Expansion of an arbitry vector using the eigenvectors of a matrix

First we choose a matrix. This one is self adjoint.

```
aa = {{1, -1, 3}, {-1, 0, -2}, {3, -2, 4}}
```

$$\begin{pmatrix} 1 & -1 & 3 \\ -1 & 0 & -2 \\ 3 & -2 & 4 \end{pmatrix}$$

```
Det[aa]
```

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Here is our arbitrary vector. We wish to expand it in terms of the eigenvalues

```
z = {-4, 3, 2}
```

$\{-4, 3, 2\}$

First get the eigenvalues

```
evals = Eigenvalues[aa]
```

$\left\{-1, 3-\sqrt{13}, 3+\sqrt{13}\right\}$

Then get the corresponding eigenvectors

```
evecs = Eigenvectors[aa]
```

$$\begin{pmatrix} -1 & 1 & 1 \\ \frac{2\left(-3+\sqrt{13}\right)}{-11+3\sqrt{13}} & -\frac{-5+\sqrt{13}}{-11+3\sqrt{13}} & 1 \\ \frac{2\left(3+\sqrt{13}\right)}{11+3\sqrt{13}} & -\frac{5+\sqrt{13}}{11+3\sqrt{13}} & 1 \end{pmatrix}$$

Now solve the adjoint system for the eigenvalues (not for this problem)

Here is how we write the expansion for the z vector,

```
Table[α[i], {i, 1, 3}] . {xvec[1], xvec[2], xvec[3]}
```

$$\text{xvec}(1)\,\alpha(1)+\text{xvec}(2)\,\alpha(2)+\text{xvec}(3)\,\alpha(3)$$

Now calculate this

```
expand1 = Table[α[i], {i, 1, 3}] . evecs
```

$$\left\{-\alpha(1)+\frac{2\left(-3+\sqrt{13}\right)\alpha(2)}{-11+3\sqrt{13}}+\frac{2\left(3+\sqrt{13}\right)\alpha(3)}{11+3\sqrt{13}},\ \alpha(1)-\frac{\left(-5+\sqrt{13}\right)\alpha(2)}{-11+3\sqrt{13}}-\frac{\left(5+\sqrt{13}\right)\alpha(3)}{11+3\sqrt{13}},\ \alpha(1)+\alpha(2)+\alpha(3)\right\}$$

Now we have the equation, z =Sum[α[i] x[i]] for the expansion. We evaluate the α[i]'s by forming the inner product of both sides of the equation with the adjoint eigenvectors (which are the same as the regular eigenvectors).

Here is the right side

```
expand2 = evecs . expand1
```

$$\left\{3\,\alpha(1)-\frac{2\left(-3+\sqrt{13}\right)\alpha(2)}{-11+3\sqrt{13}}-\frac{\left(-5+\sqrt{13}\right)\alpha(2)}{-11+3\sqrt{13}}+\alpha(2)-\frac{\left(5+\sqrt{13}\right)\alpha(3)}{11+3\sqrt{13}}-\frac{2\left(3+\sqrt{13}\right)\alpha(3)}{11+3\sqrt{13}}+\alpha(3),\right.$$

$$\alpha(1)+\alpha(2)+\alpha(3)+\frac{2\left(-3+\sqrt{13}\right)\left(-\alpha(1)+\frac{2\left(-3+\sqrt{13}\right)\alpha(2)}{-11+3\sqrt{13}}+\frac{2\left(3+\sqrt{13}\right)\alpha(3)}{11+3\sqrt{13}}\right)}{-11+3\sqrt{13}}-\frac{\left(-5+\sqrt{13}\right)\left(\alpha(1)-\frac{\left(-5+\sqrt{13}\right)\alpha(2)}{-11+3\sqrt{13}}-\frac{\left(5+\sqrt{13}\right)\alpha(3)}{11+3\sqrt{13}}\right)}{-11+3\sqrt{13}},$$

