

Examples of Variance

Ingredients:

- X a geometric random variable with
- parameter p
- $q = 1 - p$

Here

$$m(i) = pq^{i-1}, \quad i = 1, 2, \dots$$

Know:

$$E(X) = \frac{1}{p}$$

In computing that we showed

$$\sum_{i=1}^{\infty} i q^i = \frac{q}{(1-q)^2}$$

$$E(X^2) = \sum_{i=1}^{\infty} i^2 pq^{i-1}$$

$$= p \frac{d}{dq} \sum_{i=1}^{\infty} i q^i$$

$$= p \frac{d}{dq} \frac{q}{(1-q)^2}$$

$$= p \left(\frac{1}{(1-q)^2} + \frac{2q}{(1-q)^3} \right)$$

So

$$\begin{aligned} E(X^2) &= p \left(\frac{(1-q) + 2q}{(1-q)^3} \right) \\ &= p \frac{1+q}{(1-q)^3} = \frac{2-p}{p^2} \end{aligned}$$

since $q = 1 - p$ and

$$\begin{aligned} V(X) &= E(X^2) - \frac{1}{p^2} \\ &= \frac{2}{p^2} - \frac{1}{p} - \frac{1}{p^2} \\ &= \frac{1}{p^2} - \frac{1}{p} = \frac{1-p}{p^2} = \frac{q}{p^2} \end{aligned}$$

Variance of geometric distribution

$$V(X) = \frac{q}{p^2}$$

where X is geometric with parameter p , $q = 1 - p$.

Ingredients:

- X a Poisson random variable with
- parameter λ

Here

$$m(k) = \frac{\lambda^k}{k!} e^{-\lambda}, \quad k = 0, 1, 2, \dots$$

and

$$E(X) = \lambda$$

$$\begin{aligned} E(X^2) &= \sum_{k=0}^{\infty} k^2 \frac{\lambda^k}{k!} e^{-\lambda} \\ &= \sum_{k=1}^{\infty} k \frac{\lambda^k}{(k-1)!} e^{-\lambda} \\ &= \lambda \sum_{k=1}^{\infty} k \frac{\lambda^{k-1}}{(k-1)!} e^{-\lambda} \\ &= \lambda \sum_{j=0}^{\infty} (j+1) \frac{\lambda^j}{j!} e^{-\lambda} \end{aligned}$$

We have

$$\begin{aligned} E(X^2) &= \lambda \sum_{j=0}^{\infty} (j+1) \frac{\lambda^j}{j!} e^{-\lambda} \\ &= \lambda \left(\sum_{j=0}^{\infty} j \frac{\lambda^j}{j!} e^{-\lambda} + \sum_{j=0}^{\infty} \frac{\lambda^j}{j!} e^{-\lambda} \right) \end{aligned}$$

The first series on the right is just $E(X)$ so

$$E(X^2) = \lambda \left(\lambda + e^{-\lambda} \sum_{j=0}^{\infty} \frac{\lambda^j}{j!} \right) = \lambda^2 + \lambda$$

and

$$\begin{aligned} V(X) &= E(X^2) - \lambda^2 \\ &= \lambda^2 + \lambda - \lambda^2 \\ &= \lambda \end{aligned}$$

Variance of Poisson distribution

$$V(X) = \lambda$$

where X is Poisson with parameter λ .