Synchonization of self-sustained thermostatic oscillations in interacting chambers

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ABSTRACT

This is an experimental study of the synchronization of temperature oscillations in chambers due thermal interaction between them. The temperatures of the chambers are regulated by individual thermostats. The experimental facility consists of eight chambers arranged in a ring, each one of which can conduct heat to its neighbors on either side through separating walls. The temperature of each chamber is measured by a sensor and the information is fed to a PC that controls a light bulb to provide heat in each chamber. The control is thermostatic, i.e., the bulb goes off when the temperature rises above an upper limit, and comes on when it falls below a lower limit. Experiments were performed with two different separators between the chambers and three different heating powers. The effect of initial conditions was also studied. The results show that when the separators are conductive, the light bulbs become frequency synchronized; however, the phases are different for each chamber but the differences are constant. The lamps go on at one point on the ring, and an "on" wave travels around the ring in both directions until the entire ring is on. After this an "off" wave starts at the same point and travels in the same way until all bulbs are off.

There are some differences in this pattern depending on the dead band of the thermostats. For wooden separators no steady state oscillations are reached and the phases are essentially independent of each other.

NOMENCLATURE

A_C	convection area to outside [m^2]
A_s	conduction area of separator [m^2]
c	specific heat of wood [J/kgK]
d	thickness of separator [m]
h	convective heat transfer coefficient to laboratory [J/m^{2}K]
k	thermal conductivity of separator [J/mK]
M	mass [kg]
q	power of light bulb [W]
t	time [s]
T_i	temperature of chamber i [°C]
T_U	upper limit temperature [°C]
T_L	upper limit temperature [°C]
T_∞	outside temperature [°C]

1 INTRODUCTION

It has long been known that mechanical oscillators that interact weakly with each other may eventually enter into synchronization. An early example of this was the synchronization of pendulum clocks mentioned by Huygens [1]. In that case, it was observed that two clocks placed on the same table eventually entered into anti-phase lock-step; the interaction was through the table. In recent years, a number of examples of this phenomenon have been found drawn from natural as well as from man-made systems [2–4]. Synchronization can be of different types, for example, in frequency or phase. For weakly coupled systems with self-sustained oscillations, synchronization can occur even when the uncoupled frequencies are slightly different from each other. The resulting coupled oscillations are, of course, at the same frequency. Synchronization is also possible for chaotic [5] or forced systems [6], but we will not refer to those subjects here. Experimental examples in non-chaotic fluid systems have been found in coupled salt-water oscillators [7], reacting systems [8], and organ pipes [9].

There has not been much done with regard to synchronization in thermal systems. These systems can be very complex with many individual subsystems. It is common, however, to disregard the coupling and to design and operate the subsystems independently. Subsystem oscillations in variables such as temperatures, flow velocities and pressures often occur, and thus synchronization is possible if there is some form of weak coupling between the subsystems. Cai et al. [10, 11] experimentally demonstrated the existence of synchronization between secondaries in a thermal-hydraulic loop. Oscillations induced by thermostatic control of the temperature in individual loops eventually became synchronized even though the temperature settings of the thermostats controlling each loop were slightly different.

The objective of the present work is to examine the possibility of synchronization of temperatures in chambers that communicate with each other through wall conduction. The motivation is the following. The temperature in a single room with a thermostatically-controlled heater oscillates between the lower and upper limits set by the thermostat. The single room thus acts as a self-sustained oscillator. Now consider a string of adjacent rooms, such as in an office building, each with a thermostatically-controlled heater. If the rooms are all identical, and if the lower and upper limits of the thermostats are all the same, the frequencies of oscillation of all the rooms should also be the same. In addition, if there is infinite thermal resistance between adjacent rooms, the phases of the oscillations should be independent, depending only on the initial conditions. On the other hand if there is some thermal conductance, however slight, over the course of time a temperature difference will be built up that can drive heat transfer from one to the other. The phases of the temperature oscillations would then be affected. The objective of this study to determine what the effect is and what kind of dynamic synchronization pattern, if any, is set up. Temperature synchronization between rooms in large office buildings is of concern from the point of view of the instantaneous capacity of the heating system to deliver heat. If the temperatures in all the rooms are in phase, the instantaneous demand will be high at times and low at others.

