# Examples of Variance

# Geometric distribution

Ingredients:

- $X$ a geometric random variable with
- parameter $p$
- $q = 1 - p$

Here

$$m(i) = pq^{i-1}, \quad i = 1, 2, \ldots$$

Know:

$$E(X) = \frac{1}{p}$$

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In computing that we showed

$$\sum_{i=1}^{\infty} i\, q^i = \frac{q}{(1-q)^2}$$

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$$= p \left( \frac{1}{(1-q)^2} + \frac{2q}{(1-q)^3} \right)$$

So

$$E(X^2) = p\left(\frac{(1-q)+2q}{(1-q)^3}\right)$$
$$= p\,\frac{1+q}{(1-q)^3} = \frac{2-p}{p^2}$$

since $q = 1 - p$ and

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\begin{aligned}
V(X) &= E(X^2) - \frac{1}{p^2} \\
&= \frac{2}{p^2} - \frac{1}{p} - \frac{1}{p^2} \\
&= \frac{1}{p^2} - \frac{1}{p} = \frac{1-p}{p^2} = \frac{q}{p^2}
\end{aligned}
$$

## Variance of geometric distribution

$$V(X) = \frac{q}{p^2}$$

where $X$ is geometric with parameter $p$, $q = 1 - p$.

# Poisson distribution

Ingredients:

- $X$ a Poisson random variable with
- parameter $\lambda$

Here

$$m(k) = \frac{\lambda^k}{k!}\, e^{-\lambda}, \quad k = 0, 1, 2, \ldots$$

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- $X$ a Poisson random variable with
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and

$$E(X) = \lambda$$

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\begin{aligned}
E(X^2) &= \sum_{k=0}^{\infty} k^2 \frac{\lambda^k}{k!} e^{-\lambda} \\
&= \sum_{k=1}^{\infty} k \frac{\lambda^k}{(k-1)!} e^{-\lambda} \\
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$$E(X^2) = \lambda \left( \lambda + e^{-\lambda} \sum_{j=0}^{\infty} \frac{\lambda^j}{j!} \right) = \lambda^2 + \lambda$$

and

$$\begin{aligned} V(X) &= E(X^2) - \lambda^2 \\ &= \lambda^2 + \lambda - \lambda^2 \\ &= \lambda \end{aligned}$$

### Variance of Poisson distribution

$$V(X) = \lambda$$

where $X$ is Poisson with parameter $\lambda$.