf{D} \tag{11}$$

the following equations for the eigenvalues of the perturbed system are obtained:

$$(\mathbf{\Lambda} + \epsilon \mathbf{H})\mathbf{D} = \lambda'_j \mathbf{D} \tag{12}$$

where

$$\mathbf{\Lambda} = diag[\lambda_{h0}, \lambda_{\alpha 0}]; \qquad \mathbf{H} =$$

$$\rho \omega_{h\alpha 0} b^3 \begin{bmatrix} 0 & \frac{-i(\omega_{h\alpha 0} H_3^* + \lambda_{\alpha 0} H_2^*)}{(2 m_h \omega_{h0}^D)} \\ \frac{-i(\omega_{h\alpha 0} A_4^* + \lambda_{h0} A_1^*)}{(2 I_\alpha \omega_{\alpha 0}^D)} & 0 \end{bmatrix} \tag{13}$$

and ω_{h0}^D=$\omega_{h0}\sqrt{1-\xi_{h0}^2}$ and $\omega_{\alpha 0}^D$=$\omega_{\alpha 0}\sqrt{1-\xi_{\alpha 0}^2}$.

The solution of these equations provides

$$\lambda'_j = \frac{1}{2}(\lambda_{h0} + \lambda_{\alpha 0}) \mp \frac{1}{2}[(\lambda_{h0} - \lambda_{\alpha 0})^2 + 4\epsilon^2 H_{12} H_{21}]^{1/2} \tag{14}$$

$$\lambda'_j = -\xi'_j \omega'_j + i\omega'_j \sqrt{1 - \xi'^2_j} \tag{15}$$

It is clear that the eigenvalues and eigenvectors with coupled self-excited forces can be readily determined in terms of the eigenvalues and eigenvectors without the coupled self-excited forces and the coupled flutter derivatives H_2^*, H_3^*, A_1^* and A_4^* in a closed form.

3. CRITERION OF EIGENVALUE LOCI VEERING

Based on perturbation solution presented in Eq. 14, it is obvious that only when

$$\Delta = [(\lambda_{h0} - \lambda_{\alpha 0})^2 + 4\epsilon^2 H_{12} H_{21}]^{1/2} = 0 \tag{16}$$

the two adjacent eigenvalues of heaving and torsional branches are equal. This means that both the frequency and damping ratio loci of heaving and torsional branches intersect at the same wind velocity. Since in general $|\Delta|$ is not necessarily equal to zero, the frequencies or damping ratios of these two adjacent complex modes will be generally different from each other.

The off-diagonal terms H_{12} and H_{21} result in the interaction of the two adjacent modes. $H_{12} = H_{21} = 0$ means that the two adjacent modes have no interaction and it leads to $\lambda'_j = \lambda_{j0}$ and $\mathbf{\Phi}'_j = \mathbf{\Phi}_{j0}$.

When $|\lambda_{h0} - \lambda_{\alpha 0}| \gg 2\epsilon|(H_{12}H_{21})^{1/2}|$, i.e., the interaction between adjacent modes is weak, we have $\lambda'_j \approx \lambda_{j0}$ and $\mathbf{\Phi}'_j \approx \mathbf{\Phi}_{j0}$.

When $|\lambda_{h0} - \lambda_{\alpha 0}| \approx 2\epsilon|(H_{12}H_{21})^{1/2}|$, i.e., the interaction between adjacent modes is strong, the eigenvalues will be significantly influenced by the mode interaction effect indicated by $2\epsilon(H_{12}H_{21})^{1/2}$, and the eigenvectors $\mathbf{\Phi}'_h$ and $\mathbf{\Phi}'_\alpha$ result from a significant hybridization of $\mathbf{\Phi}_{h0}$ and $\mathbf{\Phi}_{\alpha 0}$. In particular, when $\lambda_{h0} = \lambda_{\alpha 0}$, we have $\Delta = 2\epsilon(H_{12}H_{21})^{1/2}$ and $\omega'_h = |\lambda'_h| < |\lambda^*_{h0}| = |\lambda^*_{\alpha 0}| < |\lambda'_\alpha| = \omega'_\alpha$. It means that the effect of the mode interaction is to separate the eigenvalues.

