

HEAT TRANSFER ENHANCEMENT DUE TO VIBRATING WALLS

ABSTRACT. This paper studies heat transfer enhancement due to vibrating walls. In the problem investigated, a Newtonian viscous fluid confined within an infinite channel with flexible walls is considered. The fluid properties are constant. The channel walls are kept at two different temperatures. The walls undergo transverse motions in the form of standing waves with small amplitudes. The hydrodynamic and thermal governing equations are solved using a regular perturbation expansion with the amplitude as a small parameter.

Heat transfer enhancement due to non-linear streaming is studied. Temperature distributions and Nusselt numbers are computed for a wide range of frequencies and channel half-widths.

The heat rate through the channel is observed to increase or decrease depending on the dimensionless channel half-width. A critical channel half-width of 2 was found. The effect of the vibrations was observed to diminish for large channel half-widths. A heat rate minimum was found at a channel half-width of 5. Increasing the Prandtl number generally enhanced the heat rate. However, at a Prandtl number of unity a singularity was found. The heat rate oscillated strongly in the vicinity of this singularity.

1. INTRODUCTION

This paper studies heat transfer enhancement due to vibrating solid boundaries. Flow in regions bounded by moving boundaries commonly occurs in many engineering applications as well as in nature. Heat and mass transfer in the presence of moving walls have significant implications in many of these applications, such as biological micro-scale experiments, heat exchangers, mixing devices, etc. The present investigation examines the effect on heat transfer in a channel due to non-linear streaming generated by transversely vibrating walls.

An oscillating wall next to a fluid at rest induces an unsteady motion of the fluid. While the time-averaged velocity of the wall displacement is inherently zero, nonlinear interactions of the oscillatory flow with itself provoke a time independent flow that is added to the unsteady part. This non-linear feature of unsteady flows is commonly called steady streaming, a phenomenon that has been subject to many investigations (Telionis, 1981, Riley, 2001). Streaming generated by oscillating walls have been extensively studied in the past for different reasons (e.g. Lyne, 1971, Jaffrin & Shapiro, 1971, Hung & Brown, 1976,

Kaneko & Honji, 1978, Pozrikidis, 1987, Li & Brasseur, 1993, Volfson & Vinals, 2001). The mixing properties of streaming flows in channels and cavities has also been addressed several times (e.g. Russel, 1999, Selverov & Stone, 2002, Yi & Hu, 2002, Carlsson, Sen, & Löfdahl, 2002).

Oscillatory motions generally enhance the heat and mass transfer in a fluid. For instance, Ni and Rama Rao (2003) reviewed different concepts of mixing enhancements through pulsations and oscillations. Go and Pak (2001) studied heat transfer enhancement using flow-induced vibrations of a microfin array and Yang (2003) numerically studied heat transfer enhancement using vortex generators. Moreover, heat transfer enhancements can be achieved without forced oscillations. Corrugated walls may induce secondary motions that mixes the fluid, thus increasing the heat and mass transfer (e.g. Sawyers, 1998, Wang & Chen, 2002, Comini & Savino, 2002).

We discuss in this paper the effect on the heat transfer in channel flows having walls vibrating as standing waves. The vibrations give rise to cellular streaming flows that advectively affect the heat transfer. Structural vibrations naturally occur at frequencies and modal shapes determined by the eigenfrequencies and eigenmodes of the specific structure, respectively. In solid structures, standing waves are the most common type of vibration; in practical applications flexible walls vibrating as standing waves are also relatively easy to achieve compared to most other types of wall deformations.

An analytical study using perturbation methods is performed to investigate the heat transfer characteristics of a viscous Newtonian fluid confined in a two-dimensional infinitely long channel in which the solid walls vibrate transversely as standing waves. The slope of the vibrating wall is assumed small and accordingly used as a perturbation parameter. Flow fields are obtained from Carlsson et al. (2002) and the corresponding temperature distributions are investigated. We solve the zeroth-order, time-dependent first- and time-averaged second-order equations. To evaluate the streaming effect on the heat transfer a Nusselt number is computed and compared to the zeroth-order steady conduction through the channel. The dependence on the dimensionless parameters of the problem is investigated

2. PROBLEM FORMULATION

Consider a two-dimensional infinitely long channel with its boundaries located at $\pm y_w$, where the region $-y_w < y < y_w$ is occupied by