2 EXPERIMENTAL FACILITY AND PROCEDURE

A multiple-chamber wooden test facility for the study of synchronization between cavities was designed and built. It was decided to have eight chambers; it is symmetrical and easy to build, and eight channels are also easy to sense and control. Fig. 1 shows a schematic of the facility that was constructed. The chambers are located in the form of a ring. The walls are made of 8 mm thick wood (particle board), and are glued to the bottom board while the top is simply placed on. The removable separators between the chambers are 400 mm in height and 252 ± 2 mm in width (the ± value here and in what follows are the standard deviation from the mean of the measured values). Two different materials were used for the separators: 0.5 mm thickness aluminum and 3 mm thickness laminated wood.

Fig. 2 shows some details of one of these chambers, including its nominal internal dimensions and the locations of a light bulb and temperature sensor. Air leaks between the chambers or to and from the laboratory are prevented by tape around the bottom and vertical edges and foam gaskets at the top. Each chamber is heated with a light bulb that can be turned on or off, and a sensor that measures the temperature at a fixed point within the chamber. The temperature sensor is a Dallas Semiconductor DS1822 Econo 1-Wire Digital

Thermometer[1] that has a resolution of 0.0625°C and an accuracy of 2°C.

As shown in Fig. 3, a Microchip 16F877 microprocessor is connected to relays controlling the light bulbs, to the DS1822 temperature sensors, and to a PC through an RS232 serial port. Software in the PC enables maximum and minimum temperatures for each chamber to be set to control the light bulbs. When the temperature rises above a prescribed upper limit T_U, the lamp switches off, and similarly it switches on when the temperature falls below a lower limit T_L. The software also enables the first time that each light bulb goes on to be specified, so that they can be all be started together or staggered in some pre-determined manner. The PC records the temperature of each one of the eight chambers every 10 seconds.

The experiments were carried out in a basement laboratory. The air temperature in the laboratory away from the experiment was measured by a Airflow Development Ltd. Model TA2 Thermometer to be 23.0 ± 0.5°C during the time of the experiments. The usual procedure was to let the experiment run for 10 to 22 hours, depending on the length of time it took to stabilize; the larger deadbands took longer. The data were monitored near the end to make sure that they were not likely to change. After downloading the data, the top was lifted from the experiment and kept off for about an hour to cool the chambers. The next run was performed only when the the temperatures at all chambers were equal and below 24°C.

The phenomenon of synchronization can be experimentally observed only if it takes place even when all chambers are not absolutely identical, because that cannot be completely achieved in practice. In the present case, differences arise for many reasons. Since the structure is of wood, the actual dimensions are slightly different for each chamber to within a few mm. The energy put out by the light bulbs are also slightly different. Lastly, the accuracy and precision of the digital thermometers play a role. A standard deviation of 0.2°C was found between the eight thermometers in uncorrected measurements of the room

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[1] Detailed specifications are in http://pdfserv.maxim-ic.com/en/ds/DS1822.pdf.

temperature. To minimize the role played by this, adjustments were made in the software to make the readings all equal to within the thermometer resolution when measuring the room temperature. After calibration, the room temperature measured by the eight digital thermometers was found to be 22.80 ± 0.06°C, confirming that the measurements were accurate to within the resolution of the digital thermometers.

3 NONDIMENSIONAL GROUPS

There are several parameters in the experiment that are fixed or outside our control. One is the thermal capacity of the wooden body of the eight-chamber set-up Mc, where M is its mass and c is its specific heat. Another is the thermal resistance due to convection between the chambers and the laboratory, hA_c, where h is the convective heat transfer coefficient, and A_c is the corresponding heat transfer area. A third is the room temperature in the laboratory, T_∞.

Some of the other parameters can be changed between runs. To observe the effect of thermal resistance between chambers, two different separators, one of wood and the other of metal, are used here. The thermal resistance of the separators between the chambers is kA_s/d, where k is the thermal conductivity of the separator material, A_s is the area for conduction, and d is the thickness of each separator. The light bulbs can also be changed, thus changing the heat rate, q. Through software in the PC the lower and upper limits of the temperature, T_L and T_U, at which the lamps come on and go off, respectively, can be set. For a given run, all eight separators and light bulbs as well as the temperature limits T_L and T_U are the same for each chamber.