$$\left.\alpha(1)+\alpha(2)+\alpha(3)+\frac{2\left(3+\sqrt{13}\right)\left(-\alpha(1)+\frac{2\left(-3+\sqrt{13}\right)\alpha(2)}{-11+3\sqrt{13}}+\frac{2\left(3+\sqrt{13}\right)\alpha(3)}{11+3\sqrt{13}}\right)}{11+3\sqrt{13}}-\frac{\left(5+\sqrt{13}\right)\left(\alpha(1)-\frac{\left(-5+\sqrt{13}\right)\alpha(2)}{-11+3\sqrt{13}}-\frac{\left(5+\sqrt{13}\right)\alpha(3)}{11+3\sqrt{13}}\right)}{11+3\sqrt{13}}\right\}$$

```
expand3 = Simplify[expand2]
```

$$\left\{3\,\alpha(1),\ \frac{2\left(-13+\sqrt{13}\right)\alpha(2)}{-11+3\sqrt{13}},\ \frac{2\left(13+\sqrt{13}\right)\alpha(3)}{11+3\sqrt{13}}\right\}$$

Here is the left side

```
left1 = evecs . z
```

$$\left\{9,\ 2-\frac{3\left(-5+\sqrt{13}\right)}{-11+3\sqrt{13}}-\frac{8\left(-3+\sqrt{13}\right)}{-11+3\sqrt{13}},\ 2-\frac{8\left(3+\sqrt{13}\right)}{11+3\sqrt{13}}-\frac{3\left(5+\sqrt{13}\right)}{11+3\sqrt{13}}\right\}$$

Now get the α[i]'s by equating the two sides and solving

```
eq = left1 == expand3
```

$$\left\{9,\ 2-\frac{3\left(-5+\sqrt{13}\right)}{-11+3\sqrt{13}}-\frac{8\left(-3+\sqrt{13}\right)}{-11+3\sqrt{13}},\ 2-\frac{8\left(3+\sqrt{13}\right)}{11+3\sqrt{13}}-\frac{3\left(5+\sqrt{13}\right)}{11+3\sqrt{13}}\right\} == \left\{3\,\alpha(1),\ \frac{2\left(-13+\sqrt{13}\right)\alpha(2)}{-11+3\sqrt{13}},\ \frac{2\left(13+\sqrt{13}\right)\alpha(3)}{11+3\sqrt{13}}\right\}$$

Note that each equation contains only 1 α[i].

```
ans = Solve[eq, Table[α[i], {i, 1, 3}]]
```

$$\left\{\left\{\alpha(1)\to 3,\ \alpha(2)\to -\frac{-17+5\sqrt{13}}{2\left(-13+\sqrt{13}\right)},\ \alpha(3)\to -\frac{17+5\sqrt{13}}{2\left(13+\sqrt{13}\right)}\right\}\right\}$$

Now let's check the result

```
test1 = Table[α[i], {i, 1, 3}] . evecs
```

$$\left\{-\alpha(1)+\frac{2\left(-3+\sqrt{13}\right)\alpha(2)}{-11+3\sqrt{13}}+\frac{2\left(3+\sqrt{13}\right)\alpha(3)}{11+3\sqrt{13}},\ \alpha(1)-\frac{\left(-5+\sqrt{13}\right)\alpha(2)}{-11+3\sqrt{13}}-\frac{\left(5+\sqrt{13}\right)\alpha(3)}{11+3\sqrt{13}},\ \alpha(1)+\alpha(2)+\alpha(3)\right\}$$

Now substitute

```
test2 = test1 /. ans[[1]]
```

$$\left\{-3-\frac{\left(-3+\sqrt{13}\right)\left(-17+5\sqrt{13}\right)}{\left(-13+\sqrt{13}\right)\left(-11+3\sqrt{13}\right)}-\frac{\left(3+\sqrt{13}\right)\left(17+5\sqrt{13}\right)}{\left(13+\sqrt{13}\right)\left(11+3\sqrt{13}\right)},\right.$$
$$3+\frac{\left(-5+\sqrt{13}\right)\left(-17+5\sqrt{13}\right)}{2\left(-13+\sqrt{13}\right)\left(-11+3\sqrt{13}\right)}+\frac{\left(5+\sqrt{13}\right)\left(17+5\sqrt{13}\right)}{2\left(13+\sqrt{13}\right)\left(11+3\sqrt{13}\right)},$$
$$\left.3-\frac{-17+5\sqrt{13}}{2\left(-13+\sqrt{13}\right)}-\frac{17+5\sqrt{13}}{2\left(13+\sqrt{13}\right)}\right\}$$

```
test3 = FullSimplify[test2]
```

$\{-4, 3, 2\}$

Recall z,

```
z
```

$\{-4, 3, 2\}$

It, of course, matches.