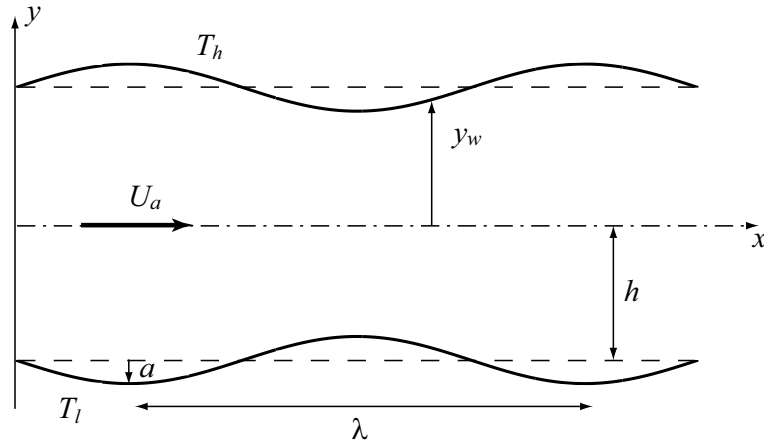


FIGURE 1. Sketch of vibrating wall.

an incompressible Newtonian viscous fluid. To allow the solid boundaries to vibrate transversely in the form of a standing wave, the wall coordinate is set to

$$(1) \quad y_w = h + \frac{\epsilon}{k} \cos(kx) \cos(\omega t),$$

where $k = 2\pi/\lambda$ is the wavenumber, λ is wavelength of the wavy surface, $\omega = 2\pi f$ is the radian frequency, and f is the frequency of vibration. The dimensionless parameter ϵ is the slope of the wall, that is $\epsilon = ak$, where a is the amplitude of vibration. The upper and lower channel walls are kept at constant temperatures, respectively, T_h and T_l . Natural convection is avoided since T_h is larger than T_l . Figure 1 displays a sketch of the wall displacement. Note that the wall motion is strictly in the y direction. The flow produced by the wall motion is periodic in x as well as symmetric with respect to the channel centreline.

A dimensionless temperature θ is defined as $(\theta = T - T_l)/\Delta T$, where $\Delta T = T_h - T_l$. If velocities are scaled with $\epsilon\omega/k$, lengths with k , time with ω^{-1} , we obtain the dimensionless parameters

$$(2) \quad \epsilon = ak, \quad H = hk, \quad \alpha^2 = \frac{\omega}{\nu k^2} \quad \text{and} \quad Pe = \alpha^2 Pr,$$

where $Pr = \nu/\sigma$, ν being the kinematic viscosity of the fluid and σ is the thermal diffusivity. These parameters represent the slope of the surface, the channel half-width, the wall frequency (or a Reynolds number), and the Peclet number, respectively. The two-dimensional

equation governing viscous flow in the channel is

$$(3) \quad \partial_t \Delta \psi + \epsilon (\partial_y \psi \partial_x \Delta \psi - \partial_x \psi \partial_y \Delta \psi) - \alpha^{-2} \Delta^2 \psi = 0.$$

where ψ represents the stream function defined by $\mathbf{u} = (\partial_y \psi, -\partial_x \psi)$, $\Delta = \partial_x^2 + \partial_y^2$. The temperature distribution is governed by

$$(4) \quad \partial_t \theta + \epsilon (\partial_y \psi \partial_x \theta - \partial_x \psi \partial_y \theta) - Pe^{-2} \Delta \theta = 0.$$

For the velocity the following boundary conditions should hold at the wall

$$\partial_y \psi(x, y = \pm y_w k, t) = 0, \quad \partial_x \psi(x, y = \pm y_w k, t) = \pm \cos x \sin t, \quad (5a, b)$$

and for the temperature

$$\theta(x, y = y_w k, t) = 1, \quad \theta(x, y = -y_w k, t) = 0. \quad (6a, b)$$

2.1. Coordinate transformation. To avoid difficulties imposed by the presence of irregular and time-dependent boundaries the physical domain is mapped into a rectangular domain (Carlsson et al., 2002, Carlsson, Sen, & Löfdahl, 2003). The irregular and time-dependent position of the surface is thus dealt with by transforming the coordinate system so that it follows the motion of the wall. Accordingly the new coordinates are

$$\xi = x, \quad \eta = y \frac{H}{H + \epsilon \cos(x) \cos(t)} \quad \text{and} \quad \tau = t, \quad (7a, b, c)$$

The y coordinate is compressed by the transverse wall motion.