In summary, at the wind velocity where the two eigenvalues are close to each other, if the frequency loci of $\omega^*_j = |\lambda^*_j|$ intersect, the frequency loci of $\omega'_j = |\lambda'_j|$ of the perturbed system may intersect or veer depending on the level of mode interaction. When the interaction between these two adjacent modes is significant, i.e.,

$$d = |\lambda_{h0} - \lambda_{\alpha 0}|/[2\epsilon|(H_{12}H_{21})^{1/2}|] \leq O(1) \tag{17}$$

the frequency loci veer, otherwise these intersect. Since the effects of the mode interaction is to separate the eigenvalues, the frequency loci of $\omega'_j = |\lambda'_j|$ of the perturbed system will not intersect if the frequency loci of $\omega_j = |\lambda_j|$ do not intersect.

4. HEAVING BRANCH FLUTTER

An example 2-DOF bridge section model is considered. The self-excited forces are given in three different cases for comparison: a) All flutter derivatives are calculated through Theodorsen function; b) All flutter derivatives are same as case a except that the A_2^* and A_3^* are two times the values of case a; c) All flutter derivatives are same as case a except that the A_2^* and A_3^* are three times the values of case a.

The comparison of the eigenvalue loci calculated by the perturbation analysis and by the complex eigenvalue analysis are shown in Fig. 1. Results illustrated the accuracy of the closed form solution. Without the coupling effect, the two frequency loci intersect in these three cases. The coupled self-excited forces result in the separation of two eigenvalue loci. In cases a and b, a coupled flutter is initiated from torsional branch. The cure veering phenomenon exist in the frequency loci in both cases a and b. In case c, the coupled self-excited forces only slightly change the eigenvalues of this system, and the frequency loci intersect. As a result, the coupled flutter is initiated from the heaving branch in case c. Table 1 compared the coupled flutter conditions including the flutter branch, critical flutter velocity, frequency, reduced velocity, amplitude ratio and phase difference between heaving and torsional motions in the coupled flutter mode.

Figure 2 shows the interaction level (Eq. 17) for the three cases indicating the effect of coupled self-excited forces in the velocity region where the two frequency loci are close to each other. A lower value corresponds to a stronger interaction. The coupling effect decreases successively in cases a, b and c. In both cases a and b, strong interaction between heaving and torsional motions results in curve veering. The result of case c indicates that a heaving branch flutter corresponds to the intersection of frequency loci. Comparing these three cases in light of the eigenvalue loci, coupled motions corresponding to the branch where the coupled flutter is initiated, flutter condition at the critical flutter velocity, and the role of the self-excited forces to the system dynamics, it is concluded that the heaving branch coupled flutter is physically consistent with the generally observed torsional branch coupled flutter.

5. CURVE VEERING OF A MULTIMODE COUPLED FLUTTER

A long span suspension bridge with a center span of nearly 2000 m is chosen as another example to illustrate curve veering. Calculations with different mode combinations have been conducted, and it was found that analysis including mode 2, 9, 10 (these are the fundamental vertical symmetric bending, second lateral symmetric bending and fundamental torsional symmetric mode, respectively) resulted in a critical flutter velocity U_{cr} of 65.3 m/s which is close to that based on the first 15 natural modes with U_{cr} of 66.5 m/s. The eigenvalue loci are also similar with a coupled flutter initiated from the complex mode branch 9 as shown in Fig. 3. The corresponding mode shapes of complex mode branches 9 and 10 are shown in Fig. 4 in terms of the amplitude ratios of the natural modes considered. It can be noted that the modal properties of these two modes have been interchanged, while the curves of frequency loci do not intersect. As a result, the flutter mode branch, i.e., mode branch 9 corresponds to vertical bending and torsion coupled motions. The underlying physics of this multimode coupled flutter is the same as the flutter dominated by the fundamental vertical bending and torsional modes. In Fig. 3, the results without the coupled terms of mode 9 with other modes are also presented. It is noted that the veering of frequency loci is due to the coupling between modes 9 and 10.