We choose another matrix. This one is not self adjoint.

```
aa = {{1, -1, 3}, {1, 0, -2}, {3, 3, 4}}
```

$$\begin{pmatrix} 1 & -1 & 3 \\ 1 & 0 & -2 \\ 3 & 3 & 4 \end{pmatrix}$$

```
Det[aa]
```

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Here is our arbitrary vector. We wish to expand it in terms of the eigenvalues

```
z = {-1, 3, 5}
```

{−1, 3, 5}

First get the eigenvalues

```
evals = Eigenvalues[aa]
```

$$\left\{\frac{5}{3}+\frac{1}{3}\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)},\right.$$
$$\frac{5}{3}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)},$$
$$\left.\frac{5}{3}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right\}$$

Then get the corresponding eigenvectors

```
evecs = Eigenvectors[aa]
```

$$\frac{1}{3}\left(-\frac{7}{3}+\frac{1}{3}\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)+\frac{50-2\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-2^{2/3}\sqrt[3]{835\ldots}}{3\left(16+2\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+2^{2/3}\sqrt[3]{83\ldots}\right.}$$

$$\frac{1}{3}\left(-\frac{7}{3}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)+\frac{-\frac{25}{3}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{82\ldots}}{2}}}{-3+3\left(-\frac{5}{3}+\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\ldots}{}}\right.}$$

$$\frac{1}{3}\left(-\frac{7}{3}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)+\frac{-\frac{25}{3}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{82\ldots}}{2}}}{-3+3\left(-\frac{5}{3}+\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\ldots}{}}\right.}$$

Now solve the adjoint system for the eigenvalues

```
evalstrans = Eigenvalues[Transpose[aa]]
```

$$\left\{\frac{5}{3}+\frac{1}{3}\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)},\right.$$

$$\frac{5}{3}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)},$$

$$\left.\frac{5}{3}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right\}$$

and the eigenvectors

```
evecstrans = Eigenvectors[Transpose[aa]]
```

$$\frac{1}{3}\left(-\frac{7}{3}+\frac{1}{3}\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)-\frac{2\left(-68+2\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+2^{2/3}\sqrt[3]{\cdots}\right.}{9\left(14+2\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+2^{2/3}\sqrt[3]{\cdots}\right.}$$

$$\frac{1}{3}\left(-\frac{7}{3}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)-\frac{2\left(\frac{34}{3}+\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-9\sqrt{\cdots}}\right.}{3\left(-2+3\left(-\frac{5}{3}+\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\cdots}\right.\right.}$$

$$\frac{1}{3}\left(-\frac{7}{3}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)-\frac{2\left(\frac{34}{3}+\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-9\sqrt{\cdots}}\right.}{3\left(-2+3\left(-\frac{5}{3}+\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\cdots}\right.\right.}$$

Here is how we write the expansion for the z vector,

```
Table[α[i], {i, 1, 3}] . {xvec[1], xvec[2], xvec[3]}
```

$$\text{xvec}(1)\,\alpha(1)+\text{xvec}(2)\,\alpha(2)+\text{xvec}(3)\,\alpha(3)$$

Now calculate this

```
expand1 = Table[α[i], {i, 1, 3}] . N[evecs]
```

$\{0.723366\,\alpha(1) - (1.19502 + 0.819253\,i)\,\alpha(2) - (1.19502 - 0.819253\,i)\,\alpha(3),$
$-0.233392\,\alpha(1) - (0.216638 - 1.52756\,i)\,\alpha(2) - (0.216638 + 1.52756\,i)\,\alpha(3),$
$1.\,\alpha(1) + 1.\,\alpha(2) + 1.\,\alpha(3)\}$

Now we have the equation, z =Sum[α[i] x[i]] for the expansion. We evaluate the α[i]'s by forming the inner product of both sides of the equation with the adjoint eigenvectors.

