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# ANALYSIS OF THE PERFORMANCE OF IONIC LIQUIDS IN COOLING LOOPS

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## ABSTRACT

Liquids are often pumped in closed loops to transfer heat from a high temperature source to a low temperature sink. They operate at low Reynolds number when the diameter of the pipe is small, the fluid velocity is low, or when the working liquid is very viscous. Ionic liquids, though environmentally friendly, typically have viscosities much larger than water. An analytical study is made of the process for the purpose of determining what the important physical parameters of the system are that will enable the largest quantity of heat to be transferred for unit work expended. For this purpose, a loop is considered that has a pump that generates a certain pressure rise and two heat exchangers, one for heating the fluid and the other for cooling it. Laminar flow that is fully-developed hydrodynamically and thermally is assumed. The analysis is based on constant fluid properties, and analytical expressions are obtained for the heat rate and the work input.

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### Abstract

### Nomenclature

| | |
|---|---|
| $D$ | diameter of pipe |
| $k$ | fluid thermal conductivity |
| $L$ | total length of loop |
| $L_c$ | length of cooled section |
| $L_h$ | length of heated section |
| $Nu$ | Nusselt number in heat exchanger |
| $\Delta p$ | pressure increase in pump |
| $Q$ | heat transfer rate |
| $R$ | $= Q/W$ |
| $T$ | temperature of fluid |
| $T_c$ | temperature of cooling fluid |
| $T_h$ | temperature of heating fluid |
| $T_c^{in}$ | temperature fluid entering cooler |
| $T_c^{out}$ | temperature fluid leaving cooler |
| $T_h^{in}$ | temperature fluid entering heater |
| $T_h^{out}$ | temperature fluid leaving heater |
| $V$ | average fluid velocity |
| $W$ | mechanical power input |
| $x$ | local coordinate inside heat exchanger |

*Greek symbols*

| | |
|---|---|
| $\alpha, \beta_c, \beta_h$ | non-dimensional parameters |
| $\rho$ | fluid density |

$\mu$ viscosity of fluid

## 1 Introduction

Although ionic liquids have been around for about a hundred years [1], there was little interest in them until recent years because of their limitations; they were unstable in the presence of air and water [2]. The last decade has seen the production of a number of ionic liquids that are stable over a wide range of temperatures, and hence there has been a remarkable increase in interest in this liquid. Using the phrase "ionic liquid" in a scientific literature database produced 7,082 hits ranging all the way back to 1939.

Ionic liquids are salts, usually with organic cations and inorganic anions. They usually have low melting points, i.e. below 100°C, so that they can be liquid at room temperature. They also have a number of desirable properties that make them suitable for a variety of applications. Many of these salts are commercially available. Recent reviews of the subject has been written by Sun [3] and Panday [4].

Interest in ionic liquids from the perspective of their low environmental impact has mostly been a consequence of their low volatility and wide range of temperature in which they are liquid. Most of the applications so far have been related to chemistry or chemical engineering. Examples are environmentally-friendly manufacturing and chemical processes [5, 6], solvents in chemical analysis [4], synthesis of inorganic materials [7], extractions [7, 8], homogeneous and heterogeneous catalysis [9], $CO_2$ capture [10], treatment of high-level nuclear waste [11], for Hg2+ and Cd2+ removal from waste water [12], photolytic degradation of organic compounds [13], and materials for opto-electronic applications [14].

Since ionic liquids have high heat capacity, high density, high thermal and chemical stability, non-flammability and non-toxicity, they can also be potentially used in thermal engineering applications. They have been used as heat transfer fluids and for thermal storage in solar collector [15–17]. Because of their ability to absorb $CO_2$, they have also been proposed for absorption refrigeration [18–20]. Some of the material properties that play a role in these applications are thermal conductivity, heat capacity and related thermodynamic and thermophysical properties. Some of these properties can be found by molecular computation, and others by measurement.

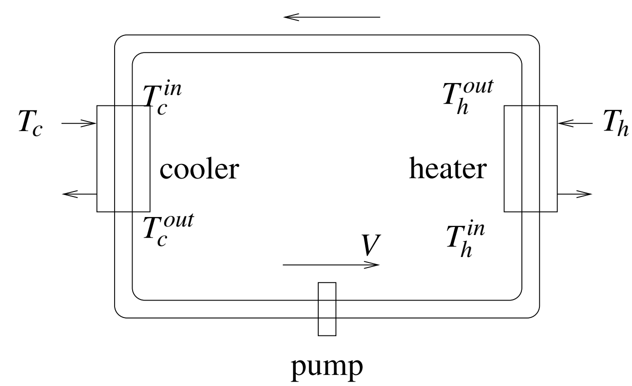

Figure 1. Schematic of closed loop.