The non-dimensional parameters in the problem can be shown to be

$$\alpha = \frac{T_U - T_L}{T_U - T_\infty}, \tag{1}$$

$$R = \frac{k_s A_s / d}{h A_c}, \tag{2}$$

$$Q = \frac{q}{h A_c (T_U - T_\infty)}. \tag{3}$$

$$\tag{4}$$

Measurements are made of the chamber temperature $T_i(t)$ $(i = 1, \ldots, 8)$, as a function of time t. In non-dimensional terms, this is

$$\theta_i(\tau) = \frac{T_i - T_L}{T_U - T_L}, \tag{5}$$

$$\tau = \frac{qt}{Mc(T_U - T_\infty)}. \tag{6}$$

4 PRELIMINARY TESTS WITHOUT CONTROL

4.1 Single chamber heated or cooled

Heating and cooling

4.2 All chambers heated or cooled

Heating and cooling

Mathematical model for heating

$$Mc\frac{dT}{dt} + hA(T - T_\infty) = Q \tag{7}$$

From this we can get three special cases:

(a) Maximum temperature reached at steady state with constant heating

$$T_{max} = T_\infty + \frac{Q}{hA}. \tag{8}$$

(b) Constant heating with $T(0) = T_0$

$$T(t) = T_{max} + (T_0 - T_{max})\, e^{-hAt/Mc}. \tag{9}$$

(c) Heating shut off with $T(0) = T_{max}$

$$T(t) = T_{max} - \frac{Q}{hA}\left(1 - e^{-hAt/Mc}\right). \tag{10}$$

From the measurements, the three parameters maximum temperature T_{max} [°C], time constant Mc/hA [s], and heat rate Q/hA [°C] can be determined.

4.3 All chambers except one heated or cooled

Heating and cooling

5 EXPERIMENTS WITH CONTROL

The following are the runs made:

A60S05

A60S75

A60S10

A60S125

A60S150

A40S05

A100S05

A60N05

W60S05

5.1 60W power, aluminum separators

The 60W nominal power light bulbs have a measured dc resistance of $62.8 \pm 0.5\ \Omega$. The upper temperature limit was held at $T_U = 45$°C and the lower one, T_L, was varied. As an initial condition, all the light bulbs were started together at the beginning of the experiment. This case was taken to the base to which the effect of changing parameters was compared.

See Figs. 4 and 5. Fig. 6 shows roughly six periods. The top figure indicates the on (white) and off (black) of the light bulbs, and the bottom the numbers that are on at any given moment.

An on-off indicator signal, $S_i(t)$, for chamber i is produced such that

$$S_i(t) = \begin{cases} 1 & \text{when light bulb is on} \\ 0 & \text{when light bulb is off} \end{cases} \tag{11}$$

The correlation matrix between chambers i and j is

$$C_{ij} = \begin{pmatrix} 1.0000 & 0.0876 & 0.0018 & 0.4723 & 0.7679 & 0.6788 & 0.3225 & 0.8595 \\ 0.0876 & 1.0000 & 0.8900 & 0.6163 & -0.1277 & -0.2278 & -0.5140 & 0.0902 \\ 0.0018 & 0.8900 & 1.0000 & 0.5302 & -0.2135 & -0.3135 & -0.5917 & 0.0044 \\ 0.4723 & 0.6163 & 0.5302 & 1.0000 & 0.2556 & 0.1556 & -0.2033 & 0.4734 \\ 0.7679 & -0.1277 & -0.2135 & 0.2556 & 1.0000 & 0.9000 & 0.5407 & 0.7811 \\ 0.6788 & -0.2278 & -0.3135 & 0.1556 & 0.9000 & 1.0000 & 0.6406 & 0.6827 \\ 0.3225 & -0.5140 & -0.5917 & -0.2033 & 0.5407 & 0.6406 & 1.0000 & 0.3235 \\ 0.8595 & 0.0902 & 0.0044 & 0.4734 & 0.7811 & 0.6827 & 0.3235 & 1.0000 \end{pmatrix} \tag{12}$$

This matrix shows the fraction of time that i and j are both on.

Including effect of initial conditions and effect of dead band.

