**Math 30650, Spring 2012**

## Review for Final

### Themes

**Note:** Only themes 6 and 7 are new since the midterm. but there have been new applications of the others.

1. Linear problems

- The set of solutions of a linear homogeneous problem is a vector space.
- Two solutions of an inhomogeneous linear problem differ by a solution of the corresponding homogeneous problem.

2. An existence and uniqueness theorem tells you

- there is a solution to a problem satisfying the hypotheses;
- there is only one solution.

3. Once you have found enough independent solutions to a homogeneous linear problem $Ly = 0$,

- you can find all solutions;
- you can find all solutions to $Ly = g$ starting with a particular solution $y_p$.

4. Good educated guesses often lead to solutions.

5. Transform a problem to a simple one, solve that, transform that solution back to a solution of the original problem.

6. Approximate a nonlinear problem by a linear one.

7. Look for solutions of a simple form; try to use them to build all solutions.

### Specific Topics

**Note:** Only topics 5-7 are new since the midterm.

1. Higher order linear ODE

- Existence, uniqueness for initial value problem
- Solutions of $n$th order homogeneous equation form an $n$ dimensional vector space
- Method of solving constant coefficient homogeneous equations

2. Numerical methods

- Euler's method, estimate for local truncation error
- Improved Euler
- Runge-Kutta
- Stability
  - Importance
  - Tests, methods of judging reliability of computer output (controlling error in dsolve(. . . ,numeric), examining graphical output)

3. Solving ODE with MATLAB

- symbolic solution using dsolve
- numerical solution using dsolve

4. Systems of first order linear ODE

- Existence, uniqueness
- The solutions of an $n \times n$ linear homogeneous system form an $n$ dimensional vector space
- Constant coefficient systems
  - diagonalizable, real eigenvalues
  - diagonalizable, complex eigenvalues
  - not diagonalizable
    * Jordan Canonical Form
    * only did real eigenvalues in this case
    * know how to find Jordan Canonical Form in $2 \times 2$ case
    * know how to use it in general case
- Trajectories
  - interpretation of eigendirections
  - how to tell direction of motion
  - behavior as $t \to \pm\infty$
- Vector field
  - use in determining direction

- stability, type of critical point at origin

- inhomogeneous system
  - Not necessarily autonomous
  - Trajectory not necessarily independent of $t$

5. Nonlinear systems of first order ODE

- Autonomous systems
  - critical points, stability
  - using the linearization to determine type, stability, when possible
- More complicated cases
  - Repeated real eigenvalues (proper node, improper node or spiral point)
  - Imaginary eigenvalues (center or spiral point, indeterminate type)
- Use of phase portraits, direction fields to determine type, stability

6. PDE, Fourier series

- Separation of variables, especially for the heat equation
- Fourier series
  - What they are
  - If it looks like one, it is
  - Convergence theorem
  - Sine series, Cosine series
- More on the heat equation
- The wave equation

7. The Laplace transform

- Definition, use in solving initial value problems for ODE
- Discontinuous forcing functions (the Heaviside function or unit step function)
- Impulse forcing functions (the delta function)