Information for the Final, Math 30750

The final will cover everything we have done in the course, so Chapter 1, sections 2.1-2.2, 2.4-2.6, 3.1-3.3, 3.5-3.6, 4.1-4.3, 4.5, 5.1-5.3, 5.6-5.8, 6.1-6.4 and the local existence and uniqueness theorem for ODEs. (A weaker version of the local existence and uniqueness is in section 7.1.1, with a much longer proof because it doesn't use the Contraction Mapping Principle.) The format of the exam will be similar to the format of Exams 1 and 2.

The following question will be on the exam.

(a) State and prove a major theorem from the course.

(b) Why is the theorem major?

Review outline

Major Terms

- real numbers (ordered (axioms O1-5) field (axioms P1-9) satisfying Axiom of Completeness)
- countable set
- convergence of a sequence
- divergence to ∞ or $-\infty$
- bounded sequence
- Cauchy sequence
- greatest lower bound, least upper bound
- limit point
- continuous function (and equivalent ϵ, δ criterion)
- limit of a function
- uniformly continuity
- partition, upper and lower sums
- Riemann integrable, Riemann integral
- jump discontinuity
- piecewise continuous
- improper Riemann integral

- differentiable, continuously differentiable, derivative
- pointwise convergence
- uniform convergence
- lim sup and lim inf
- infinite series (both series of constants and series of functions), convergence, divergence
- geometric series
- absolute and conditional convergence
- power series
- Taylor series
- metric spaces
- convergence, Cauchy sequences, completeness in metric spaces
- normed linear space, Banach space

Major Theorems

- Cauchy criterion for convergence
- Convergence of bounded monotone sequences
- Existence of sup of set bounded above
- Bolzano–Weierstrass theorem
- A continuous function on a closed bounded interval is bounded. (Theorem 3.2.1)
- A continuous function on a closed bounded interval takes its inf and sup. (Theorem 3.2.2)
- Intermediate Value Theorem
- A continuous function on a closed bounded interval is uniformly continuous. (Theorem 3.2.5)
- A continuous function on a closed bounded interval is Riemann integrable. (Theorem 3.3.1)
- A bounded monotone function on a closed bounded interval is Riemann integrable. (Theorem 3.5.1)
- Mean Value Theorem

- Fundamental Theorem of Calculus (Parts I and II)
- Taylor's Theorem
- The inverse of a continuous strictly monotone function is continuous. (Theorem 4.5.1)
- The inverse of a continuously differentiable function with nowhere 0 derivative is continuously differentiable. (Corollary 4.5.3)
- Theorems on interchanging limiting operations under appropriate hypotheses (all involving uniform convergence): limit of a sequence and continuity, limit of sequence and integration, limit of sequence and differentiation and applications of these to series of functions (Theorems 5.2.1-5.2.3, 6.3.2-6.3.3, 6.4.2)
- Comparison test
- Ratio test
- Root test
- Weierstrass M-test
- Existence of radius of convergence of power series (Theorem 6.4.1)
- Contraction Mapping Principle
- Existence and Uniqueness Theorem for ODEs