5.2 Effect of initial condition

To change the initial condition, the starting of the light bulbs was distributed uniformly over one period. Thus the second light bulb was started 9.72 minutes after the first, the third 9.72 minutes after the second, and so on.

5.3 Effect of power

5.4 Effect of separators

Heating and cooling

6 CONCLUSIONS

The present study used local control, i.e., the heating of one chamber depended only on its own temperature. This leads to a great variation in the total instantaneous demand for heating. Optimization of the instantaneous heating demand can only be made by a group or global control system such that control of the heater at one chamber is done using information from the other chambers also.

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A Physical dimensions

Chamber	D (cm)	L_1 (cm)	L_2 (cm)	H (cm)
1	23.1	39.4	20.0	40.0
2	23.0	39.4	20.0	40.0
3	23.1	39.4	20.0	40.0
4	23.1	39.4	20.0	40.0
5	23.1	39.4	20.0	40.0
6	23.15	39.4	20.0	40.0
7	23.3	39.4	20.0	40.0
8	23.4	39.4	20.0	40.0

A.1 Temperature sensor

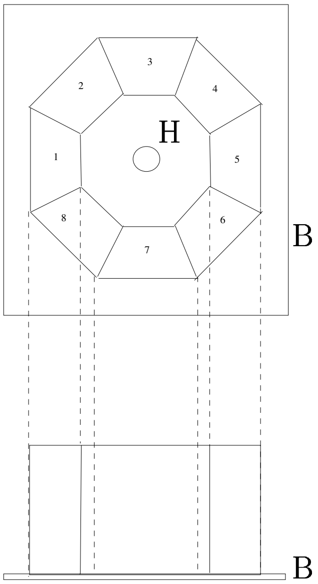

Figure 1: Schematic of six contiguous chambers. Top board has been removed; B is bottom board. H is hole through which the wires for the temperature sensors pass.

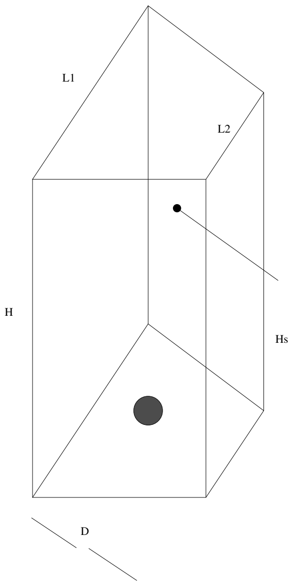

Figure 2: Schematic of a single chamber with internal dimensions; L is the light bulb and T the temperature sensor; S are removable separators between adjacent chambers; $D = 232 \pm 1$ mm, $L_1 = 394$ mm, $L_2 = 200$ mm, $H = 400$ mm, $a = 33 \pm 1$ mm, $b = 29.5 \pm 1$ mm, $c = 10 \pm 1$ mm.

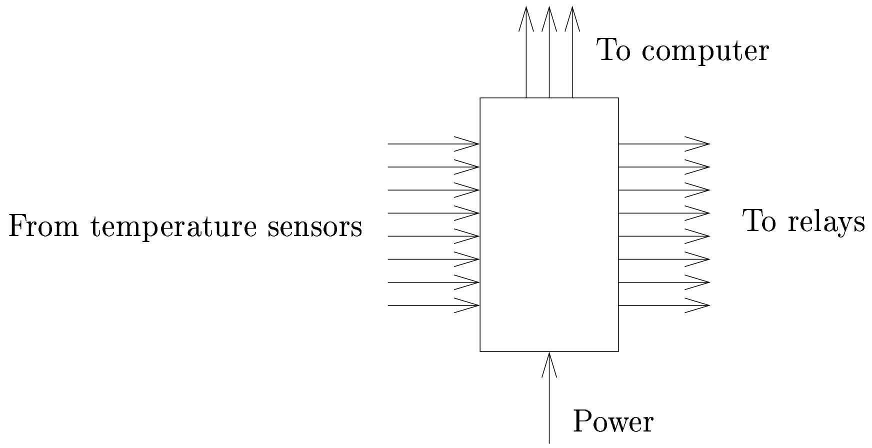

Figure 3: Schematic of connections to microprocessor.