A perturbation analysis has been conducted for predicting the eigenvalue loci of the complex modes 9 and 10 at wind velocities around 65 m/s. The perturbation solution is based on the adjacent complex modes 9 and 10 at the same wind velocity predicted by neglecting the coupled terms of mode 9 with other modes. The comparison with the complex eigenvalue analysis is shown in Fig. 5 which illustrates the accuracy of the perturbation analysis. Similar to the discussion for 2-DOF flutter, the interaction level of complex mode 9 and 10 around 65 m/s can be calculated based on the perturbation analysis and Eq. 17 and it is shown in Fig. 5. It is noted that the strong interaction of these two modes resulted in the veering of frequency loci.

6. CONCLUDING REMARKS

Proposed closed form solution of 2-D flutter analysis based on a perturbation solution clearly illustrated the influence of coupled self-excited forces on the coupled flutter. The observed heaving branch flutter corresponds to the intersection of frequency loci due to the weak aerodynamic interaction resulting from the coupled self-excited forces, and is physically consistent with the generally observed torsional branch coupled flutter. Curve veering is related to the interaction of closely spaced modes. The proposed criterion clearly illustrates the occurrence of cure veering, which is given in terms of the difference between the closely spaced eigenvalues and the effects of coupling. This discussion of curve veering has advanced our understanding of the physics of multimode coupled flutter of bridges.

ACKNOWLEDGMENTS

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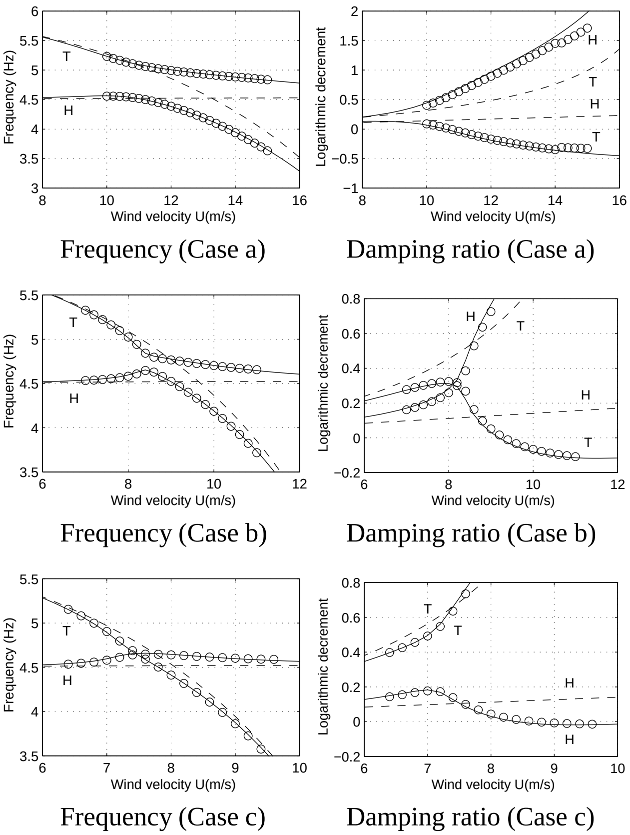
Table 1 Comparison of the flutter conditions

Case No.	a)	b)	c)
Branch	Torsion	Torsion	Heaving
U_{cr} (m/s)	10.57	9.19	8.41
U_{cr}/fB	13.71	12.87	12.12
$h/B\alpha$	0.68	1.30	2.07
$\phi(h) - \phi(\alpha)$	11.05	23.03	41.27