To enable a perturbation analysis, the small parameter ϵ must appear explicitly. Hence the expression (7b) is rewritten in powers of ϵ , yielding

$$(8) \quad \eta = y \left[1 - \frac{\epsilon}{H} \cos(x) \cos(t) + \frac{\epsilon^2}{H} (\cos(x) \cos(t))^2 - \dots \right].$$

2.2. Perturbation expansion. If the parameter ϵ is assumed small it can be employed to perturb the governing equations (Van Dyke, 1964; Nayfeh, 1981). A perturbation solution valid for $\epsilon \ll 1$ is constructed as

$$(9) \quad \psi = \psi_0 + \epsilon \psi_1 + \epsilon^2 \psi_2 \dots, \quad \theta = \theta_0 + \epsilon \theta_1 + \epsilon^2 \theta_2 \dots$$

Note that ψ_0 is of order ϵ and θ_0 is of order unity.

Equation (9) is substituted into (3) and (4), and the transformation (7) together with (8) are used to map the flow onto the rectangular domain. By that the governing equations are reduced to a set of linear equations that can be solved order by order in ϵ . In the present

analysis the first and time-averaged second-order equations are investigated which are sufficient to capture the main features of the flow in the vibrating channel.

3. FLOW FIELD

The perturbation equations governing the flow field is solved up to second order in Carlsson et al. (2002). At first order the solution is oscillating around a zero mean

$$(10) \quad \psi_0(\eta) = i \frac{-A \cosh(AH) \sinh \eta + \cosh H \sinh(A\eta)}{-A \cosh(AH) \sinh H + \cosh H \sinh(AH)} \sin(\xi) e^{i\tau} + \text{c.c.},$$

where $A = \sqrt{1 - i\alpha^2}$.

By taking the periodic time-average of the second-order equation, the steady streaming generated by the vibrating surface is calculated. The solution reads

$$(11) \quad \begin{aligned} \psi_1^{(s)}(\xi, \eta) = \sin(2\xi) & \left[a_1 \eta \cosh(2\eta) + a_2 \sinh(2\eta) + a_3 \cosh \eta + a_4 \sinh \eta \right. \\ & + a_5 \cos(\gamma\eta) \sinh(\beta\eta) + a_6 \sin(\gamma\eta) \cosh(\beta\eta) \\ & + a_7 \eta \sin(\gamma\eta) \sinh(\beta\eta) + a_8 \eta \cos(\gamma\eta) \cosh(\beta\eta) \\ & + a_9 \sin(2\gamma\eta) + a_{10} \sinh(2\beta\eta) \\ & + a_{11} \cos(\gamma\eta) \sinh(\eta + \beta\eta) + a_{12} \sin(\gamma\eta) \cosh(\eta + \beta\eta) \\ & \left. + a_{13} \cos(\gamma\eta) \sinh(\eta - \beta\eta) + a_{14} \sin(\gamma\eta) \cosh(\eta - \beta\eta) \right], \end{aligned}$$

where

$$\beta = \sqrt{R} \cos \theta, \quad \gamma = \sqrt{R} \sin \theta, \quad (12a, b)$$

and $R = (1 + \alpha^4)^{1/4}$ and $\theta = 1/2 \arctan(-\alpha^2)$. $\psi_1^{(s)}$ represents the steady part of ψ_1 . The coefficients a_1 to a_{14} depend on α and H and are not given herein.

Streamlines form closed loops creating a cellular flow pattern in the channel. Naturally, the flow strongly depends on the dimensionless parameters α and H . Regions of different cell structures appear. For $H = 1$ figure 2 depicts streamlines obtained for $\alpha^2 = 2$ and $\alpha^2 = 300$, respectively.

Four regions of distinct cell structures exist. If a critical value is defined as being the α^2 at which a new layer of flow cells is formed or destroyed, the different flow regions are clearly distinguished by critical lines in a parameter plot, figure 3. Figure 2 (a) and (b) represent regions 1 and 4, respectively. The effect of the opposite wall vanishes for high values of H , so that the critical values of α^2 are constant for