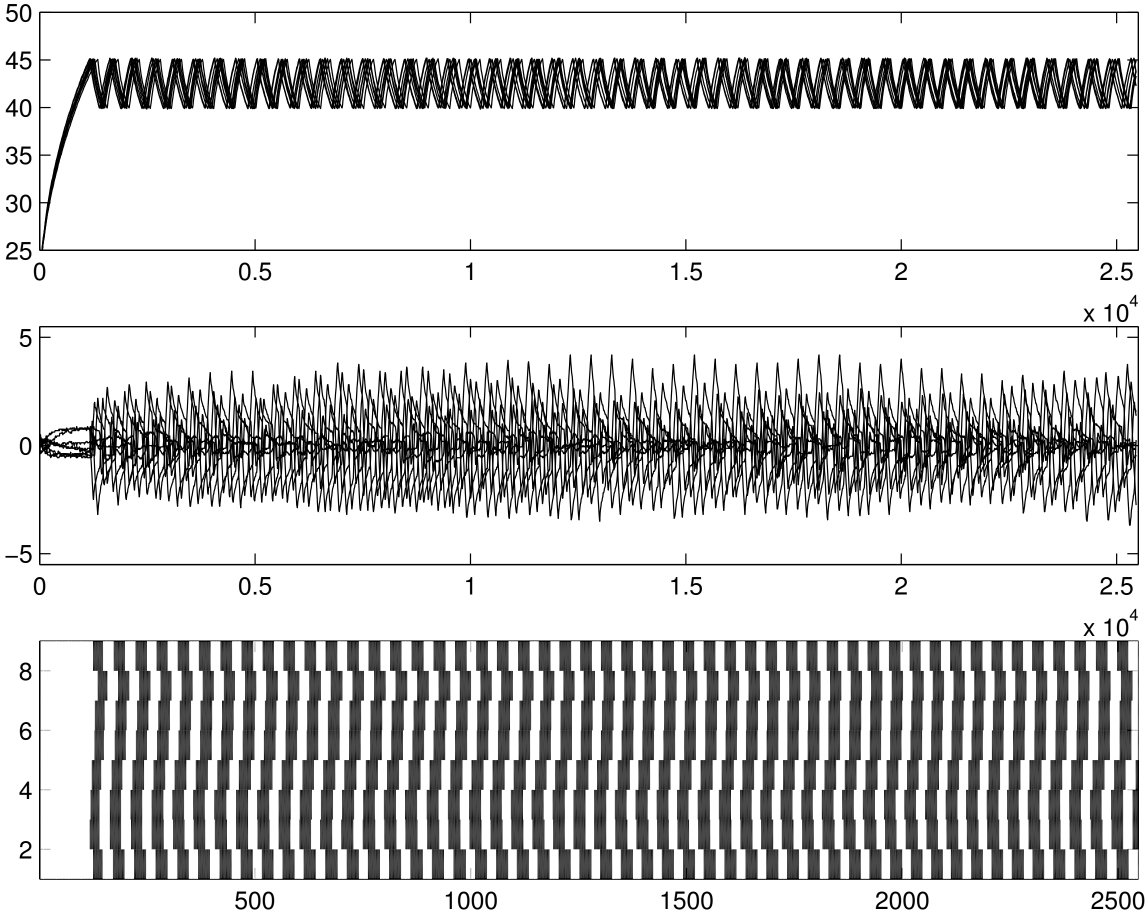

Figure 4: Display of data.

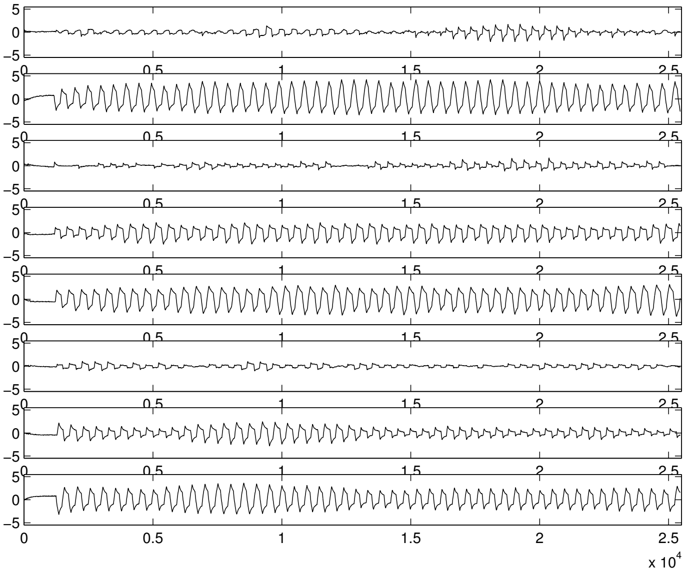

Figure 5: Temperature differences between adjacent rooms.

