ON THE BEAT PHENOMENON IN COUPLED SYSTEMS

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Abstract

The classical *beat phenomenon* has been observed in most coupled structure-damper systems. The focus of this paper is to provide a better understanding of this phenomenon, which is caused by the coupling that is introduced through the mass matrix of the combined system. However, beyond a certain level of damping in the secondary system, the *beat phenomenon* ceases to exist. This is due to coalescing of the modal frequencies of the combined system to a common frequency beyond a certain level of damping in the secondary system. Numerical and experimental results are presented in this paper to elucidate the *beat phenomenon* in combined structure-damper systems. Although this papers focusses primarily on coupled systems with liquid dampers, this is applicable to any type of coupled system, for e.g., a linear tuned mass damper or other such vibration absorber.

Introduction

The effectiveness of liquid dampers in controlling structural motions under wind and earthquake loadings has been demonstrated in theory and practice. The most commonly used liquid dampers are Tuned Liquid Dampers (TLDs) and Tuned Liquid Column Dampers (TLCDs). The TLCD is a special type of TLD that instead of sloshing relies on the oscillations of a column of liquid in a tube-like container to cancel the forces acting on the primary structure (Sakai and Takaeda, 1989). Damping in the TLCD is introduced by providing an orifice to dampen the oscillations of the liquid column. Experimental studies involving a TLCD combined with a simple structure have provided insightful understanding of the behavior of liquid damper systems. The motivation of this paper is potrayed in Figs. 1(a) and (b), which show the free vibration decay of a combined structure-TLD and -TLCD obtained by experiments. The controlled response exhibits the classical *beat phenomenon* characterized by a modulated instead of an exponential decay in the signature. The *beat phenomenon* has been discussed in many classical texts on vibration (e.g., Den Hartog, 1956). There is a transfer of energy between the coupled system, similar to the coupled penduli problem. The focus of this paper is to better understand this phenomenon for the combined structure-TLCD system.

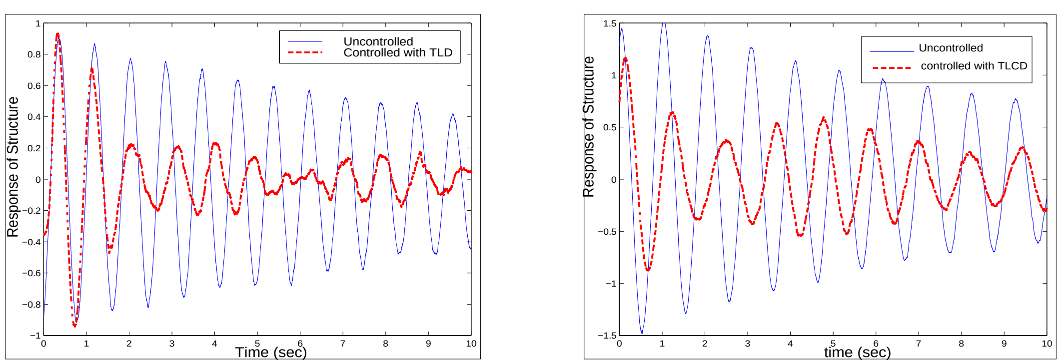

Figure 1. Uncontrolled and Controlled structural response with (a) TLD (b) TLCD.

The equations of motion of the combined single degree of freedom structure (primary system) and a TLCD (secondary system) shown in Fig. 2(a) are given by,

$$\begin{bmatrix} m_1 + m_2 & \alpha m_2 \\ \alpha m_2 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_2|\dot{x}_2| \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{1}$$

where x_1 and x_2 are the displacements of primary system and the secondary system, respectively; m_2 = mass of fluid in the tube = ρAl; c_2= nonlinear damping of the liquid damper=$1/2\rho A\xi$; k_2 = stiffness of the liquid column = $2\rho Ag$; m_1, k_1, c_1= mass, stiffness and damping in the structure; ρ= density of liquid; A = cross sectional area of the tube;ξ= headloss coefficient; α is the length ratio =b/l l = total length of the water column; and b = horizontal length of the column. Details of this system can be found in Yalla, *et al.* 1998. In the following sections, different cases of this general combined system are discussed.