It is possible to choose suitable cations and anions and hence design or tailor ionic liquids to give desirable properties for specific needs. Thus, for use as a heat transfer fluid in a closed loop, it may appear that it is desirable to have an ionic liquid that has both high conductivity for better heat transfer and low viscosity for easier pumping. But it is *a priori* not clear which of these two properties, if any, is the more important, or even how exactly each affects the heat transfer. That is the purpose of this analysis: to determine, though a very simplified analysis, the heat rate and pumping power in a closed loop as a function of the properties of the liquid.

## 2 Description of loop

A pipe of constant diameter forms a closed loop as shown in Fig. 1 in which a pump is used to drive a viscous fluid around the loop. There are two heat exchangers, one cools the fluid and the other heats it. The rest of the loop is insulated.

## 3 Governing equations

### 3.1 Momentum

The Reynolds number for ppe flow is defined as $Re = \rho VD/\mu$, where $\rho$ is the fluid density, $\mu$ is the fluid viscosity, $V$ is the average velocity in the pipe, and $D$ is the pipe diameter. We will assume that it is small enough for the flow to be laminar. Taking viscous resistance to be that corresponding to Poiseuille flow, we have

$$\Delta p \frac{\pi D^2}{4} = 8\pi\mu LV, \tag{1}$$

where $L$ is the total length of the loop, and $\Delta p$ is the pressure developed by the pump.

The mechanical power input at the pump is then

$$W = V\Delta p \frac{\pi D^2}{4}, \tag{2}$$

which can be written in terms of the pump pressure rise as

$$W = \frac{\pi D^4}{128\mu L}\Delta p^2. \tag{3}$$

### 3.2 Energy

We will assume the heat exchangers to be of concentric-tube configuration, and that the flow rates of the cooling and heating fluids are high enough for them to be at a constant temperature $T_c$ and $T_h$, respectively. Let $x$ be the coordinate along a heat exchanger, so that $x = 0$ is the inlet; $x = L_c$ is the outlet for the cooler and $x = L_h$ for the heater.

For the heater, for example, energy balance at any $x$ location gives

$$\rho \frac{\pi D^2}{4} V c \frac{dT}{dx} = h\pi D \left(T_h - T\right), \tag{4}$$

from which

$$T = T_h - \left(T_h - T_h^{in}\right) \exp\left(-\frac{4hx}{\rho DVc}\right). \tag{5}$$

Thus

$$T_h^{out} = T_h - \left(T_h - T_h^{in}\right) \exp\left(-\frac{4hL_h}{\rho DVc}\right). \tag{6}$$

Similarly, for the cooler

$$T_c^{out} = T_c + \left(T_c^{in} - T_c\right) \exp\left(-\frac{4hL_c}{\rho DVc}\right). \tag{7}$$

Aside from the heat exchangers, the rest of the loop is assumed to be insulated so that

$$T_c^{in} = T_h^{out}, \tag{8}$$
$$T_h^{in} = T_c^{out}. \tag{9}$$

From these equations, the four heat exchanger inlet and outlet temperatures can be calculated to be

$$T_c^{in} = F_h T_h + E_h F_c T_c, \tag{10}$$
$$T_c^{out} = E_c F_h T_h + F_c T_c, \tag{11}$$
$$T_h^{in} = E_c F_h T_h + F_c T_c, \tag{12}$$
$$T_h^{out} = F_h T_h + E_h F_c T_c, \tag{13}$$

where

$$E_{h,c} = \exp\left(-\frac{4hL_{h,c}}{\rho DVc}\right), \tag{14}$$
$$F_{h,c} = \frac{1 - \exp\left(-\frac{4hL_{h,c}}{\rho DVc}\right)}{1 - \exp\left(-\frac{4h(L_c+L_h)}{\rho DVc}\right)}. \tag{15}$$

This gives the heat transfer rate at the cooler and heater as

$$\begin{aligned} Q &= \rho \frac{\pi D^2}{4} V c \left(T_c^{in} - T_c^{out}\right) \\ &= \rho \frac{\pi D^2}{4} V c \left(T_h^{out} - T_h^{in}\right) \\ &= \rho \frac{\pi D^2}{4} V c \left\{ \left(F_h - E_c F_h\right) T_h - \left(F_c - E_h F_c\right) T_c \right\}. \end{aligned} \tag{16}$$