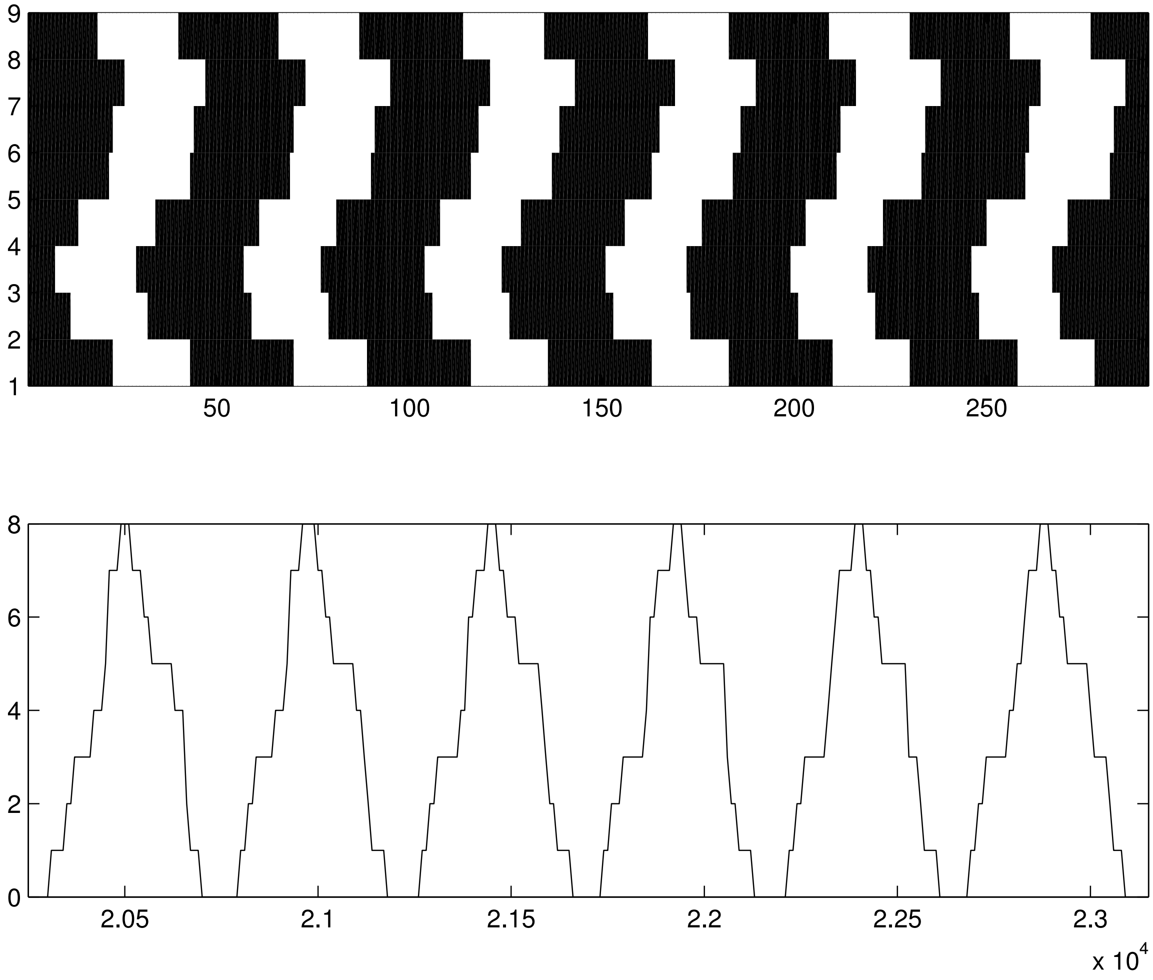

Figure 6: Number of rooms at any given moment that are on.