Case 1: Undamped Combined System

The coupled equations of motion without damping in the primary and secondary system can be obtained from Eq. 1 by dropping c_1 and c_2,

$$\begin{bmatrix} 1+\mu & \alpha\mu \\ \alpha & 1 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2}$$

where μ is the mass ratio=m_2/m_1; and ω_1 and ω_2 are the natural frequencies of the structure and damper respectively. Figure 2(b) shows the time histories of the displacement of the undamped primary system for α=0 and 0.6. As expected, when coupling is present between the two systems, the displacement signature is amplitude modulated. The modal frequencies of this system are given by:

$$\varpi_{1,2} = \sqrt{\frac{\omega_1^2 + \omega_2^2(1+\mu) \pm \Pi}{2(1+\mu-\alpha^2\mu)}} \tag{3}$$

where $\Pi^2 = (\omega_1^2 - \omega_2^2(1+\mu))^2 + 4\omega_1^2\omega_2^2\alpha^2\mu$.

To understand this phenomenon better, one can consider the solution of the system of equations given in Eq. 2. After some mathematical manipulation the displacement of the primary system for the initial conditions, $x_1(0) = x_0$; $x_2(0) = 0$; $\dot{x}_1(0) = 0$ and $\dot{x}_2(0) = 0$, is given by:

$$x_1(t) = x_0\cos(\omega_B t/2)\cos(\omega_A t/2) \tag{4}$$

where $\omega_A = \varpi_1 + \varpi_2$ and $\omega_B = \varpi_2 - \varpi_1$, which means that the resulting function is an amplitude-modulated harmonic function with a frequency equal to ω_B and the amplitude varying with a frequency of ω_A.

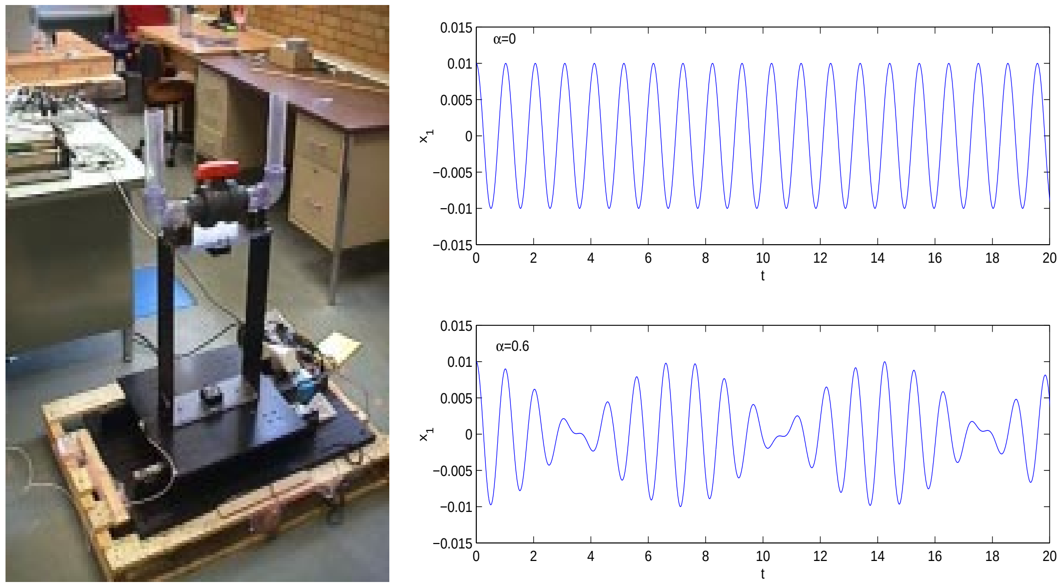

Figure 2. (a) Experimental Set-up of coupled Structure-TLCD system (b) Time histories of primary system displacement for α=0 **and** 0.6

Case 2: Linearly Damped Structure with Undamped Secondary System

In this section a linearly damped primary system with undamped secondary system is considered. Accordingly, the equations of motion are given by:

$$\begin{bmatrix} 1+\mu & \alpha\mu \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 2\omega_1\zeta_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{5}$$

Figure 3(a) shows the effect of damping in the primary system on the response of the structure. As the damping increases, the response dies out in an exponential decay. However, the *beat phenomenon* still exists. This poses difficulty in the estimation of system damping from free vibration response time histories.