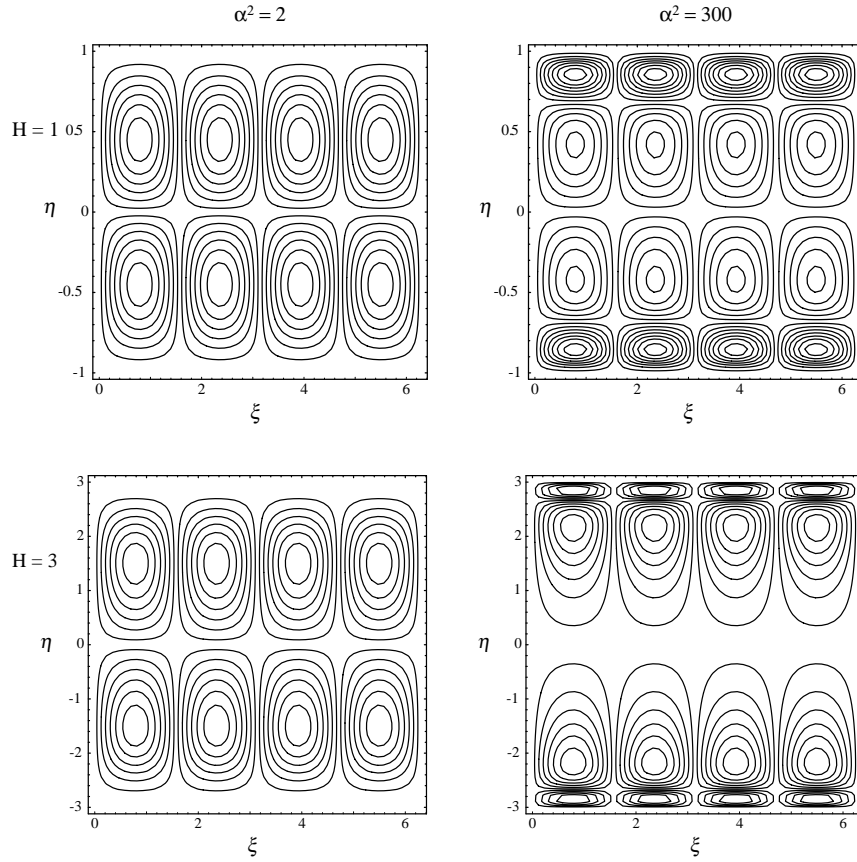


FIGURE 2. Streamlines showing the time-averaged second-order flow field for; (a) $\alpha^2 = 2$, $H = 1$; (b) $\alpha^2 = 300$, $H = 1$; (c) $\alpha^2 = 2$, $H = 3$; (d) $\alpha^2 = 300$, $H = 3$. $\eta = H$ is at vibrating wall.

$H \rightarrow \infty$. For large values of H the cellular streaming is confined to the wall regions of the channel. Region 1 and 3 represent such solutions.

These critical lines represent where in parameter space the solution changes its structure. The change from one structure to another is a smooth process and not as dramatic as figure 3 may suggest.

4. HEAT-TRANSFER

Mixing of passive scalars in a still fluid is an effect of molecular diffusion. The effectiveness of this mixing depends on the heat diffusion coefficients. For a flowing fluid momentum transfer within the fluid generally enhances mixing, an effect denoted advective mixing.

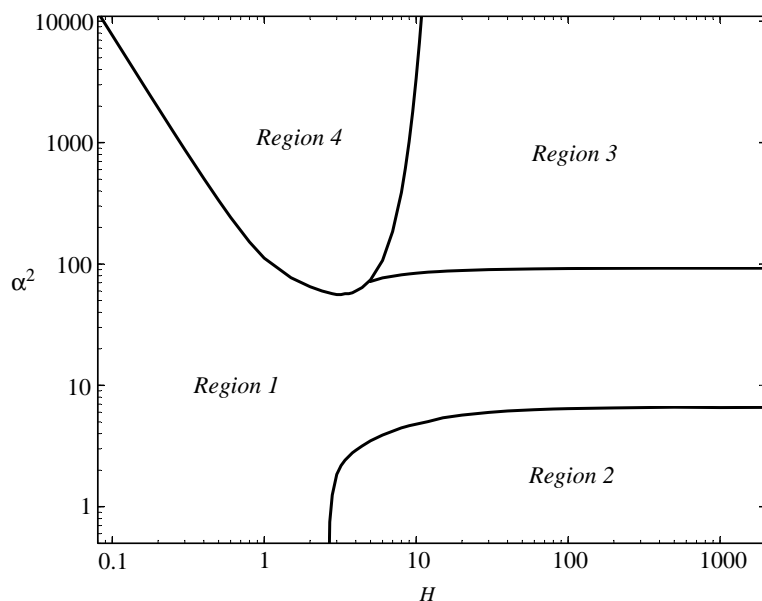


FIGURE 3. α^2 as a function of H . Solid lines are critical values of α^2 . The four regions represent different types of cellular flows in channel: cells in single layer in region 1, two layers of cells in region 2, three-cell layers in region 3, and two layers of cells in region 4.

The impact of advective mixing in comparison to molecular diffusion is governed by Pe . In this section we examine the effect of the vibrating walls on the mixing properties of the fluid. The perturbed energy equation is solved order by order.