Here is the right side

```
expand2 = Chop[ExpandAll[N[evecstrans]]] . expand1
```

$\{0.762659\,(0.723366\,\alpha(1) - (1.19502 + 0.819253\,i)\,\alpha(2) - (1.19502 - 0.819253\,i)\,\alpha(3)) +$
$0.409026\,(-0.233392\,\alpha(1) - (0.216638 - 1.52756\,i)\,\alpha(2) - (0.216638 + 1.52756\,i)\,\alpha(3)) +$
$1.\,(1.\,\alpha(1) + 1.\,\alpha(2) + 1.\,\alpha(3)),\ (-1.35501 - 0.664297\,i)$
$(0.723366\,\alpha(1) - (1.19502 + 0.819253\,i)\,\alpha(2) - (1.19502 - 0.819253\,i)\,\alpha(3)) +$
$(0.0849607 - 2.0589\,i)$
$(-0.233392\,\alpha(1) - (0.216638 - 1.52756\,i)\,\alpha(2) - (0.216638 + 1.52756\,i)\,\alpha(3)) +$
$1.\,(1.\,\alpha(1) + 1.\,\alpha(2) + 1.\,\alpha(3)),\ (-1.35501 + 0.664297\,i)$
$(0.723366\,\alpha(1) - (1.19502 + 0.819253\,i)\,\alpha(2) - (1.19502 - 0.819253\,i)\,\alpha(3)) +$
$(0.0849607 + 2.0589\,i)$
$(-0.233392\,\alpha(1) - (0.216638 - 1.52756\,i)\,\alpha(2) - (0.216638 + 1.52756\,i)\,\alpha(3)) +$
$1.\,(1.\,\alpha(1) + 1.\,\alpha(2) + 1.\,\alpha(3))\}$

```
expand3 = Chop[Simplify[expand2]]
```

$\{1.45622\,\alpha(1),\ (5.20172 + 2.47976\,i)\,\alpha(2),\ (5.20172 - 2.47976\,i)\,\alpha(3)\}$

Here is the left side

```
left1 = N[evecstrans . z]
```

$\{5.46442,\ 6.6099 - 5.5124\,i,\ 6.6099 + 5.5124\,i\}$

Now get the α[i]'s by equating the two sides and solving

```
eq = left1 == expand3
```

$\{5.46442, 6.6099 - 5.5124\, i, 6.6099 + 5.5124\, i\} ==$
$\{1.45622\, \alpha(1), (5.20172 + 2.47976\, i)\, \alpha(2), (5.20172 - 2.47976\, i)\, \alpha(3)\}$

Note that each equation contains only 1 α[i].

```
ans = Solve[eq, Table[α[i], {i, 1, 3}]]
```

$\{\{\alpha(1) \to 3.75247 + 0.\, i, \alpha(2) \to 0.623764 - 1.35709\, i, \alpha(3) \to 0.623764 + 1.35709\, i\}\}$

Now let's check the result

```
test1 = Table[α[i], {i, 1, 3}] . evecs
```

$$\Bigg\{\left(\frac{1}{3}\left(-\frac{7}{3}+\frac{1}{3}\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)+\frac{50-2\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-2^{2/3}\sqrt[3]{835+9\sqrt{8269}}}{3\left(16+2\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+2^{2/3}\sqrt[3]{835+9\sqrt{8269}}\right)}\right)\alpha(1)+$$

$$\left(\frac{1}{3}\left(-\frac{7}{3}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)+\left(-\frac{25}{3}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)\Bigg/\left(-3+3\left(-\frac{5}{3}+\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)\right)\right)$$

$\alpha(2) +$

$$\left(\frac{1}{3}\left(-\frac{7}{3}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)+\right.$$

$$\left(-\frac{25}{3}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)\Big/$$

$$\left(-3+3\left(-\frac{5}{3}+\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+\right.\right.$$

$$\left.\left.\left.\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)\right)\right)$$

$$\alpha(3),$$

$$-\frac{\left(50-2\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-2^{2/3}\sqrt[3]{835+9\sqrt{8269}}\right)\alpha(1)}{3\left(16+2\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+2^{2/3}\sqrt[3]{835+9\sqrt{8269}}\right)}-$$

$$\left(\left(-\frac{25}{3}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)\alpha(2)\right)\Big/$$

$$\left(-3+3\left(-\frac{5}{3}+\right.\right.$$

$$\left.\left.\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)\right)-$$

$$\left(\left(-\frac{25}{3}-\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}-\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)\alpha(3)\right)\Big/$$

$$\left(-3+\right.$$

$$\left.3\left(-\frac{5}{3}+\frac{1}{6}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{835}{2}-\frac{9\sqrt{8269}}{2}}+\frac{1}{6}\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(835+9\sqrt{8269}\right)}\right)\right),$$

$$\alpha(1)+\alpha(2)+\alpha(3)\}$$

Now substitute

```
test2 = Chop[test1 /. ans[[1]]]
```

$\{-1., 3., 5.\}$

Recall z,

```
z
```

$\{-1, 3, 5\}$

It, of course, matches.