At this stage, the effect of decreasing the beat frequency on the response signal can be further examined. Figure 3(b) shows that as ω_B approaches zero, T_B (the time period of the beat frequency) becomes very large. As a result, due to the damping in the primary system, the response dies out before the next peak of the beat cycle arises. Therefore, the response resembles that of a damped single degree of freedom (SDOF) system.

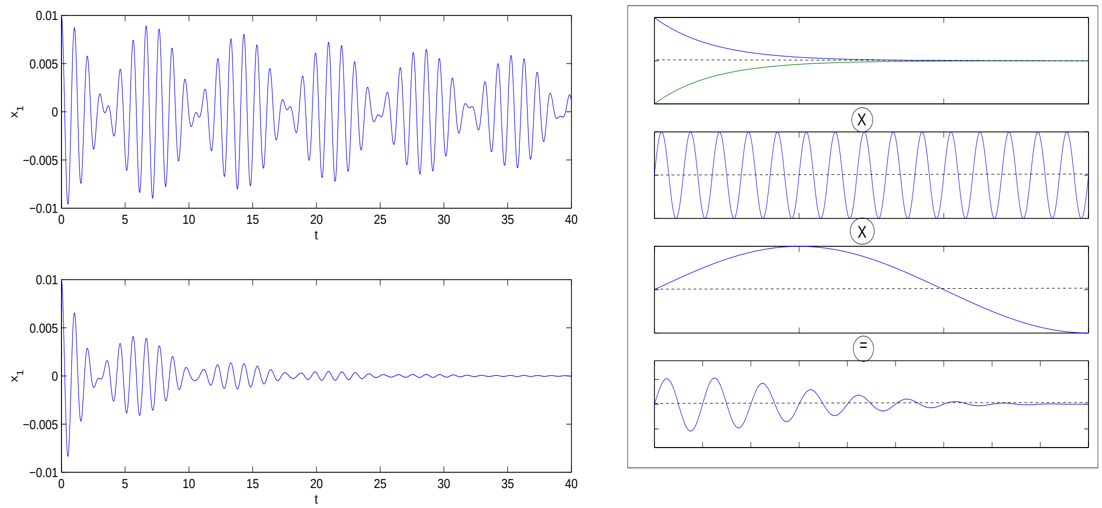

Figure 3. (a) Time histories for ζ_1=0.5 % and ζ_1=5% (b) Anatomy of the damped response

Case 3: Damped Primary and Secondary System

In this section, consider the system represented by Fig. 2(a), where now an orifice imparts damping into the system. Equation 1 is numerically simulated for the free vibration case at different levels of the headloss coefficient (Fig. 4(a)). The figure shows an interesting behavior of the liquid damper system. In the previous section, the damping simply caused an exponential decay of the beat response. However, in this case, the *beat phenomenon* disappears after a certain level of the headloss coefficient. Since an analytical solution is not convenient for this equation due to the quadratic nonlinearity in the damping matrix, a linearized version of this system is generally considered. Therefore, Eq. 1 is recast as:

$$\begin{bmatrix} 1+\mu & \alpha\mu \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 2\omega_1\zeta_1 & 0 \\ 0 & 2\omega_2\zeta_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{6}$$

In order to further validate the observations made in this paper, a simple experiment was conducted using the experimental setup shown in Fig. 2(a). The TLCD was designed with a variable orifice, to effectively change the headloss coefficient. At Φ= 0 degrees, the valve is fully opened and the headloss is increased with an increase in the angle of rotation, Φ. In Fig. 4(b), clearly at low headloss coefficients, there is an obvious *beat* pattern but as the headloss coefficient is increased, the *beat phenomenon* disappears and an exponentially decaying signature is obtained.