4.1. Perturbation solutions. At zeroth order the heat transfer is due solely to molecular diffusion and a linear steady temperature profile is formed in the channel as

$$(13) \quad \theta_0 = \frac{1}{2} \left(1 + \frac{\eta}{H} \right).$$

At first order θ_1 is governed by

$$(14) \quad \partial_\tau \theta_1 - Pe^{-1} \Delta_{tr} \theta_1 = -v_0 \partial_\eta \theta_0 + \eta H^{-1} \partial_\tau b \partial_\eta \theta_0.$$

where $b = \cos(\xi) \cos(\tau)$ and v_0 is derived from (10). Due to the structure of the sources in (14) we seek solutions of the form $\theta_1 = \hat{\theta}_1 \cos(\xi) e^{-i\tau}$. These solutions are oscillatory in nature and do not directly effect the net heat rate through the channel. The amplitude

function is obtained as

$$\begin{aligned}
 \widehat{\theta}_1 = & -i \frac{\eta}{H} \frac{Pe}{2H(1-iPe)} \\
 & + \frac{1}{2H} \frac{A \cosh AH \sinh \eta}{\cosh H \sinh AH - A \cosh AH \sinh H} \\
 (15) \quad & - \frac{Pe}{2H(Pe - \alpha^2)} \frac{\cosh H \sinh A\eta}{\cosh H \sinh AH - A \cosh AH \sinh H}.
 \end{aligned}$$

At second order the heat rate is oscillating around a steady mean value. The temperature thus may be divided into a steady and unsteady part $\theta_2 = \theta_2^{(s)} + \theta_2^{(u)}$, where s and u denote steady and unsteady, respectively. If this expression is inserted into the second-order temperature equation and the result is periodically time-averaged the following equation appears for $\theta_2^{(s)}$

$$\begin{aligned}
 -Pe^{-1} \Delta_{tr} \theta_2^{(s)} = & \langle u_0 \partial_\xi \theta_1 \rangle \\
 & + \langle (-\eta H^{-1} b \partial_\tau b + \eta \partial_\xi b u_0 + H^{-1} b v_0 - v_1) \partial_\eta \theta_0 \rangle \\
 (16) \quad & + \langle (\eta \partial_\tau b - v_0) \partial_\eta \theta_1 - 2Pe^{-1} (H^{-1} b \partial_\eta^2 + \eta \partial_\xi b \partial_\xi^2) \rangle,
 \end{aligned}$$

where $\langle \cdot \rangle$ imply a periodic time-average. Equation (16) can be solved and the solution has the form

$$(17) \quad \theta_2^{(s)} = A_1(\eta) + A_2(\eta) \cos(2\xi).$$

A_1 and A_2 are functions depending on η . Both of them contain 28 coefficients and are thus relatively lengthy and not given in the paper.

4.2. Temperature distribution. Second-order temperatures are determined by the motion of the wall, first-order temperatures, first-order flow fields and the streaming flow at second order. Which of these affects the temperature the most is determined by the mutual strength of the source terms in equation 16. Figure 4 (*a*, *b*) display constant temperature lines for the two flow cases shown in figure 2. In figure 4 (*a*) Pe is low implying that both conduction and advection are important for the solution. Heat is transported from the upper to the lower wall and the temperature distribution is not symmetric around the channel centreline. The flow field in figure 2 (*a*) is responsible for the advective transport and the similarities between the flow and temperature solutions are obvious.

Increasing α increases Pe , thus, the importance of advection is enhanced. Figure 4 (*b*) shows the temperature distribution corresponding to the flow field in figure 2 (*b*). Conduction is less pronounced and the structure of the temperature solution is mainly determined by the