Frequency (Case a) Damping ratio (Case a)

Frequency (Case b) Damping ratio (Case b)

Frequency (Case c) Damping ratio (Case c)

Fig. 1 Comparison of eigenvalue loci (– – and — are calculated by the complex eigenvalue analysis w/ and w/o coupled self-excited forces; circle dots are calculated by the perturbation analysis)

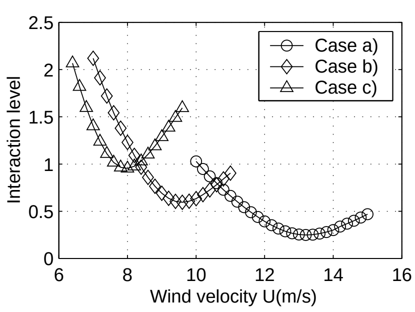
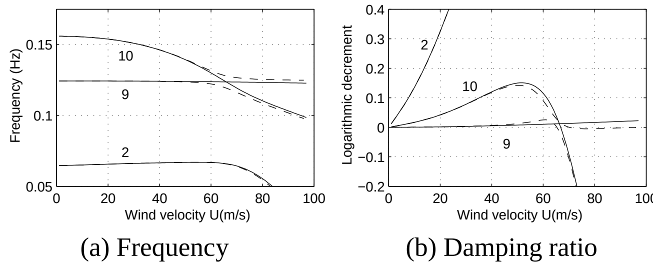
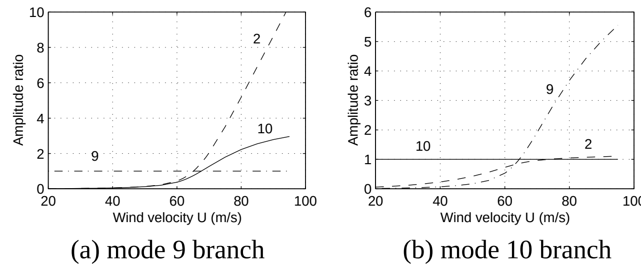


Fig. 2 Interaction level of heaving and torsional branches



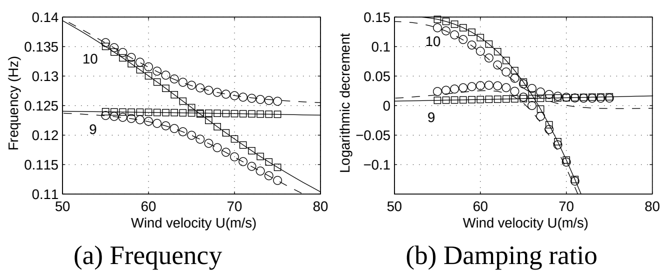
(a) Frequency (b) Damping ratio

Fig. 3 Eigenvalue loci of the suspension bridge (– – and — are w/ and w/o coupled terms of mode 9 with other modes)



(a) mode 9 branch (b) mode 10 branch

Fig. 4 Amplitudes of natural modes in different complex mode branches



(a) Frequency (b) Damping ratio

Fig. 5 Comparison of eigenvalue loci of the suspension bridge (– – and — are calculated by complex eigenvalue analysis w/ and w/o coupled terms of mode 9 with other modes; circle and square dots are calculated by perturbation analysis)

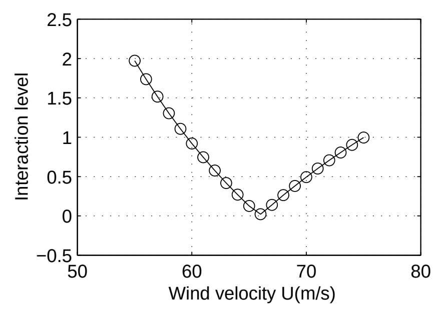


Fig. 6 Interaction level of adjacent complex 9 and complex 10 branches for the suspension bridge