Figure 5 explains the disappearance of the *beat phenomenon* due to coalescing of the modal frequencies after a certain value of ξ is reached. The resulting beat frequency approaches zero and hence *beat phenomenon* ceases to exist. This is similar to a previous case where there was no *beat phenomenon* for coupling term α=0, in which case the beat frequency was also zero.

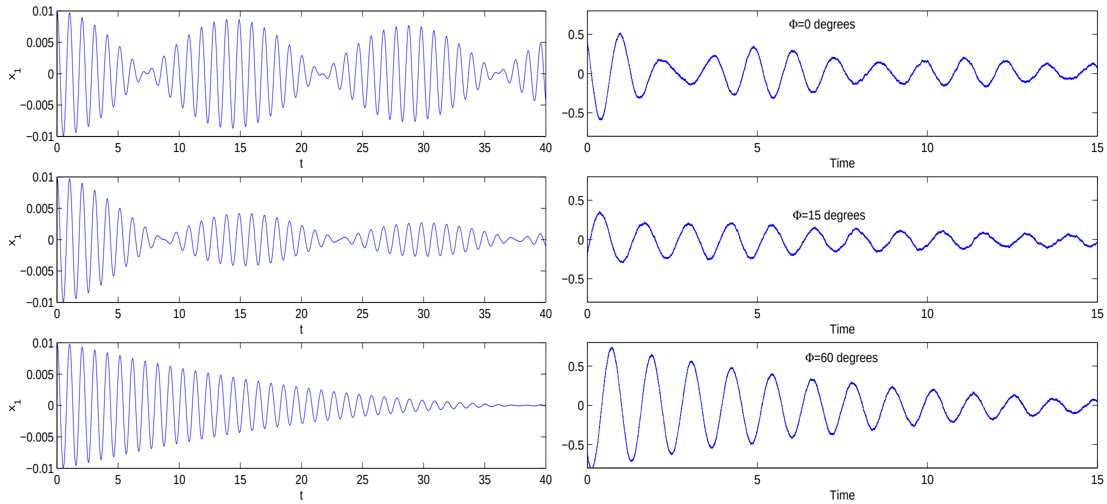

Figure 4. (a) Time histories of response for ξ=0.2, 2 and 50 (b) Experimental results for Φ =0, 15, 60 degrees

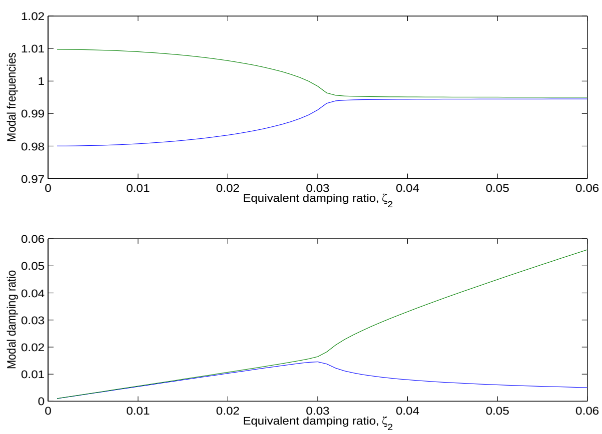

Figure 5. Modal frequencies and damping ratios

Conclusions

Like coupled mechanical systems, the combined structure-liquid damper system exhibits the *beat phenomenon* due to the coupling in the mass matrix of the combined system. The free vibration response resembles an amplitude modulated signal. The beat frequency of the modulated signature is given by the difference in the modal frequencies of the coupled system. However, beyond a certain level of damping in the secondary system (liquid damper), the *beat phenomenon* ceases to exist. This is attributed to the coalescing of the modal frequencies of the combined system to a common frequency over that range of damping in the secondary system.

Acknowledgements

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