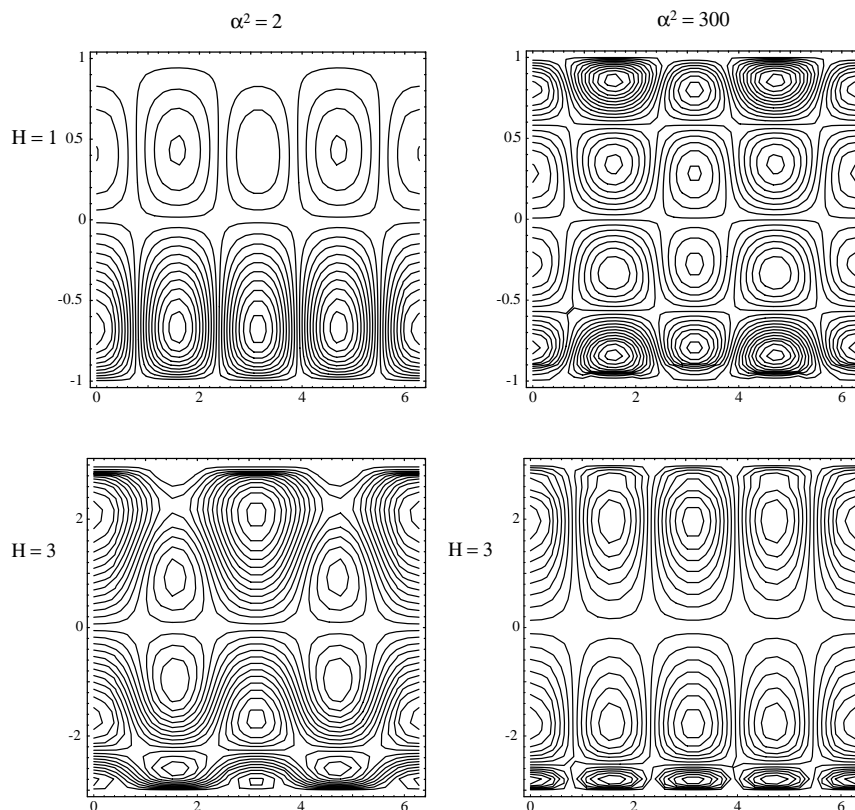


FIGURE 4. Isotherms showing the time-averaged second-order temperature field for $H = 1$. (a) $\alpha^2 = 2$; (b) $\alpha^2 = 300$. $\eta = H$ is at vibrating wall.

streaming flow. This is indicated by the more symmetric structure of the temperature distribution, and by comparing figures 4 (b) and 2 (b).

Increasing the channel half-width implies that the hydrodynamic effects on the channel flow draw nearer to the walls. This effect is most pronounced for larger values of α as shown in figures 2 (c, d). Figures 4 (c, d) display the corresponding temperature distributions.

4.3. Heat-rate. To evaluate the effect of the vibrating channel walls on the heat rate through the channel, the nusselt number Nu is computed. Nu is the dimensionless heat rate. At zeroth order Nu_0 is unity and steady. The oscillatory solution at first order does not contribute to the net heat rate and is not discussed. However, at second order a net heat rate is generated. To eliminate variations in the direction of the channel Nu is integrated over one wavelength of the vibrating

surface, hence

$$(18) \quad \overline{Nu}_2 = 2\pi^2 \int_0^{2\pi} \partial_\eta \theta_2^{(s)} d\xi.$$

where π^2 is a result of the coordinate transformation.

Figure 5 exhibits \overline{Nu}_2 as a function of α^2 for a fluid with $Pr = 10$. The solutions are always oscillation as a function of α and the oscillations depend strongly on Pr , generally increasing their amplitude with increasing Pr .

Two classes of solutions are observed for the heat rate as α is increased. Depending on H , \overline{Nu}_2 either increases or decreases. For H less than 2, \overline{Nu}_2 increases with increasing α as shown by the solid line in figure 5 (a). The dashed line shows a decreasing \overline{Nu}_2 for $H = 10$. At the critical value, $H_c = 2$, the oscillatory nature of the solution dominates the behavior of \overline{Nu}_2 as demonstrated in figure 5 (b).

Figure 6 exhibits \overline{Nu}_2 as a function of H . For the fully viscous solution, α is extremely low, \overline{Nu}_2 is always positive as shown by the dotted line. The dashed line is obtained for an α of unity and in this case \overline{Nu}_2 can attain both positive and negative values. The introduction of inviscid forces thus allows the heat rate to decrease due to the vibrations. The solid line in figure 6 represents the curve for $\alpha = 10$. Viscous forces are now confined to the wall and the inviscid part of the solution dominates. A critical channel half-width, H_c , of 2 is always obtained for sufficiently large values of α . Variations in Pr do not generally affect H_c . However, if Pr is large the solution may experience large oscillations.

For $H < 2$ the vibrations strongly influences the hydrodynamics over the full channel half-width irrespective of α . The scale of mixing is comparable to the channel half-width and the heat transfer is increased. Above H_c the hydrodynamic effects are drawn towards the wall regions of the channel. Fluid mixing is therefore confined to these wall regions. This has the remarkable effect of decreasing the heat transfer through the channel. However, if α is extremely small viscous effects prevents this decrease in heat rate. Thus, It is the inviscid part of the solution together with a sufficient channel half-width that is responsible for the decrease in \overline{Nu}_2 .

This insulating effect of the vibrations is, however, diminished at sufficiently large values of H as demonstrated in figure 6. Consequently, a minimum is formed at $H = 4.5$.