Solution of a system of equations by eigenvector expansion

We choose again a matrix

```
aa = N[{{1, -1, 3, -2}, {2, 0, -3, 1}, {1, 1, 1, 1}, {0, 2, 3, 1}}]
```

$$\begin{pmatrix} 1. & -1. & 3. & -2. \\ 2. & 0 & -3. & 1. \\ 1. & 1. & 1. & 1. \\ 0 & 2. & 3. & 1. \end{pmatrix}$$

Note that I use the N[ ] command to make every thing more compact and faster.

```
bb = N[{2, -1, 0, 3}]
```

{2., −1., 0, 3.}

```
Det[aa]
```

−8.

So we are safe as far as getting a solution

Let's find the solution, voila!!

```
LinearSolve[aa, bb]
```

{0.5, 5.5, −1., −5.}

But this is a graduate course so we have to take an circuitous journey

First get the eigenvalues

```
evals = Eigenvalues[aa]
```

{2.34844, −1.61094, 1.13125 + 0.913717 ⅈ, 1.13125 − 0.913717 ⅈ}

Then get the corresponding eigenvectors

```
evecs = Eigenvectors[aa]
```

$$\begin{pmatrix} -0.103588 & 0.224444 & -0.517766 & -0.819031 \\ -0.233492 & 0.748595 & 0.0398965 & -0.61927 \\ -0.262965 - 0.000374187\, i & -0.443896 + 0.302719\, i & 0.327908 + 0.0207981\, i & 0.730897 + 0.\, i \\ -0.262965 + 0.000374187\, i & -0.443896 - 0.302719\, i & 0.327908 - 0.0207981\, i & 0.730897 + 0.\, i \end{pmatrix}$$

Now solve the adjoint system for the eigenvalues

```
evalstrans = Eigenvalues[Transpose[aa]]
```

{2.34844, −1.61094, 1.13125 + 0.913717 ⅈ, 1.13125 − 0.913717 ⅈ}

and the eigenvectors

```
evecstrans = Eigenvectors[Transpose[aa]]
```

$$\begin{pmatrix} 0.526338 & -0.0557749 & 0.821282 & -0.212964 \\ -0.270849 & -0.0567133 & 0.820598 & -0.500043 \\ 0.126938 + 0.286499\, i & -0.441882 + 0.0767949\, i & 0.638645 + 0.\, i & -0.540878 - 0.0151906\, i \\ 0.126938 - 0.286499\, i & -0.441882 - 0.0767949\, i & 0.638645 + 0.\, i & -0.540878 + 0.0151906\, i \end{pmatrix}$$

Now recall the expansion for the b vector

```
Table[betax[i] = evecstrans⟦i⟧.bb / (evecstrans⟦i⟧.evecs⟦i⟧), {i, 1, Length[aa]}]
```

{−1.4773, −5.46583, −1.09097 − 4.05442 *i*, −1.09097 + 4.05442 *i*}

This then leads to the expansion for x, get the alfa's

```
Table[alfax[i] = betax[i] / evals⟦i⟧, {i, 1, Length[aa]}]
```

{−0.629058, 3.39293, −2.33554 − 1.69759 *i*, −2.33554 + 1.69759 *i*}

Now construct the x vector, which s the solution

$$\text{exes} = N\left[\sum_{i=1}^{\text{Length[aa]}} \text{alfax}[i]\ \text{evecs}[\![i]\!]\right]$$

{0.5 + 0. *i*, 5.5 + 0. *i*, −1. + 0. *i*, −5. + 0. *i*}

```
LinearSolve[aa, bb]
```

{0.5, 5.5, −1., −5.}