Math 40480, Spring 2015

Summary

Concepts

- **Main concept—analytic function**: What is an analytic function? Four points of view
  - complex differentiable
  - satisfies the Cauchy–Riemann equations
  - locally has a complex power series expansion
  - conformal (where the derivative is nonzero)
- conformal map
- harmonic function
- isolated singularities—three types
  - removable, Riemann's Removable Singularity Theorem
  - pole
  - essential singularity
- order of zero or pole
- residues, methods for computing them

Results about analytic functions

- Cauchy Integral Theorem
- Cauchy Integral Formula
- Morera's Theorem
- Liouville's Theorem
- Residue Theorem
- Principle of Analytic Continuation—If $f,\ g$ are analytic in D and are equal on a sequence of points which converge to a limit **in** D, then $f \equiv g$.
- The range of a non-constant analytic function is open.
- Argument Principle
- Rouché's Theorem

- Maximum Modulus Principle
- Schwarz's Lemma
- Riemann Mapping Theorem
- Schwarz–Christoffel Theorem
- Schwarz Reflection Principle

Results about harmonic functions

- Maximum principle
- Mean–value property
- If ϕ is analytic and u is harmonic on the range of ϕ then $u \circ \phi$ is harmonic.
- The Poisson Integral Formula in the unit disc and in the upper half plane
  - Use in solving the Dirichlet problem

Examples and applications

- Examples of analytic functions and their mapping properties
  - polynomials
  - exponential
  - logarithm
  - trigonometric functions
  - linear fractional transformations
- Evaluation of definite integrals
- Use of conformal mapping to transfer the solution of a problem on a simple domain (e.g., a disc) to a conformally equivalent domain

Mathematical culture

- $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$
- $\displaystyle\int_0^{\infty} \frac{\sin x}{x}\, dx = \frac{\pi}{2}$
- Proofs of the Fundamental Theorem of Algebra
- Your project topic