Pr expresses the ratio between viscous to thermal diffusion. To evaluate the effect of varying Pr figure 7 shows \overline{Nu}_2 for a number of

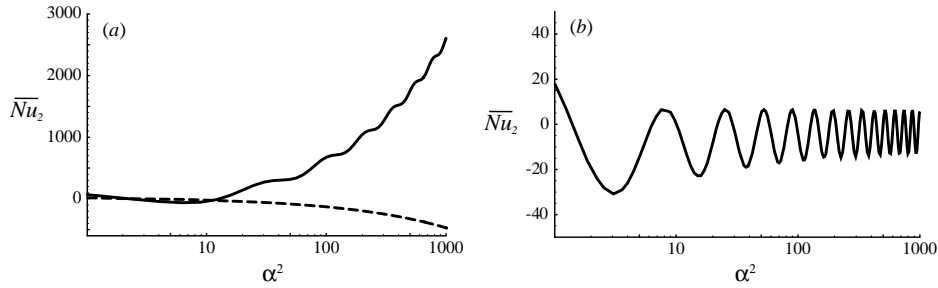


FIGURE 5. Second-order Nusselt number as a function of α^2 for $Pr = 10$. (a), solid line $H = 1$ and Dashed line $H = 10$. (b), $H = 2$

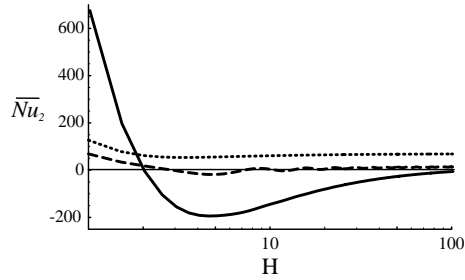


FIGURE 6. Second-order Nusselt number as a function of H for $Pr = 10$. Solid line $\alpha^2 = 1000$ and dashed line $\alpha^2 = 100$.

Pr and $H = 1$. For $H > 2$ a similar behavior is observed except for the fact that \overline{Nu}_2 decreases instead of increases as α grows.

By increasing Pr , Pe is also increased and the effect of advection is enhanced. This implies that \overline{Nu}_2 is amplified as observed in figure 7 (a). However, when the viscous and thermal diffusion is of the same order, $Pr = 1$, a singularity arises. Vigorous oscillations can be observed around the singularity. Figure 7 (b) display \overline{Nu}_2 for Prandtl numbers of 0.98 and 1.02, respectively. The singularity first appears in the first-order solution, equation 15, through the term $1/(Pe - \alpha^2)$.

If α is small, the largest effects of the vibrations on \overline{Nu}_2 are found for Prandtl numbers around unity. This is clearly demonstrated in figure 8 (a). The domination of the singularity is diminished as α is increased, however, it is still a pronounced feature as may be observed in figure 8 (b).

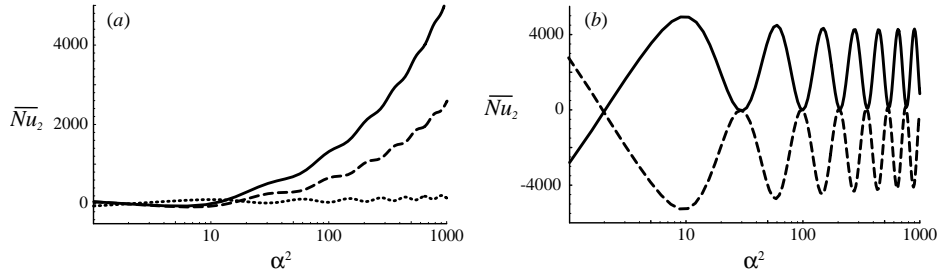


FIGURE 7. Second-order Nusselt number as a function of α^2 for $H = 1$. (a), solid line $Pr = 20$, dashed line $Pr = 10$ and dotted line $Pr = 0.5$. (b), solid line $Pr = 0.98$ and dashed line $Pr = 1.02$.

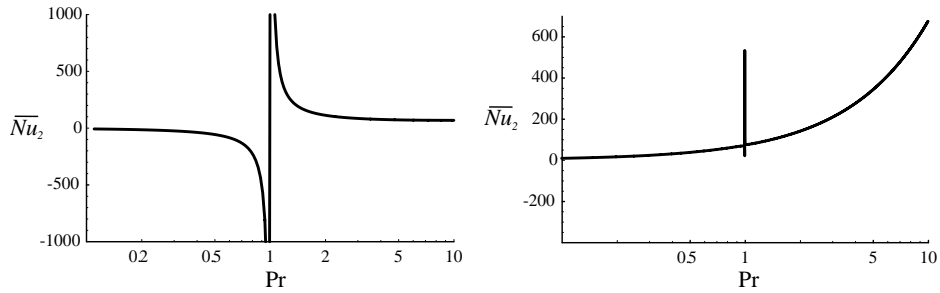


FIGURE 8. Second-order Nusselt number as a function of Pr for $H = 1$. (a) $\alpha = 1$; (b) $\alpha = 10$

5. CONCLUSIONS

We have investigated the effect on the heat transfer through a channel subjected to vibrating walls. Transverse vibrations in the form of standing waves were considered.