Substituting $V = \Delta p\, D^2/32\mu L$ from Eq. (1), and writing $h = Nu\, k/D$, where $Nu$ is the Nusselt number, we get

$$Q = \frac{\rho c \pi D^4 \Delta p}{128 \mu L}\, \frac{G_c G_h}{H}\, (T_h - T_c) \tag{17}$$

where

$$G_{c,h} = 1 - \exp\left(-\frac{128\, Nu\, k\mu L L_{c,h}}{D^4 \rho c \Delta p}\right) \tag{18}$$

$$H = 1 - \exp\left(-\frac{128\, Nu\, k\mu L (L_c + L_h)}{D^4 \rho c \Delta p}\right) \tag{19}$$

It is interesting to note that as $\Delta p \to \infty$, $Q \to Q_{max}$, where

$$Q_{max} = \pi Nu\, k \frac{L_c L_h}{L_c + L_h} (T_h - T_c)\,. \tag{20}$$

Beyond a certain pressure difference, the pumping power increases but the heat transfer rate does not; the maximum value of the heat rate only depends on the fluid thermal conductivity and not on its other properties.

From Eq. (3), the heat transfer rate per unit work input is

$$R = \frac{\rho c}{\Delta p}\, \frac{G_c G_h}{H}\, (T_h - T_c)\,, \tag{21}$$

where $R = Q/W$. This can be written as

$$R = \alpha \frac{\left(1 - e^{-\beta_c}\right)\left(1 - e^{-\beta_h}\right)}{1 - e^{-\beta_c - \beta_h}}, \tag{22}$$

where the non-dimensional parameters are

$$\alpha = \frac{\rho c\,(T_h - T_c)}{\Delta p}, \tag{23}$$

$$\beta_c = \frac{128\, Nu\, k\mu L L_c}{D^4 \rho c \Delta p}, \tag{24}$$

$$= 64\, \frac{L_c}{D}\, \frac{Nu}{f_F Re^2 Pr}, \tag{25}$$

$$\beta_h = \frac{128\, Nu\, k\mu L L_h}{D^4 \rho c \Delta p}, \tag{26}$$

$$= 64\, \frac{L_h}{D}\, \frac{Nu}{f_F Re^2 Pr}, \tag{27}$$

where the Fanning friction factor and Prandtl numbers are $f_F = D\Delta p/2\rho V^2 L$, and $Pr = \mu c/k$, respectively.

For $\beta_c, \beta_h \gg 1$, we can write

$$R \approx \alpha, \tag{28}$$

while for $\beta_c, \beta_h \ll 1$, we have

$$R \approx \alpha \frac{\beta_c \beta_h}{\beta_c + \beta_h} \tag{29}$$

$$= \frac{128 Nu\, k\mu\,(T_h - T_c)}{D \Delta p^2}\, \frac{L}{D}\, \frac{L_c}{D}\, \frac{L_h}{D}. \tag{30}$$

### 3.3 Numerical example

Consider a loop of geometry such that $D = 0.01$ m, $L = 1$ m, $L_c = L_h = 0.3$ m. Let the fluid properties (approximately those of the ionic liquid [Emim][EtSO$_4$]) be $\rho = 1.2 \times 10^3$ kg/m$^3$, $c = 1.6 \times 10^3$ J/kg K, $k = 0.29$ W/m K, $\mu = 6.24 \times 10^{-2}$ Pa s, operating under the conditions $T_h = 100$°C and $T_c = 0$°C. The Nusselt number corresponding to fully developed flow with uniform wall temperature is $Nu = 3.66$.

For $\Delta p = 200$ Pa, we get the parameters $\alpha = 9.6 \times 10^5$, and $\beta_c = \beta_h = 0.662$. The operating conditions are then $V = 0.01$ m/s, Re $= 1.926$, $Q = 48.3$ W, $W = 1.573 \times 10^{-4}$ W, $R = 3.07 \times 10^5$.

If we vary $\Delta p$, keeping all other system parameters to be the same, we can calculate and plot the variables

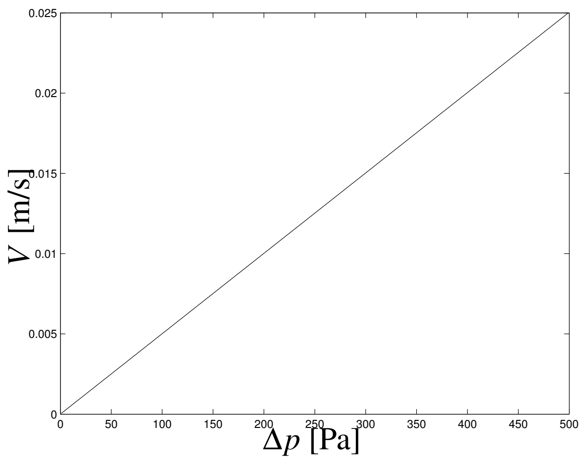


Figure 2. Fluid velocity in loop.