An analytical study of the heat rate through an infinite two-dimensional channel with flexible walls has been performed using perturbation methods with the slope of the wall as a perturbation parameter. The time-averaged heat rate was evaluated for varying α , H and Pr .

Depending on H the heat rate through the channel increases or decreases as a function of α . Below $H = 2$ an increase is observed and above the heat rate decreases. At $H = 5$ a minimum in the heat rate is found and for sufficiently large values of H the heat rate is unaffected by the vibrations.

Increasing Pr generally implies an increased heat rate. At $Pr = 1$ a singularity arises implying strong oscillations in the heat rate as α is increased.

REFERENCES

- Carlsson, F., Sen, M., & Löfdahl, L. (2002). Analytical studies of flow effects due to vibrating walls. *submitted to J. Fluid Mech.*
- Carlsson, F., Sen, M., & Löfdahl, L. (2003). Hydrodynamic effects in channel flow due to vibrating walls. *submitted to J. Fluid Mech.*
- Comini, N. C., G., & Savino, S. (2002). Convective heat and mass transfer in wavy finned-tube exchangers. *Int. J. Num. Meth. for Heat & Fluid Flows*, 12(6), 735-755.
- Go, K. S. J. L. G. Y. H. L. J. S. I., J. S., & Pak, Y. E. (2001). Heat transfer enhancement using flow-induced vibration of a microfin array. *Sensors and Actuators A-Physical*, 90(3), 232-239.
- Hung, T., & Brown, T. B. (1976). Solid-particles motion in two-dimensional peristaltic flow. *J. Fluid Mech.*, 73, 77-96.
- Jaffrin, M. Y., & Shapiro, A. H. (1971). Peristaltic pumping. *Ann. Rev. Fluid Mech.*, 3, 13-37.
- Kaneko, A., & Honji, H. (1978). Double structures of steady streaming in the oscillatory viscous flow over a wavy wall. *J. Fluid Mech.*, 93, 727-736.
- Li, M., & Brasseur, J. G. (1993). Nonsteady peristaltic transport in finite length tubes. *J. Fluid Mech.*, 248, 129-151.
- Lyne, W. H. (1971). Unsteady viscous flow over a wavy wall. *J. Fluid Mech.*, 50, 33-48.
- Nayfeh, A. H. (1981). *Introduction to perturbation techniques*. John Wiley & Sons.
- Ni, M. M. R. H. A. P. S. P. B. M. H. I., X., & Rama Rao, N. V. (2003). Mixing through oscillations and pulsations - a guide to achieving process enhancements in the chemical and process industries. *Chemical Eng. Research & Design*, 81(A3), 373-383.
- Pozrikidis, C. (1987). A study of peristaltic flow. *J. Fluid Mech.*, 180, 551-527.
- Riley, N. (2001). Steady streaming. *Ann. Rev. Fluid Mech.*, 33, 43-65.
- Russel, G. K. (1999). Structure and particle transport in second-order stokes flow. *Phys. Rev. E.*, 61, 6606-6620.
- Sawyers, D. R. (1998). Heat transfer enhancement in three-dimensional corrugated channel flow. *Int. J. Heat and Mass Transfer*, 41(22), 3559-3573.

- Selverov, K. P., & Stone, H. A. (2002). Peristaltically driven channel flows with applications toward micromixing. *Phys. Fluids*, *13*, 1387-1859.
- Telionis, D. P. (1981). *Unsteady viscous flows*. New York: Springer.
- Van Dyke, M. (1964). *Perturbation methods in fluid mechanics*. Academic Press.
- Volfson, D., & Vinals, J. (2001). Flow induced by a randomly vibrating boundary. *J. Fluid Mech.*, *432*, 387-408.
- Wang, C. C., & Chen, C. K. (2002). Forced convection in a wavy-wall channel. *Int. J. Heat and Mass transfer*, *45*, 2587-2595.
- Yang, S. J. (2003). Numerical study of heat transfer enhancement in a channel flow using an oscillating vortex generator. *Heat and Mass Transfer*, *39*(3), 257-265.
- Yi, B. H. H., M., & Hu, H. (2002). Peristaltically induced motion in a closed cavity with two vibrating walls. *Phys. Fluids*, *14*, 184-197.