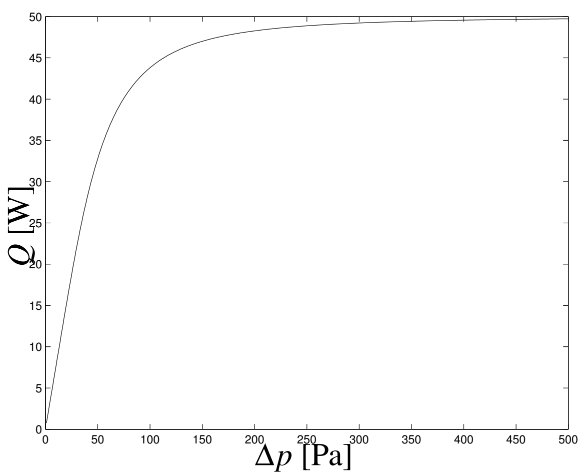


Figure 3. Heat transfer rate.

of interest. The velocity of the fluid $V$ is shown in Fig. 2 indicating that the relationship is linear. The heat rate is shown in Fig. 3; it levels off after a certain value of $\Delta p$ and there is little point in increasing $\Delta p$ beyond this. The power input is displayed in Fig. 4 and shows an increasing trend. Finally, Fig. 5 shows a decreasing trend in the ratio between the heat rate and the power.

## 4 Conclusions

A simplified analysis has been made of the flow and heat transfer around a closed loop. The flow is driven by a pump characterized by a pressure increase.

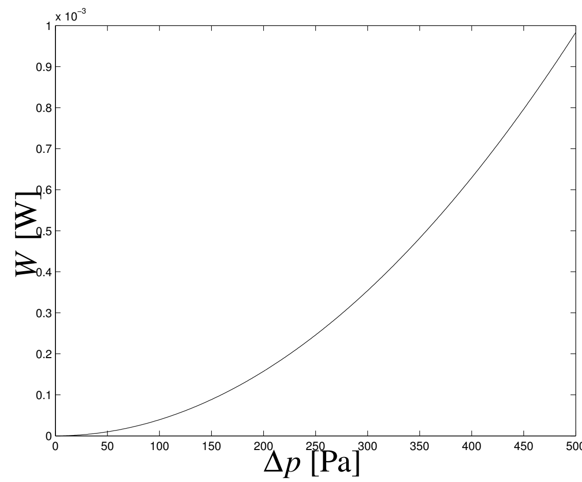


Figure 4. Mechanical power input.

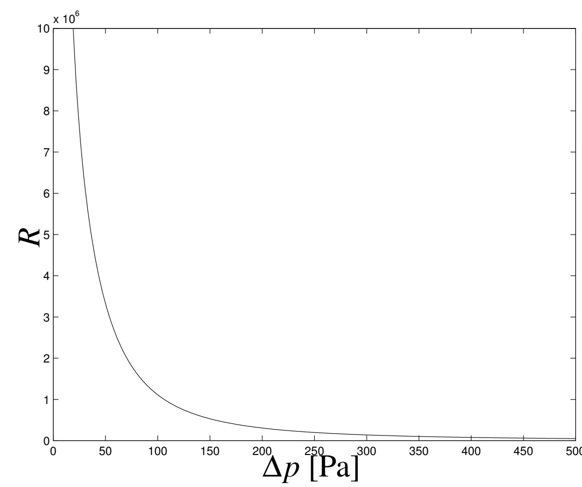


Figure 5. Ratio of heat transfer rate to mechanical power input.

The heat transfer occurs at two heat exchangers, one to heat the working fluid and the other to cool it. The heat rate and pumping power as well as the ratio of the two can be determined. Here we have presented them as a function of the pressure rise in the pump, which is a characteristic value that represents the size of the pump. The qualitative result is somewhat surprising: the viscosity and thermal conductivity occur in the ratio $R$ as a product. In other words, it is as convenient to have a higher viscosity as an increased thermal conductivity. The latter is not surprising, but the former is. The physical reason is that a higher viscosity leads to a lowering of the flow rate around the loop so that the temperature changes in the heat exchangers are high; the fluid is better able to extract heat and transport it. There is also an optimal pump pressure difference because, beyond a certain value, the pumping power increases but the heat rate does not.


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